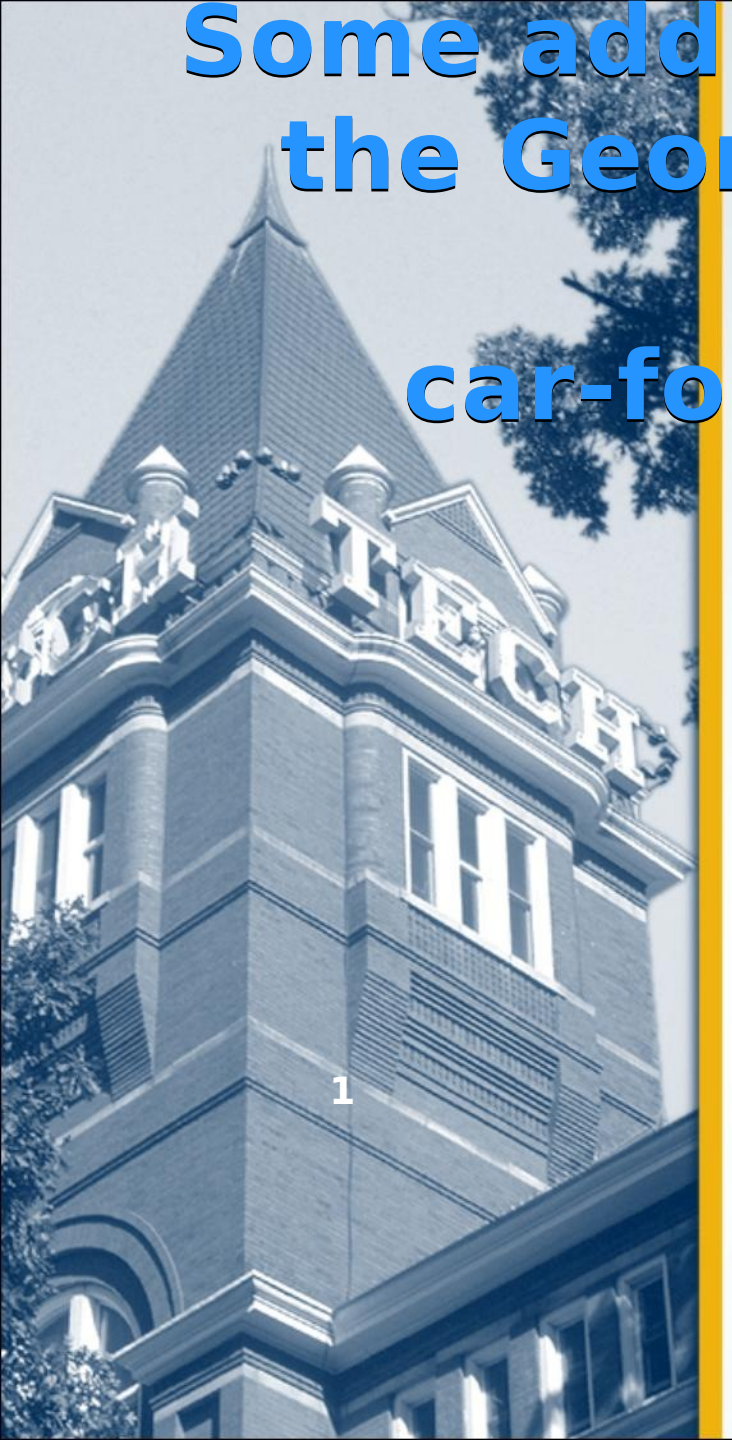


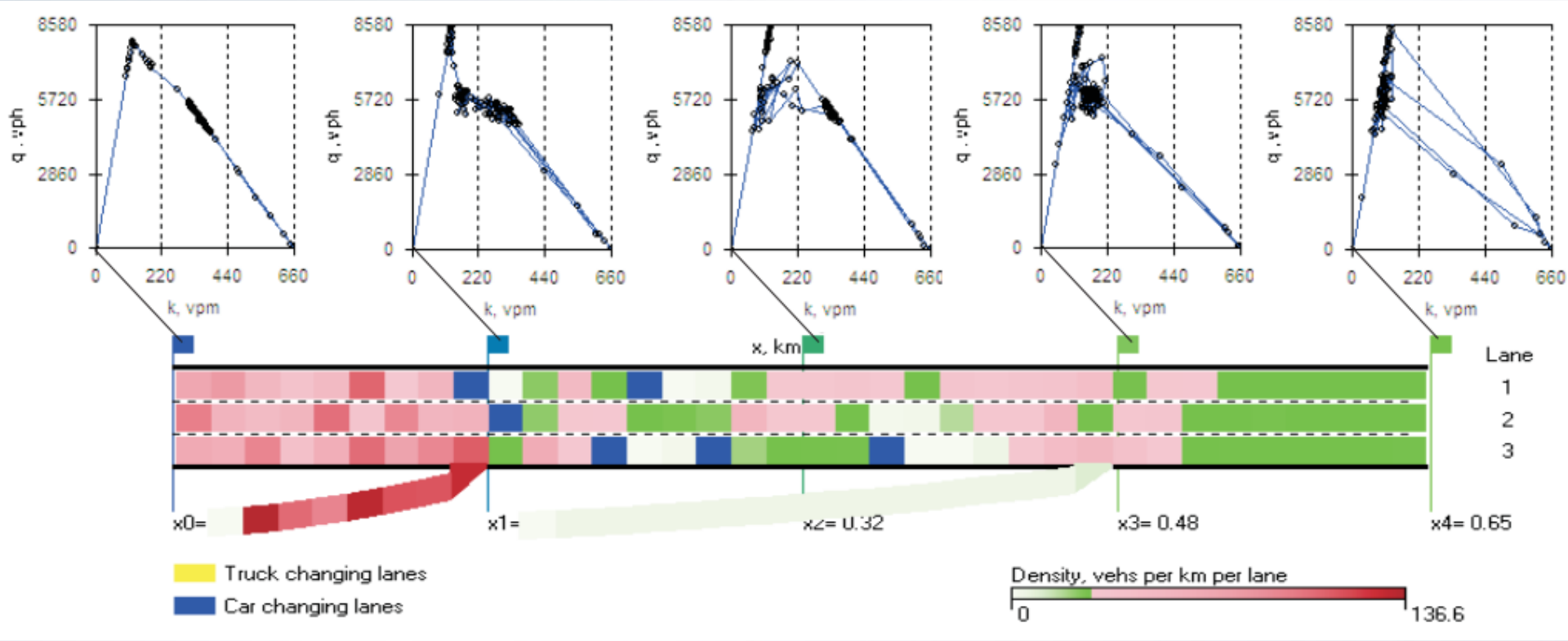
Some additional features of the Geometric Brownian Motion car-following model

Jorge A. Laval

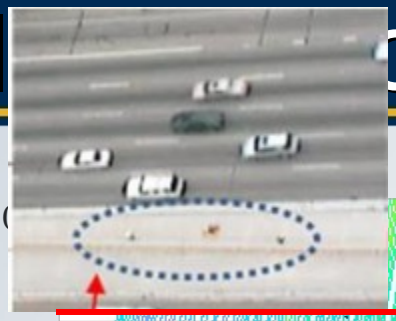
TU Delft, Dec 2016

Georgia Institute of Technology

Multilane instabilities – FD scatter



I ction: NGSIM US-101



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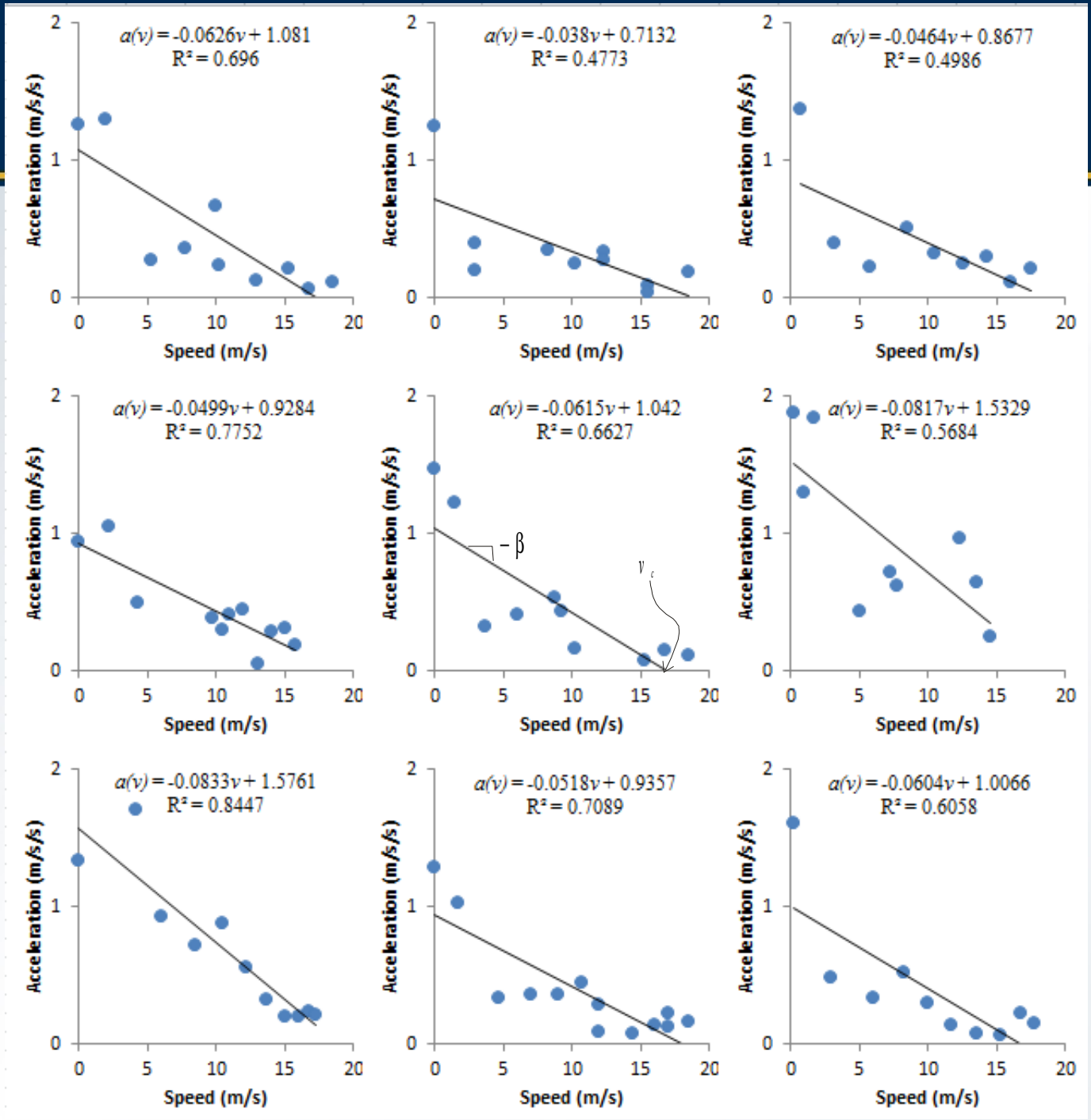
The hypothesis

- the random error in drivers acceleration processes may be responsible for most traffic instabilities:
  - Formation and propagation of oscillations
  - Oscillations growth
  - Hysteresis

- Laval, Toth, and Zhou (2015), A parsimonious model for the formation of oscillations in car-following models. *Transportation Research Part B* 70, 228-238.

Scope

- Car-following only, no lane changes
- Single lane
- Homogeneous drivers, no trucks



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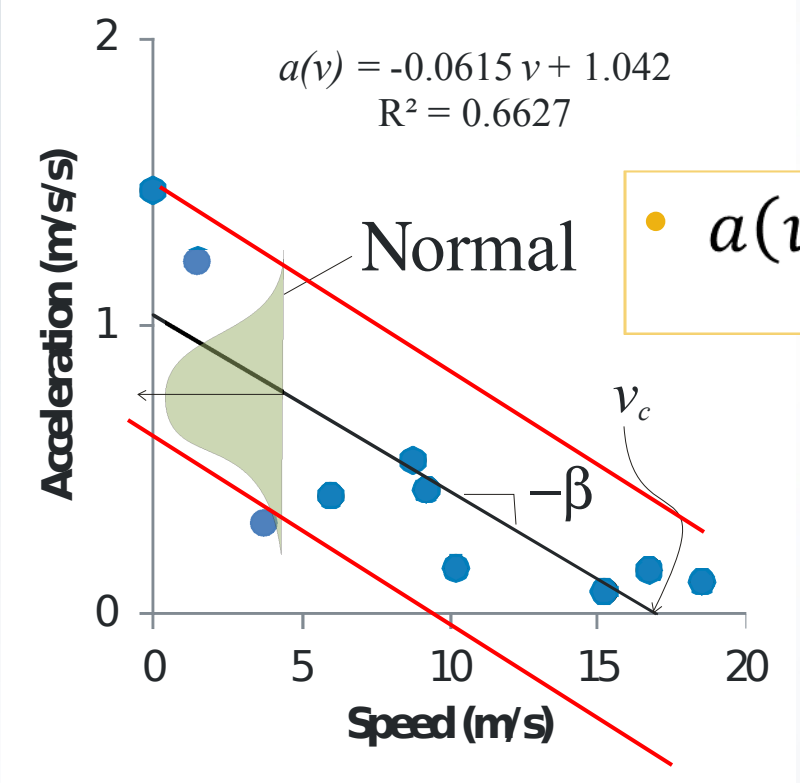
Stochastic desired accelerations

- desired acceleration ₂E vehicle downstream does not constrain the motion



- $a(v) = (v_c - v)\beta$+white noise

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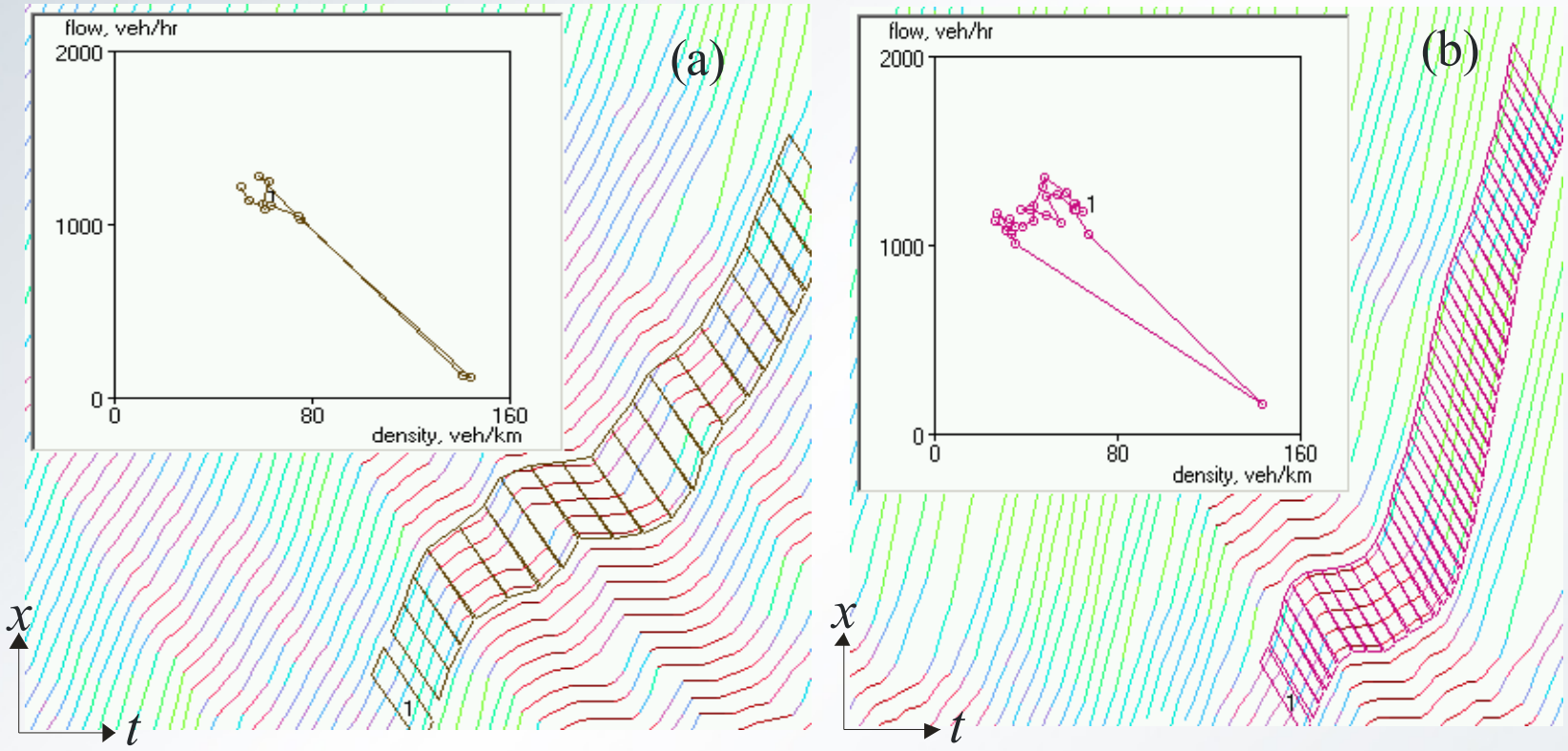
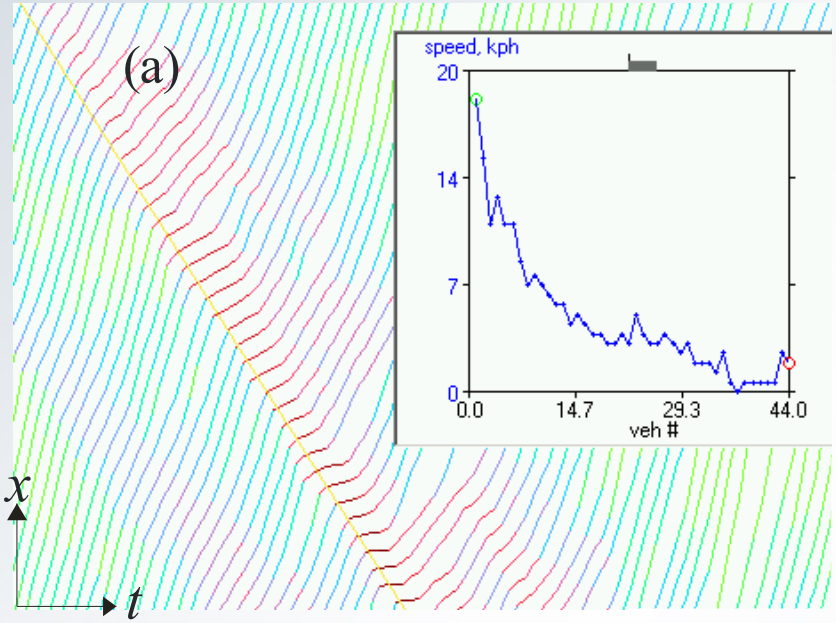
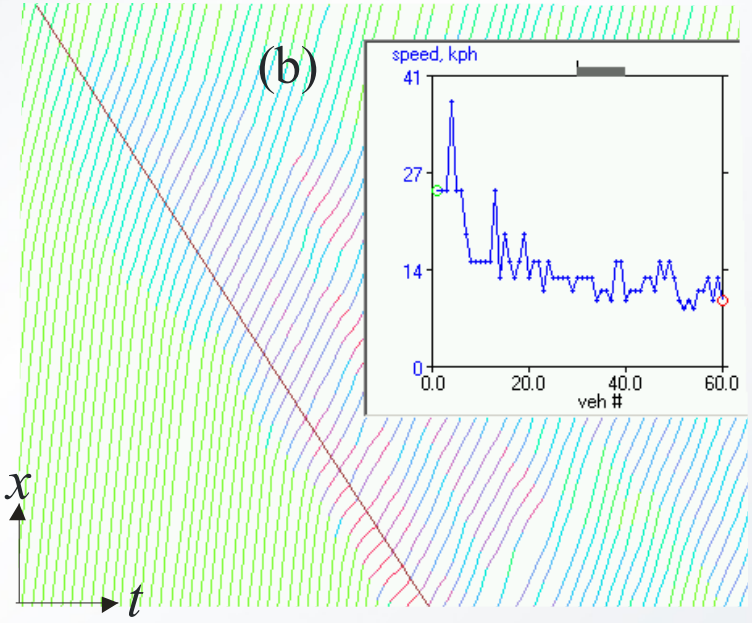
Model captures hysteresis



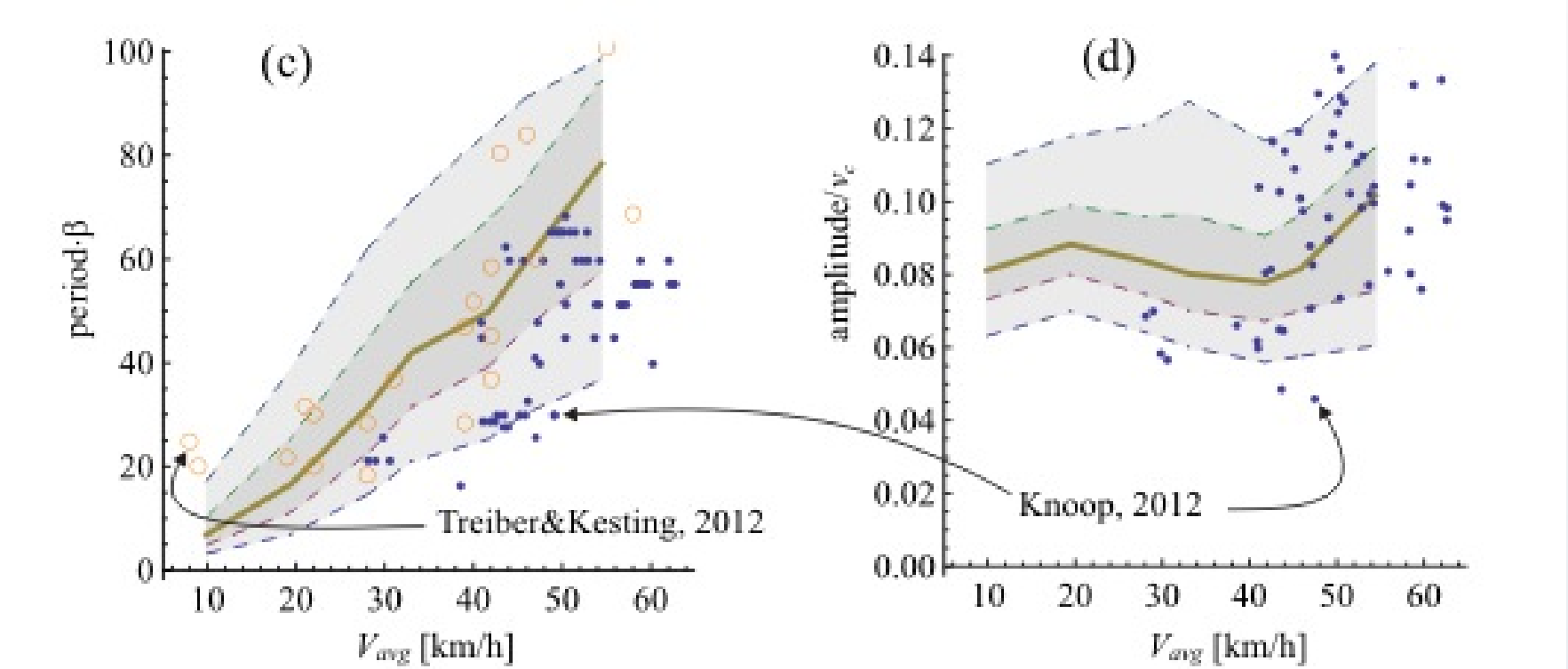
ajectory Explorer (trafficlab.ce.gatech.e

Model captures oscillation growth



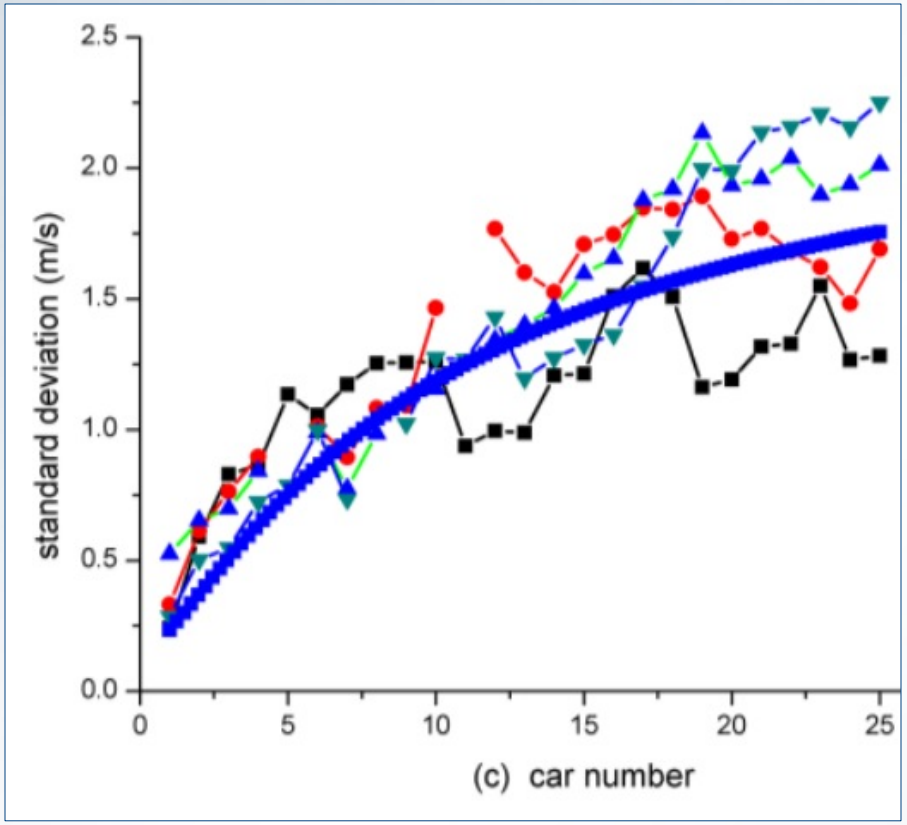


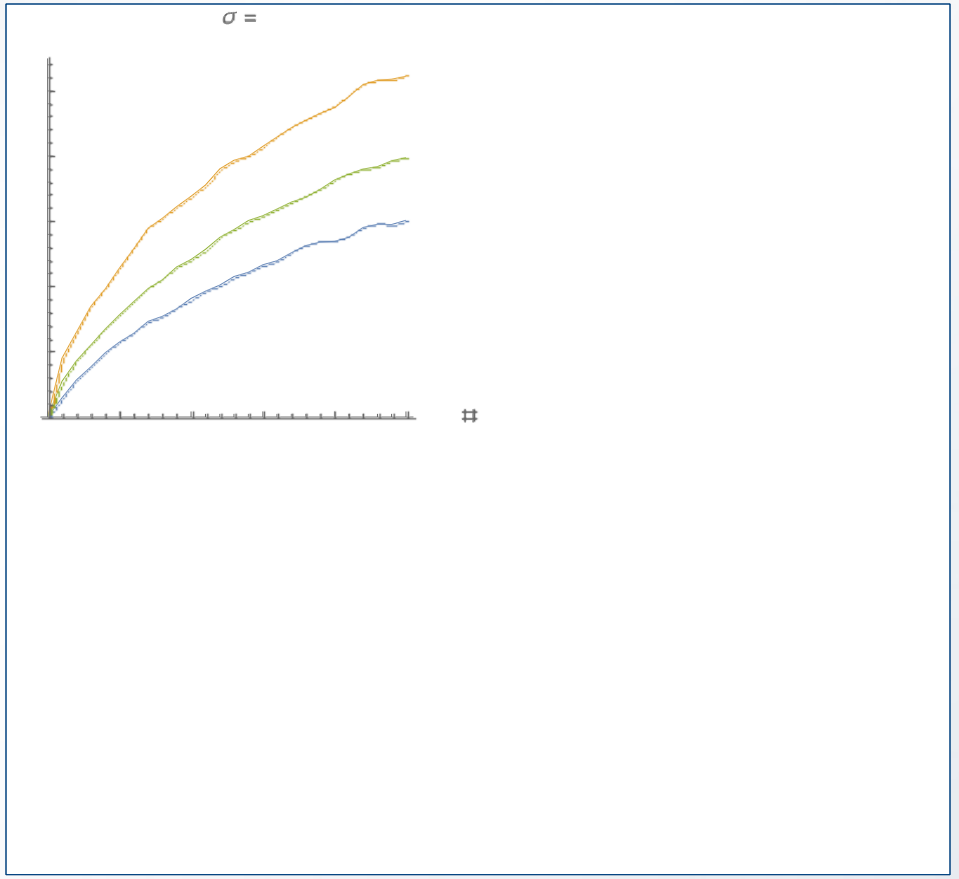
Oscillations period and amplitude



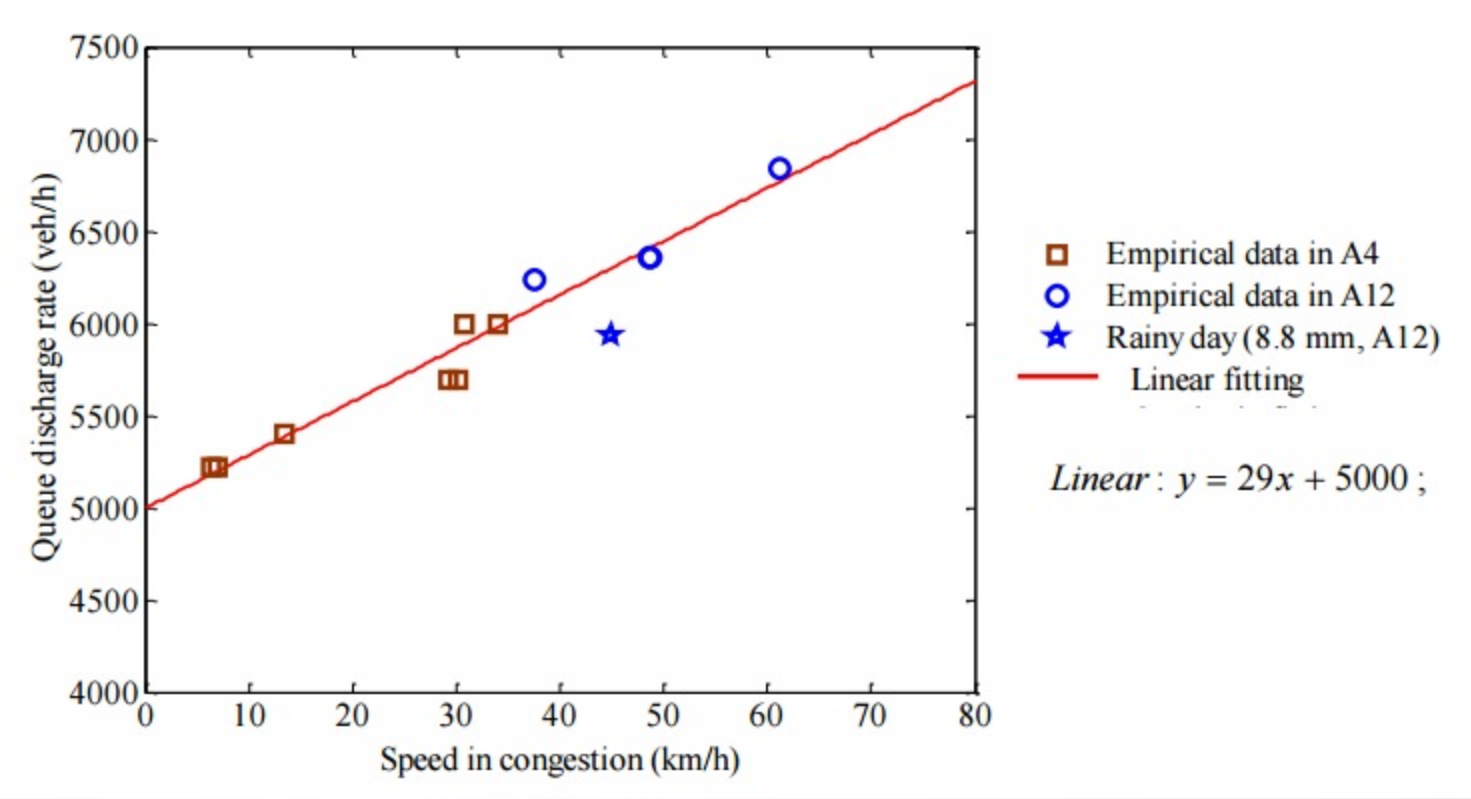
Model captures “concavity”

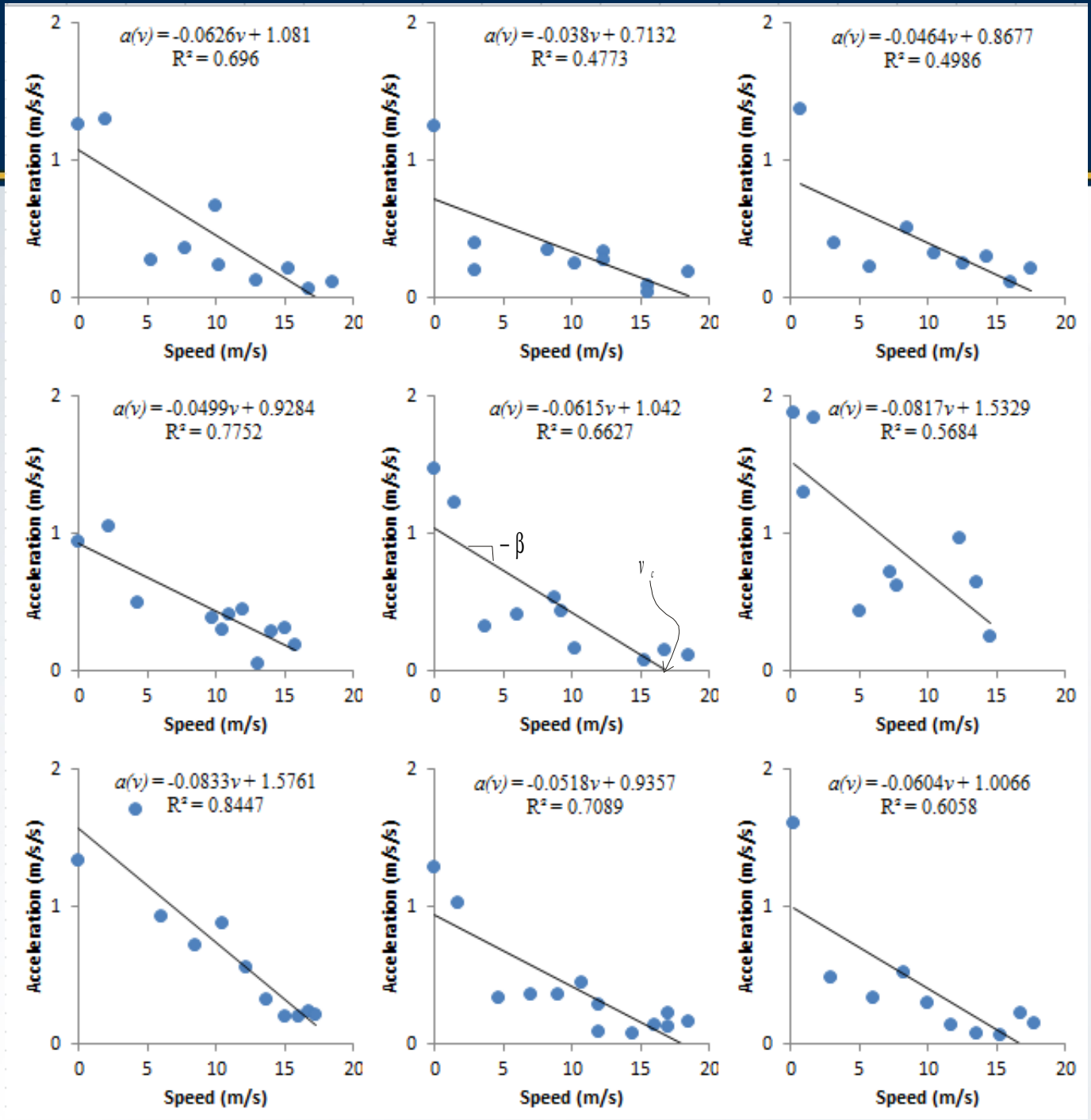
- Tian et al, Trans. Res. B (2015)
- Jian et al, PloS one (2014)





Does not capture capacity-BN speed relation





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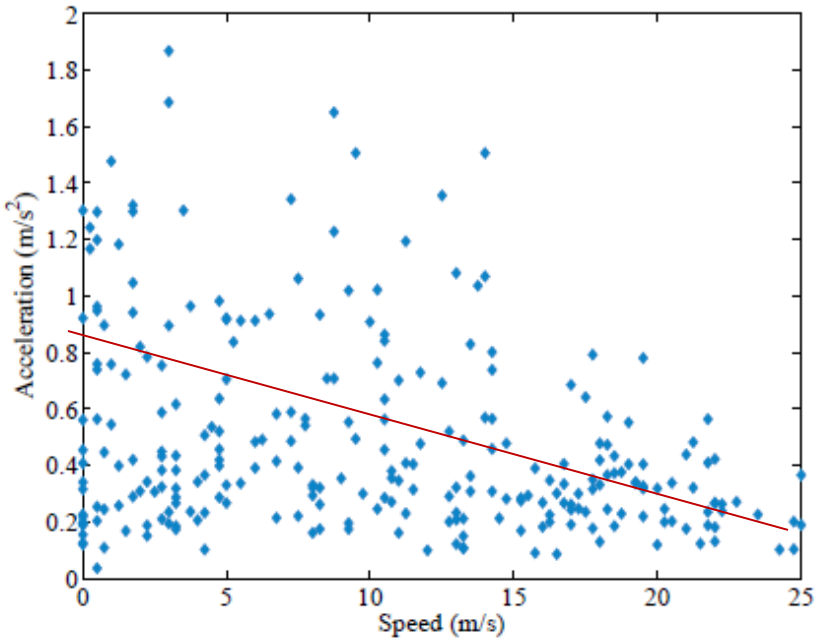
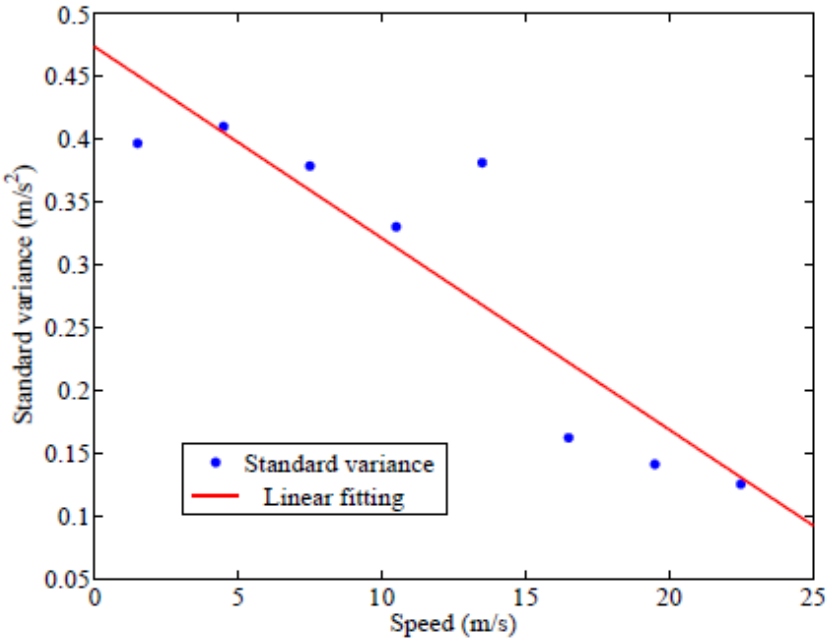
Brownian motion formulation

- desired acceleration a_E vehicle downstream does not constrain the motion



(a) Drivers' desired acceleration when traveling at different speed.



(b) Standard variance of drivers' desired accelerations at different vehicular speeds v.

Speed (m/s)

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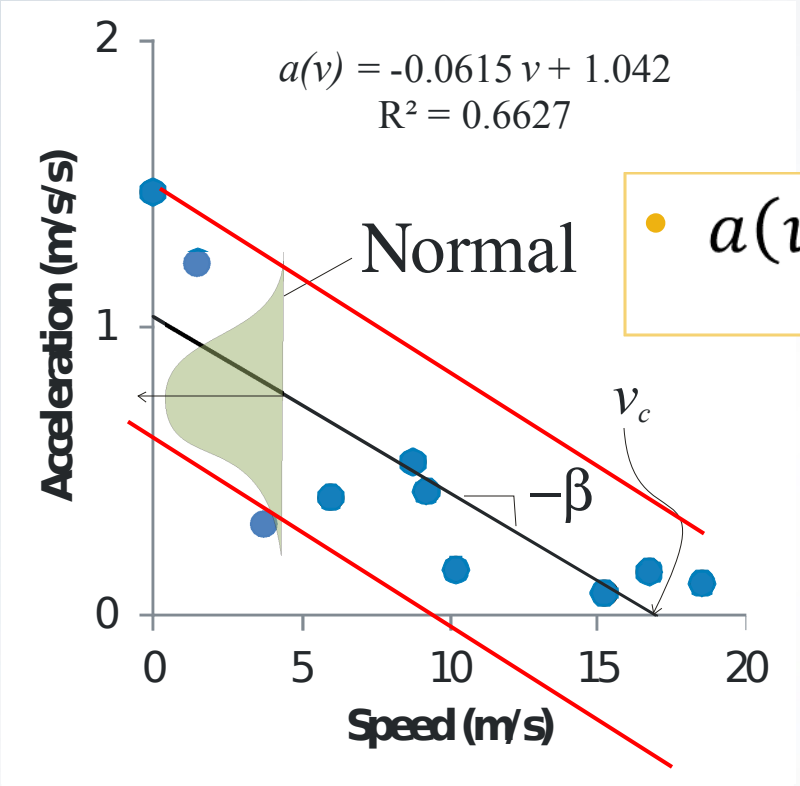
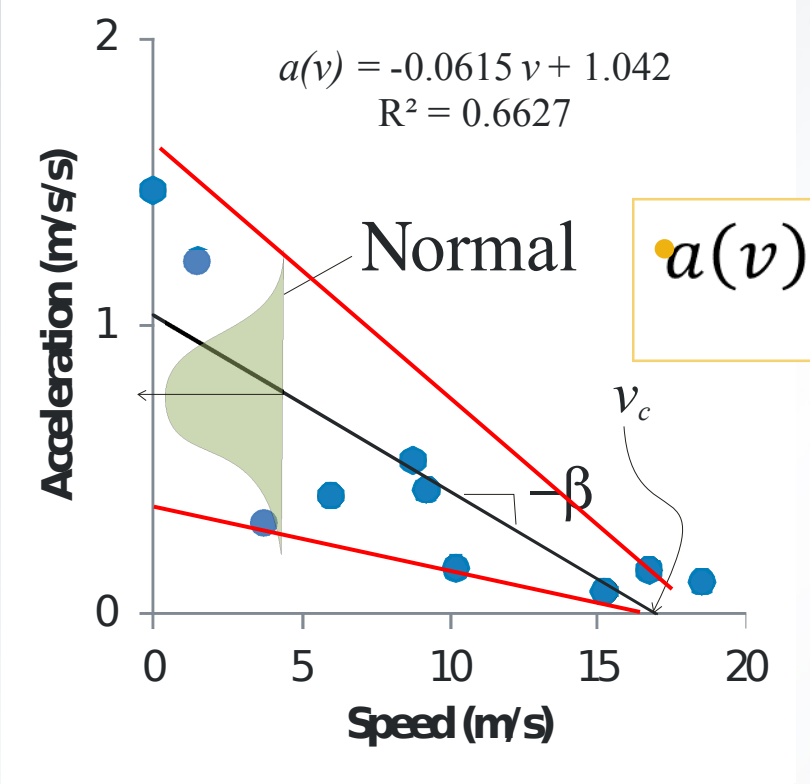
Brownian motion (BM) formulation



- $a(v) = (v_c - v)\beta$+white noise

Geometric Brownian motion formulation



$$a(v) = (v_c - v)\beta + \text{white noise}(v)$$

The BM Stochastic ODE

$\xi(t) =$ position at time t
$v(t) =$ speed at time t
$W(t) =$ standard Brownian motion,
$\sigma^2 =$ diffusion coefficient, units of $[\text{distance}]^2[\text{time}]^{-3}$

$$\begin{cases} d\xi(t) = v(t)dt, & \xi(0) = 0, \\ dv(t) = (v_c - v(t))\beta dt + \sigma dW(t), & v(0) = v_0, \end{cases}$$

Note: β can be eliminated by measuring time in units if $1/\beta$, i.e. redefining t as βt.

The GBM Stochastic ODE

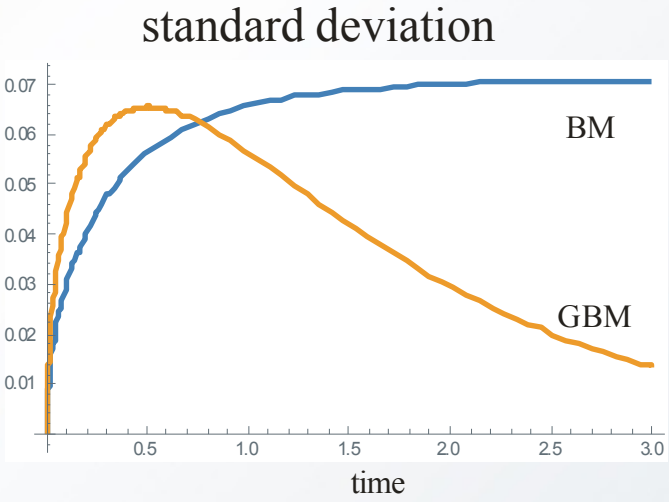
$\xi(t) =$ position at time t
$v(t) =$ speed at time t
$W(t) =$ standard Brownian motion,
$\sigma^2 =$ diffusion coefficient, units of $[\text{time}]^{-1}$

$$\begin{cases} d\xi(t) = v(t)dt, & \xi(0) = 0, \\ dv(t) = (v_c - v(t))(\beta dt + \sigma dW(t)), & v(0) = v_0, \end{cases}$$

Distribution of $v(t)$

- Normal for BM and LogNormal for GBM, with:
- Expected value, BM and GBM

$$E[v(t)] = u + e^{-t}\left(u - v_0\right)$$



- Variance, BM

$$V[v(t)] = \sigma^2\left(1 - e^{-2t}\right)/2$$

- Variance, GBM

$$V[v(t)] = e^{-2t}\left(u - v_0\right)^2\left(e^{\sigma^2 t} - 1\right)$$

Distribution of $\xi(t)$

- Normal for BM and GBM, with:
- Expected value, BM and GBM

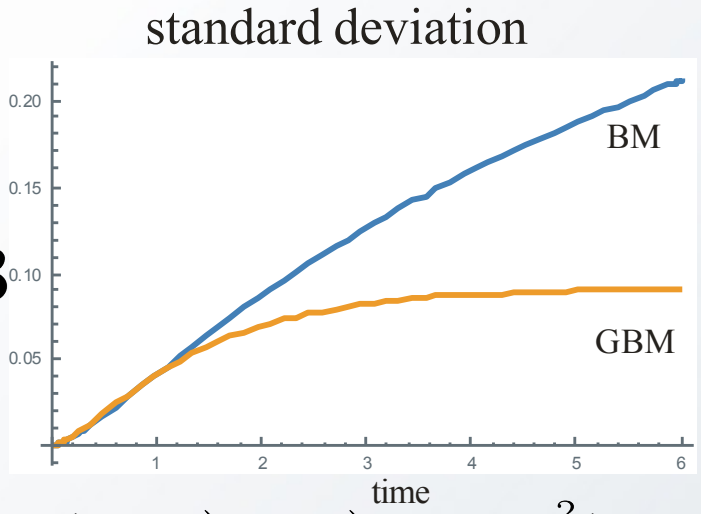
$$E[\xi(t)] = \left(t + e^{-t} - 1\right) u + v_0(\sinh(t) - \cosh(t) + 1)$$

- Variance, BM

$$V[\xi(t)] = \frac{1}{2}\sigma^2 e^{-2t}\left(e^{2t}(2t - 3\right.$$

- Variance, GBM

$$V[\xi(t)] = e^{-2t}(u - v_0)^2 \left(\frac{\sigma^2\left(e^t\left(2\sigma^2 + e^t - 4\right) + 3\right) + 2e^{\sigma^2 t} - 2}{\sigma^4 - 3\sigma^2 + 2}\right)$$



Plugin to Newell's car-following model

$$x_{i+1}(t) = \min\{\underbrace{x_{i+1}(t-\tau) + \xi_{i+1}(\tau)}_{\text{free-flow}}, \underbrace{x_i(t-\tau) - \delta}_{\text{congestion}}\}$$

where:

τ = wave trip time between two cars

δ = jam spacing,

$x_{i+1}(t)$ = position of vehicle $i+1$ at time t

$\xi_{i+1}(\tau)$ = vehicle displacement between $t-\tau$ and t

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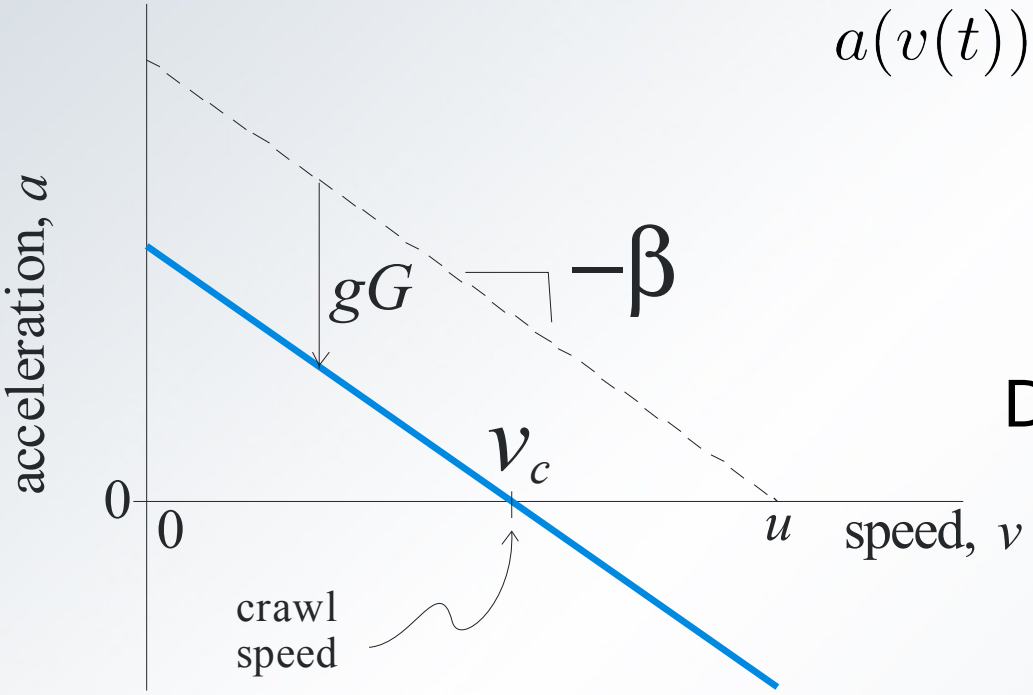
Effect of Grade

- On a 100G% upgrade

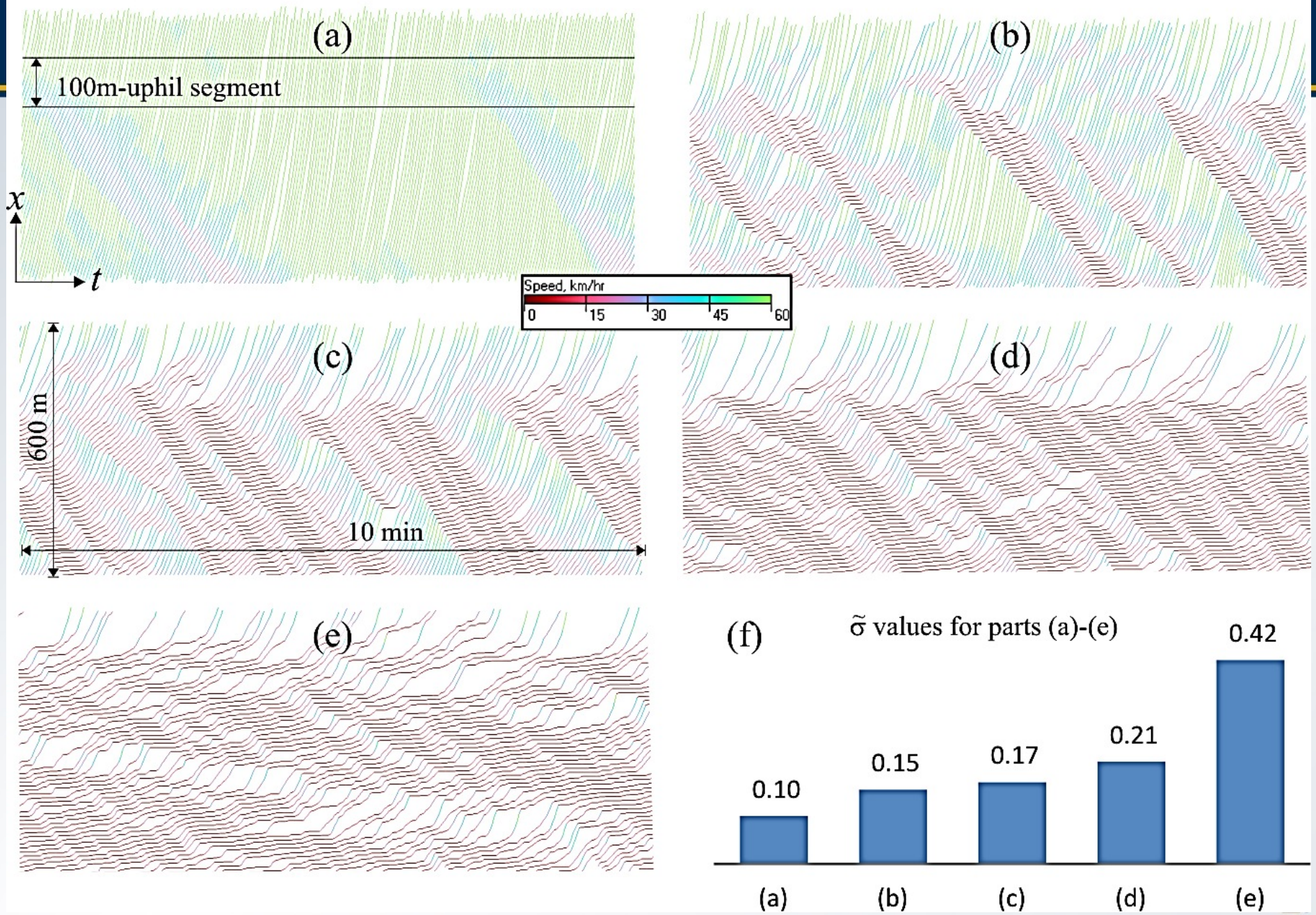


$$\begin{aligned} a(v(t)) &= (u - v(t))\beta - gG \\ &= (\underbrace{u - gG/\beta}_{v_c} - v(t))\beta \end{aligned}$$

Dimensionless parameter

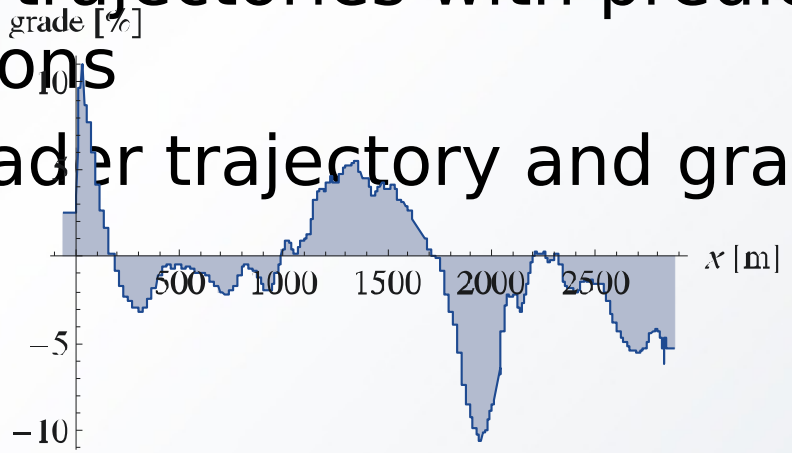
$$\tilde{\sigma}^2 = \frac{\sigma^2}{v_c^2 \beta} \quad \text{(BM)}$$

$$\tilde{\sigma}^2 = \frac{\sigma^2}{\beta} \quad \text{(GBM)}$$

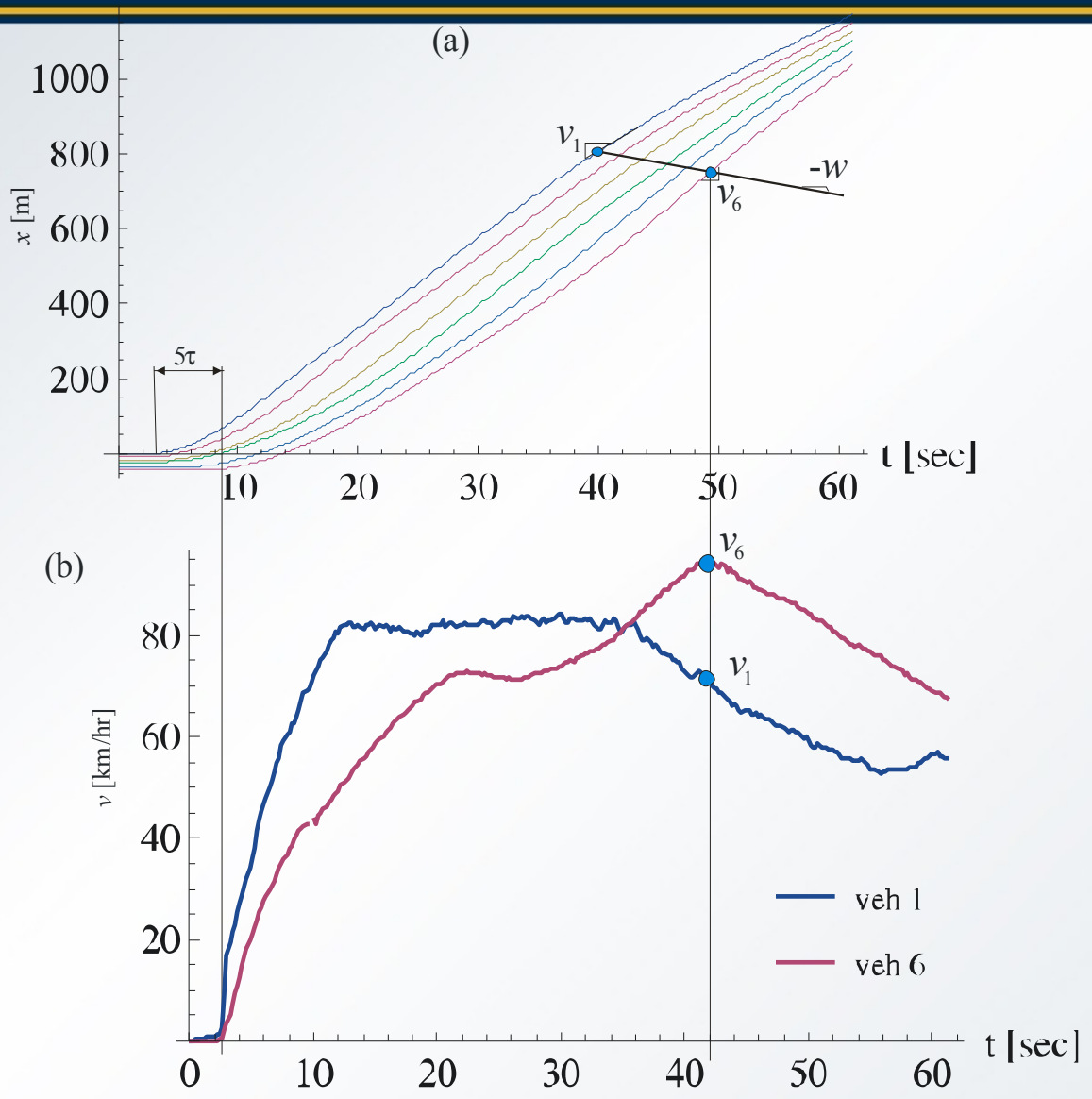

(a)
100m-uphil segment
x
t
(b)
Speed, km/hr
0
15
30
45
60
(c)
600 m
10 min
(d)
(e)
(f)
σ̃ values for parts (a)-(e)
0.10
0.15
0.17
0.21
0.42
(a)
(b)
(c)
(d)
(e)

Car-following experiment

- 6-vehicle platoon, unobstructed leader
- 5Hz GPS devices and Android app in each vehicle
- two-lane urban streets around Georgia Tech campus
- Objective:
  - compare trajectories with predicted distributions
  - given: leader trajectory and grade G=G(x)



Trailing vehicle speed peak



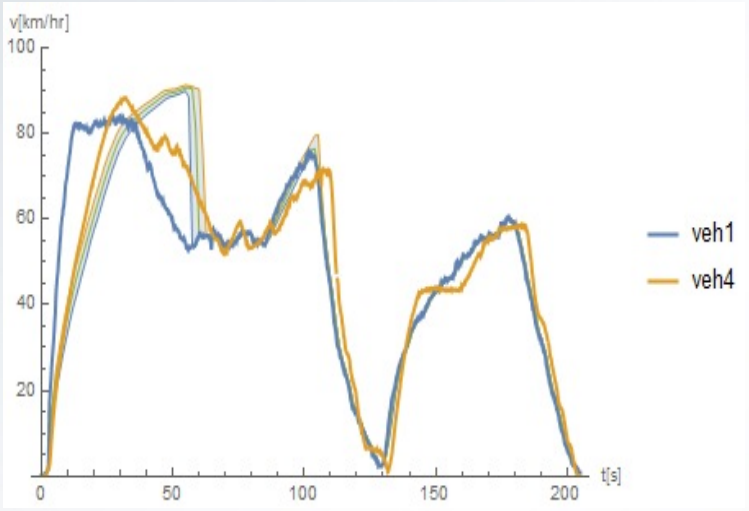
Vehicle 4

Vehicle 4, Segment 3, Small Variance

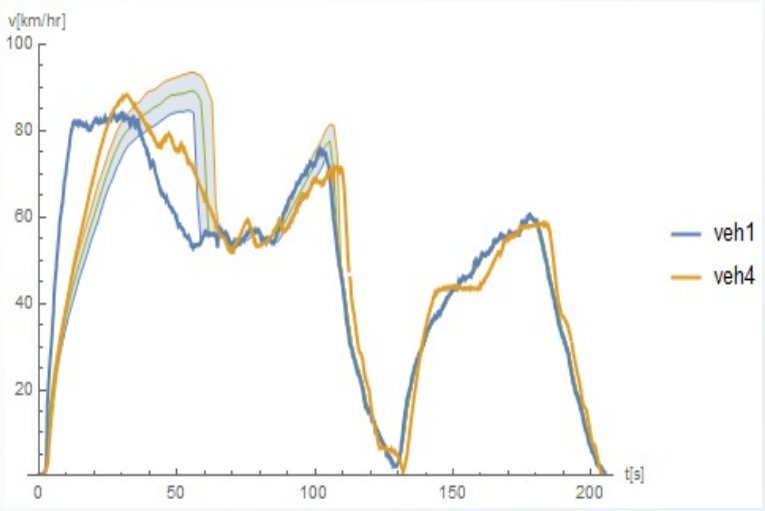
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion

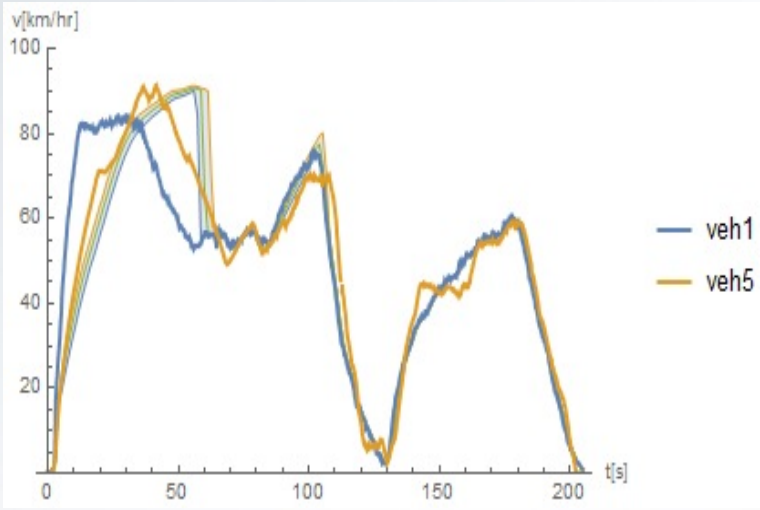


Vehicle 5, Segment 3, Small Variance

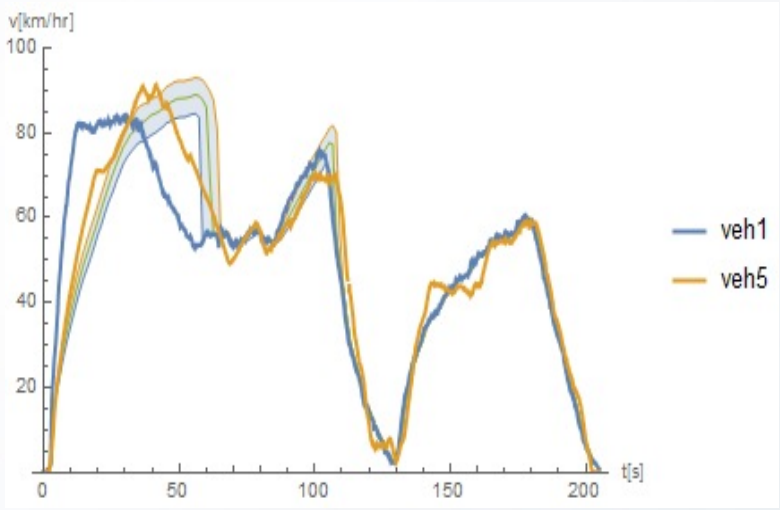
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion

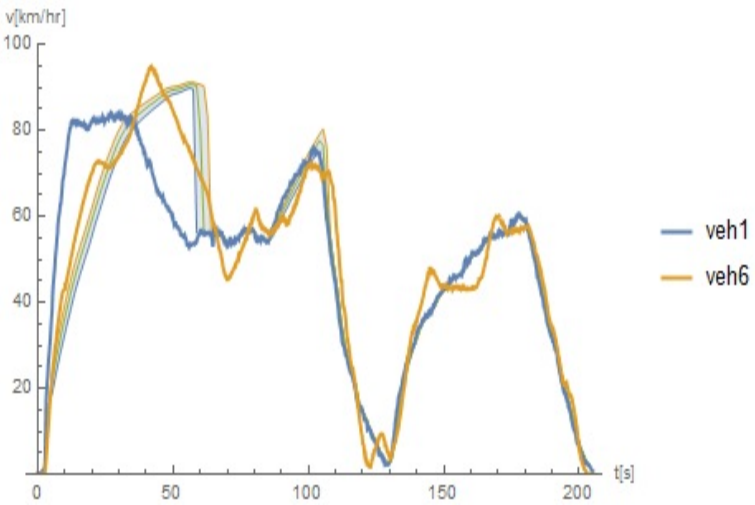


Vehicle 6, Segment 3, Small Variance

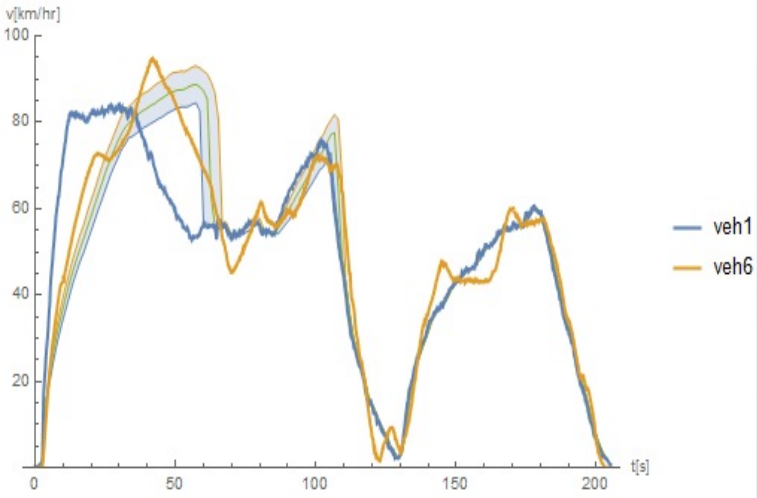
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion

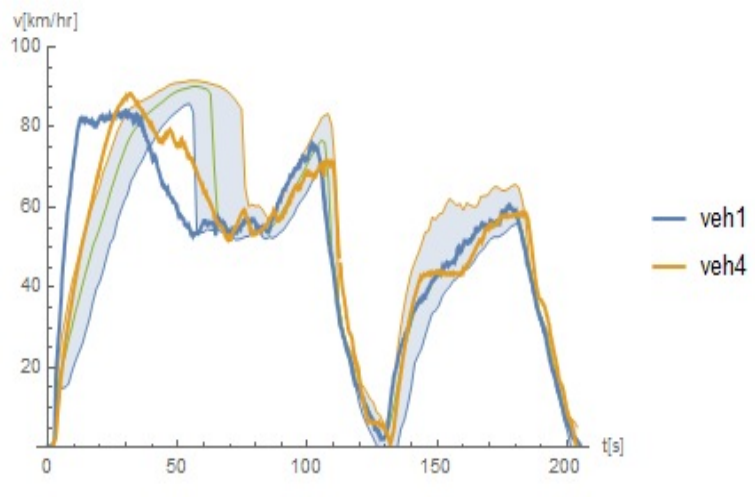


Vehicle 4, Segment 3, Large Variance

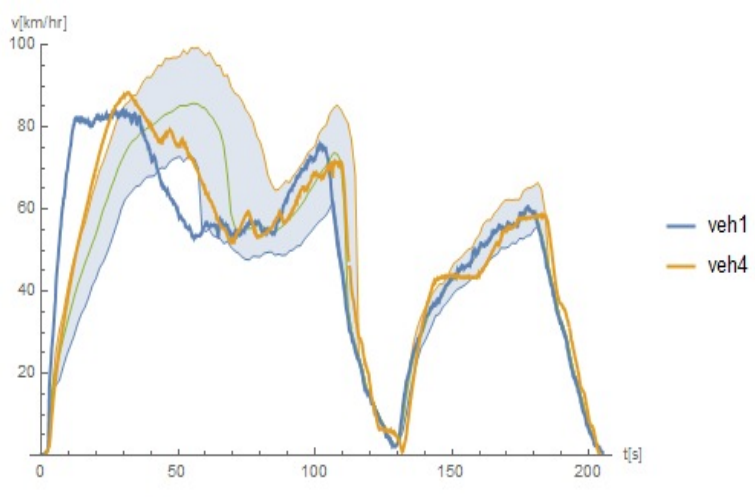
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion

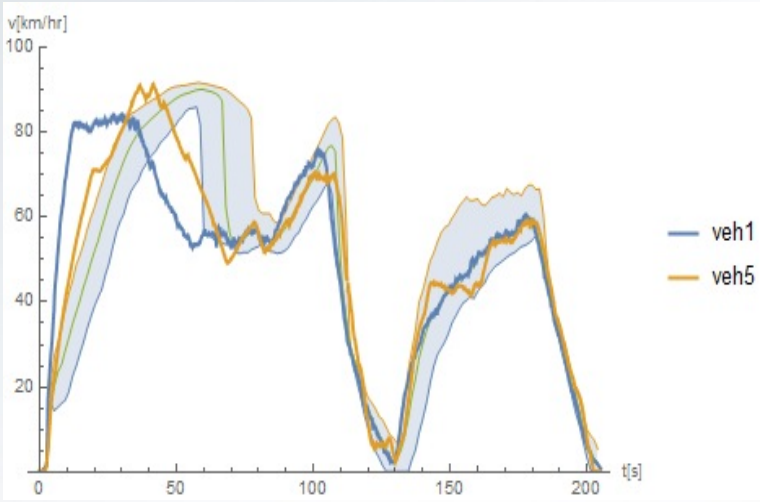


Vehicle 5, Segment 3, Large Variance

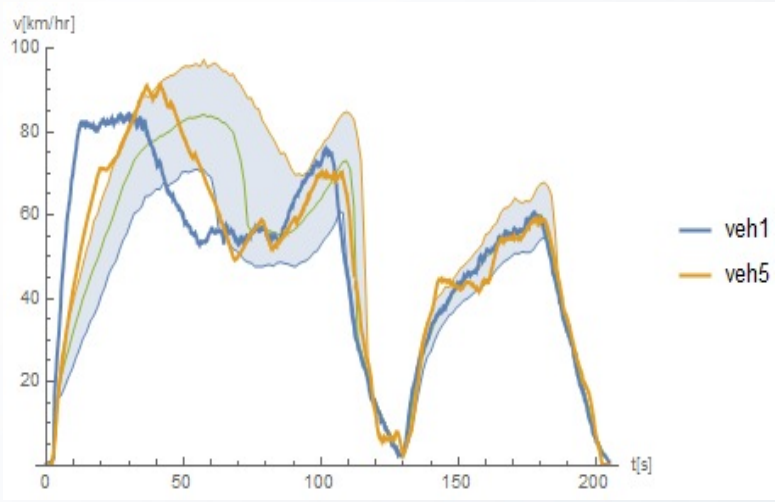
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion

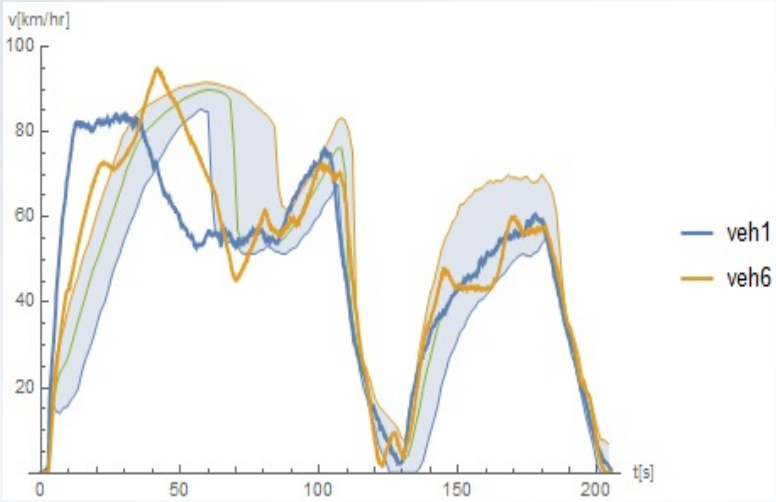


Vehicle 6, Segment 3, Large Variance

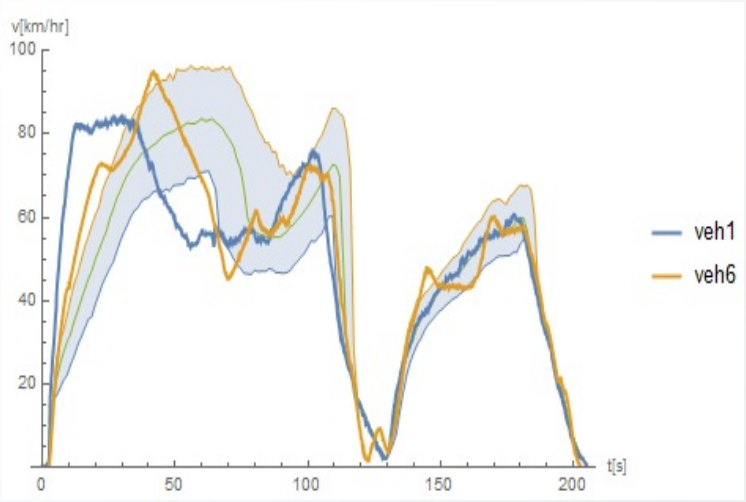
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion



Q & A

THANK YOU !

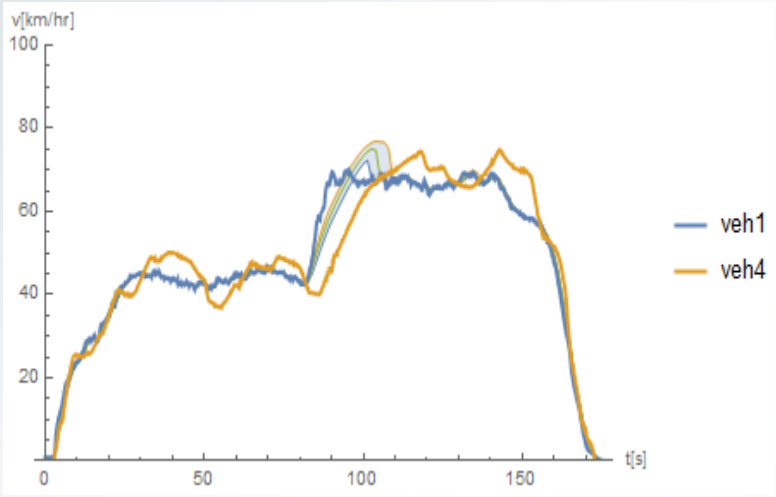
33

Vehicle 4, Segment 4, Small Variance

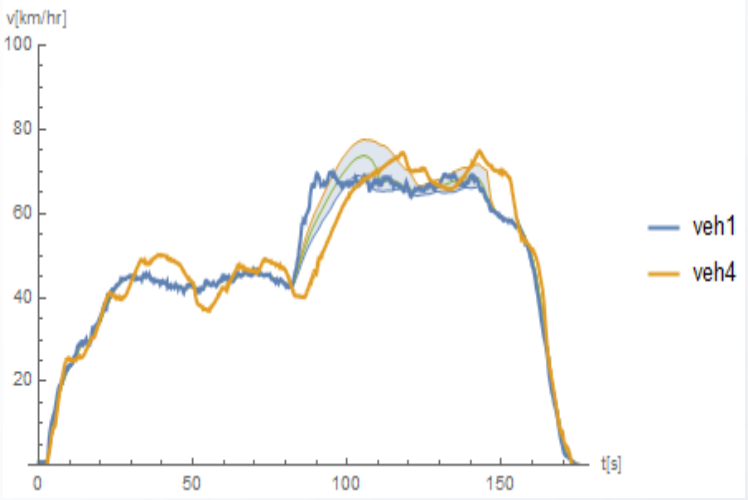
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion

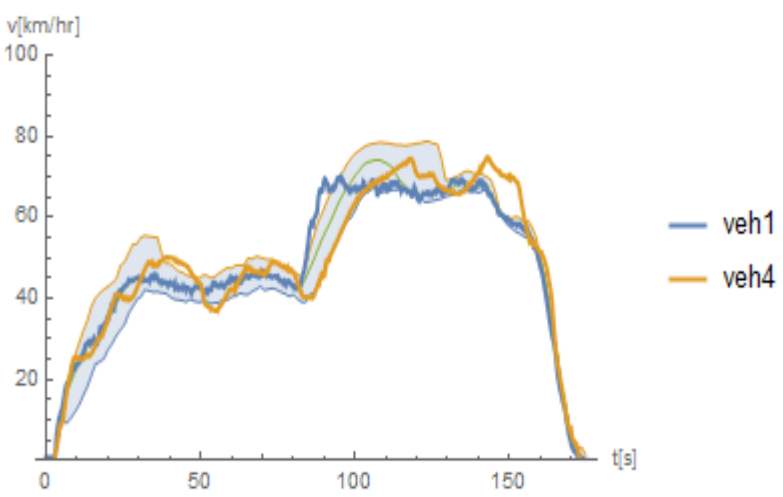


Vehicle 4, Segment 4, Large Variance

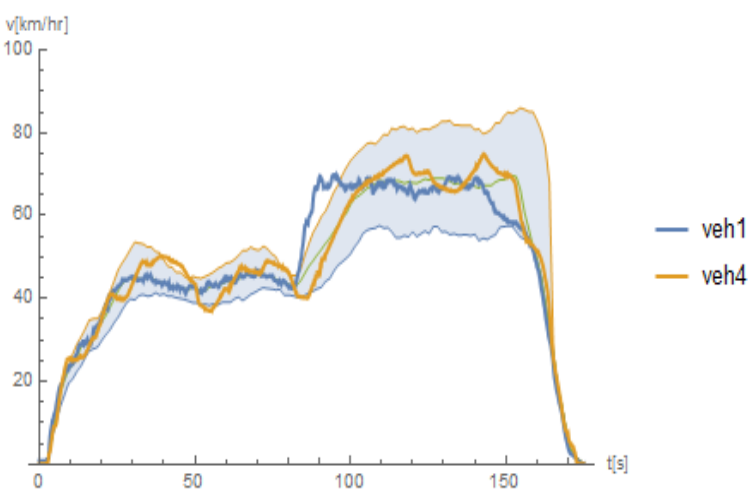
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion



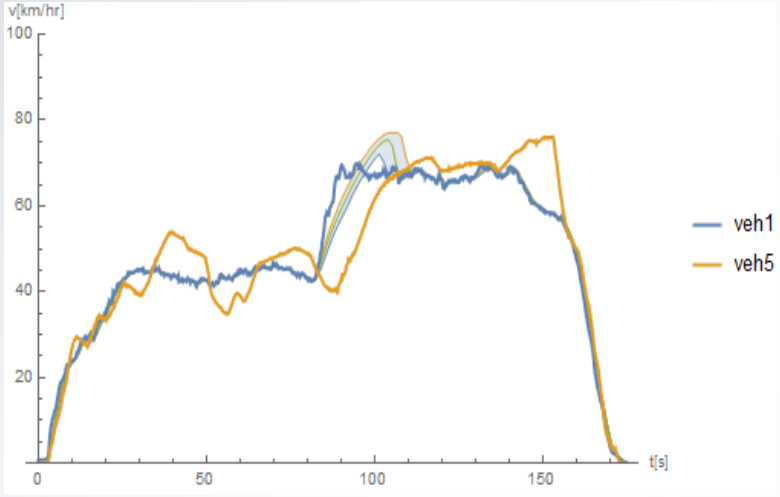
Vehicle 5

Vehicle 5, Segment 4, Small Variance

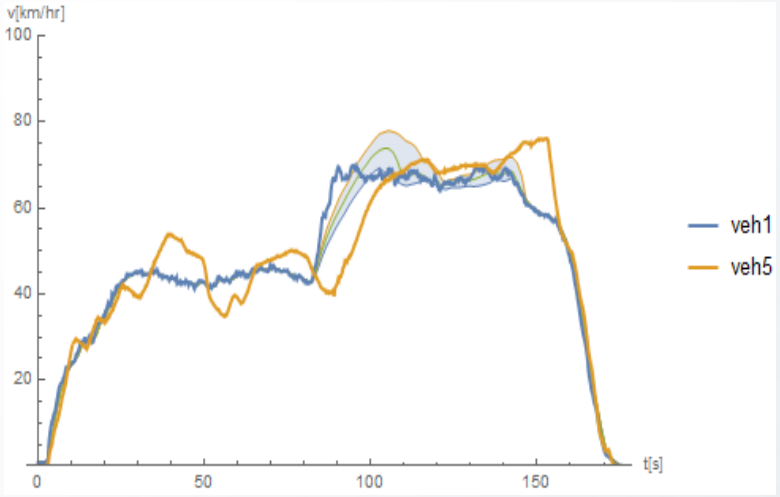
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion

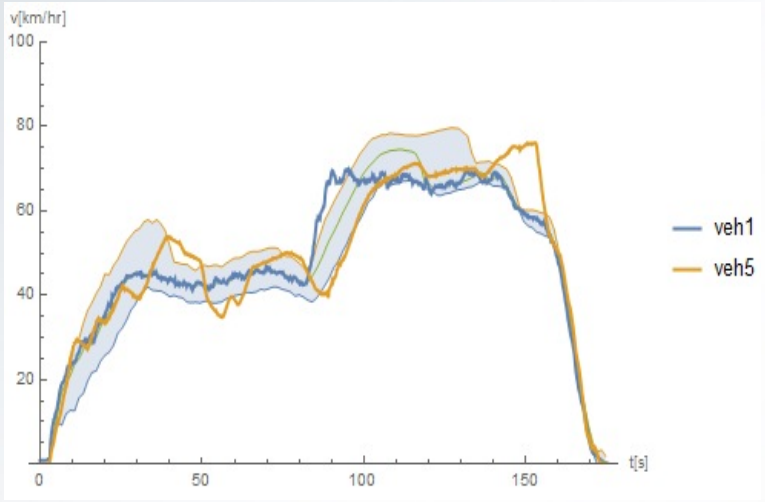


Vehicle 5, Segment 4, Large Variance

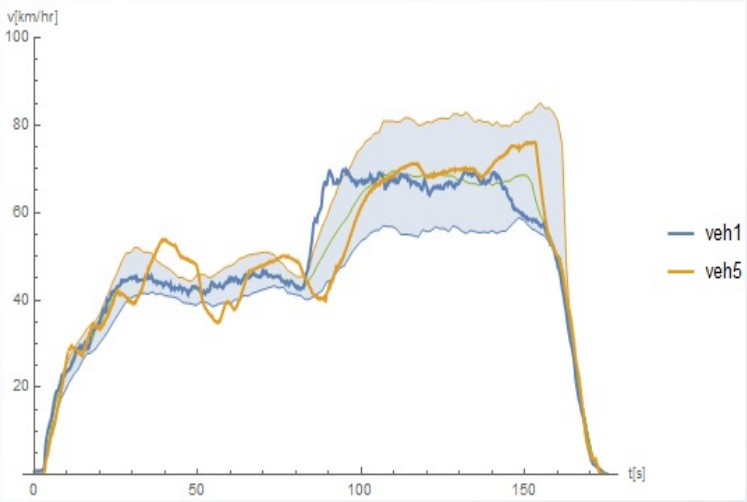
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion

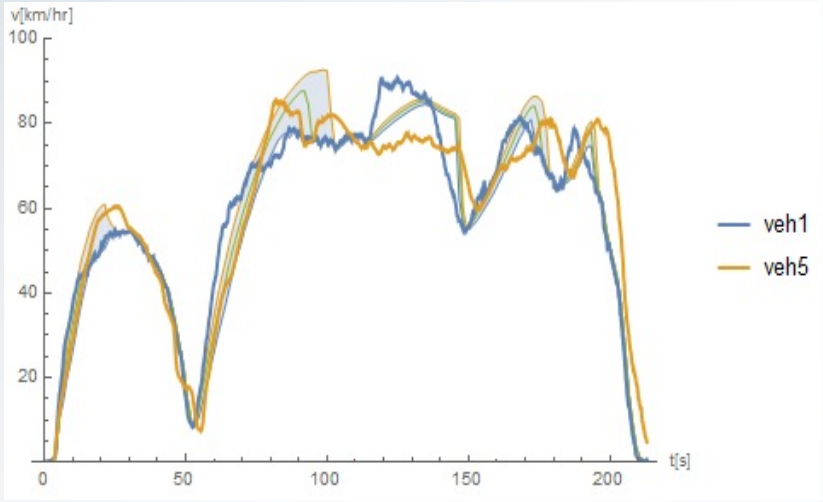


Vehicle 5, Segment 6, Small Variance

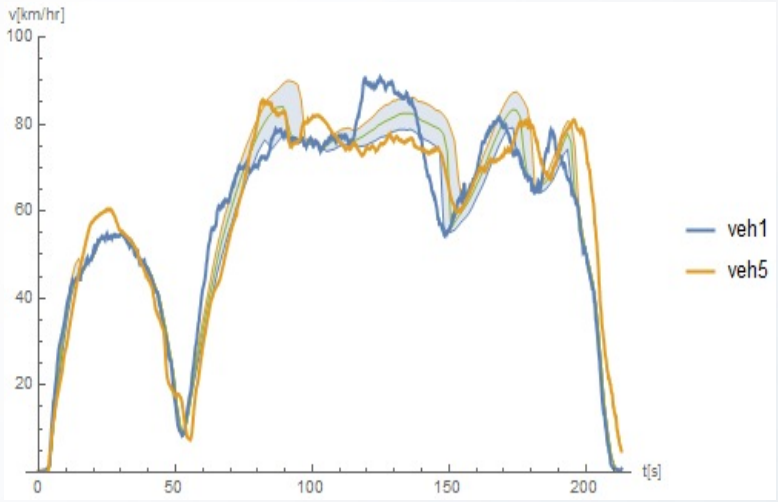
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion

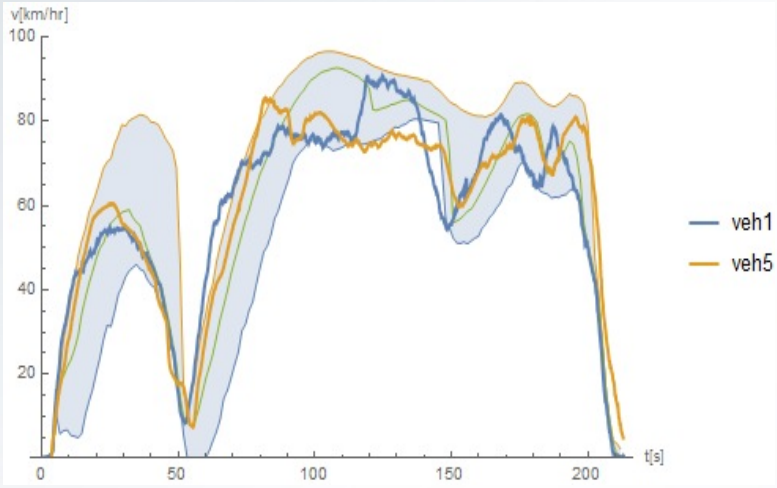


Vehicle 5, Segment 6, Large Variance

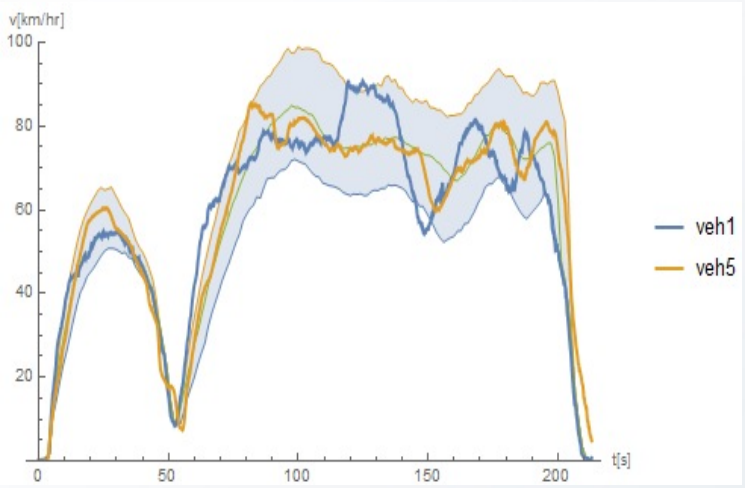
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion



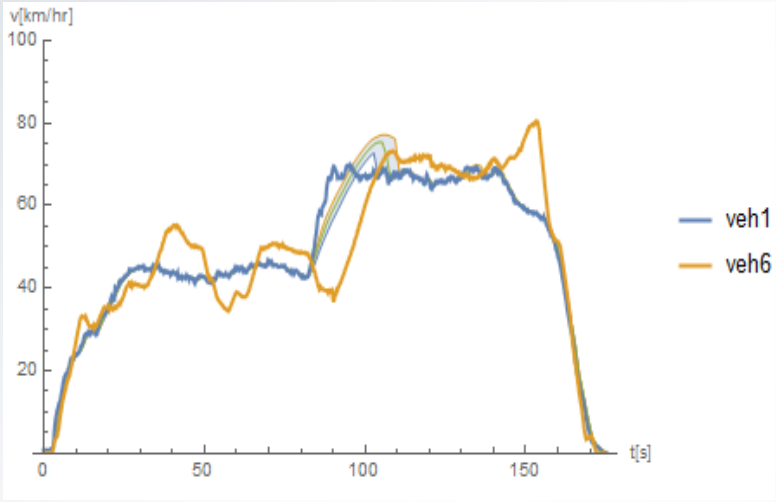
Vehicle 6

Vehicle 6, Segment 4, Small Variance

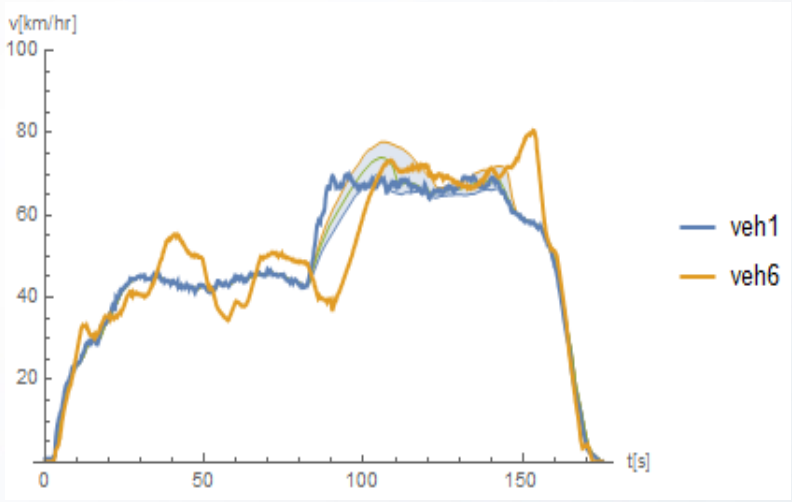
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion



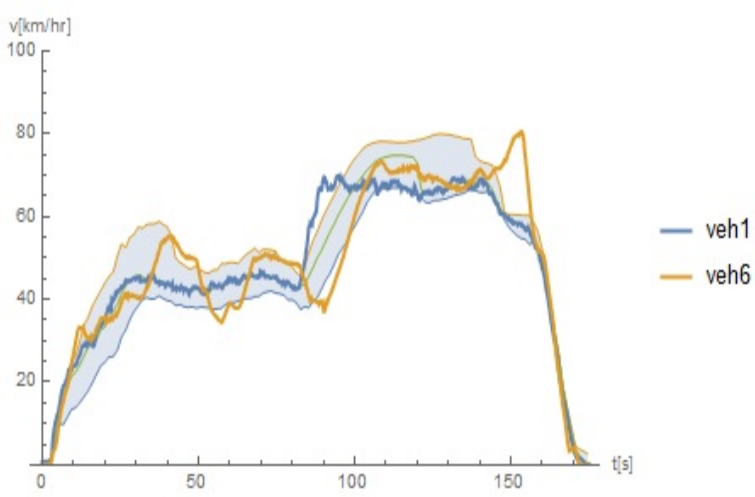
Georgia Tech

Vehicle 6, Segment 4, Large Variance

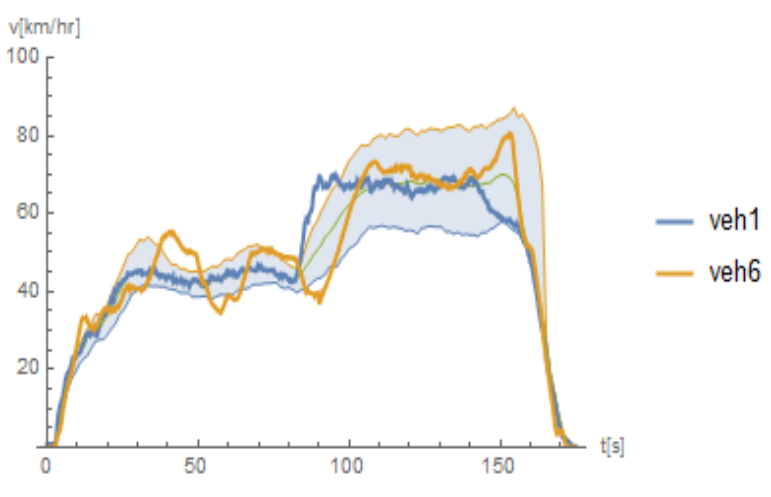
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion



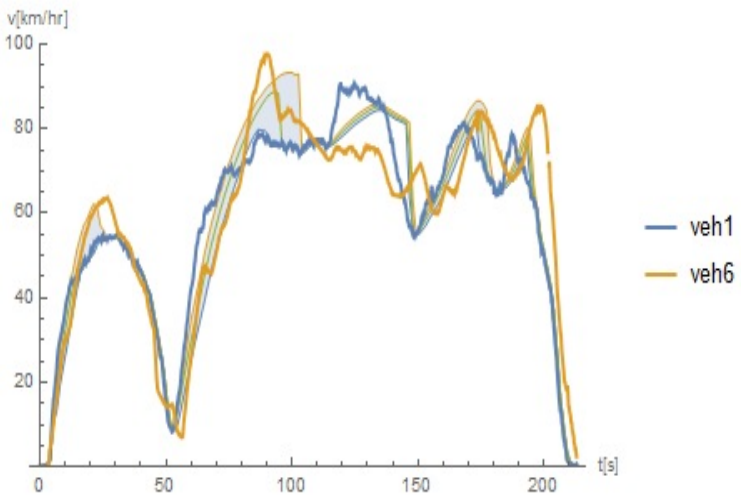
Georgia Tech

Vehicle 6, Segment 6, Small Variance

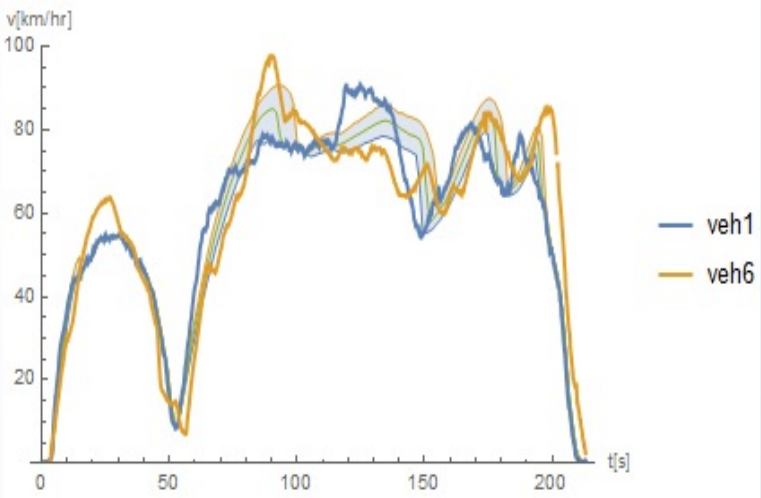
- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion



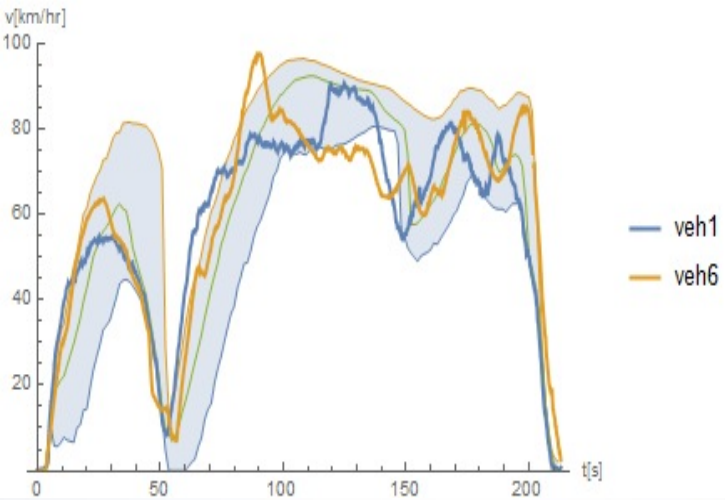
Georgia Tech

Vehicle 6, Segment 6, Large Variance

- Geometric Brownian Motion

Brownian Motion

Geometric Brownian Motion



Brownian Motion