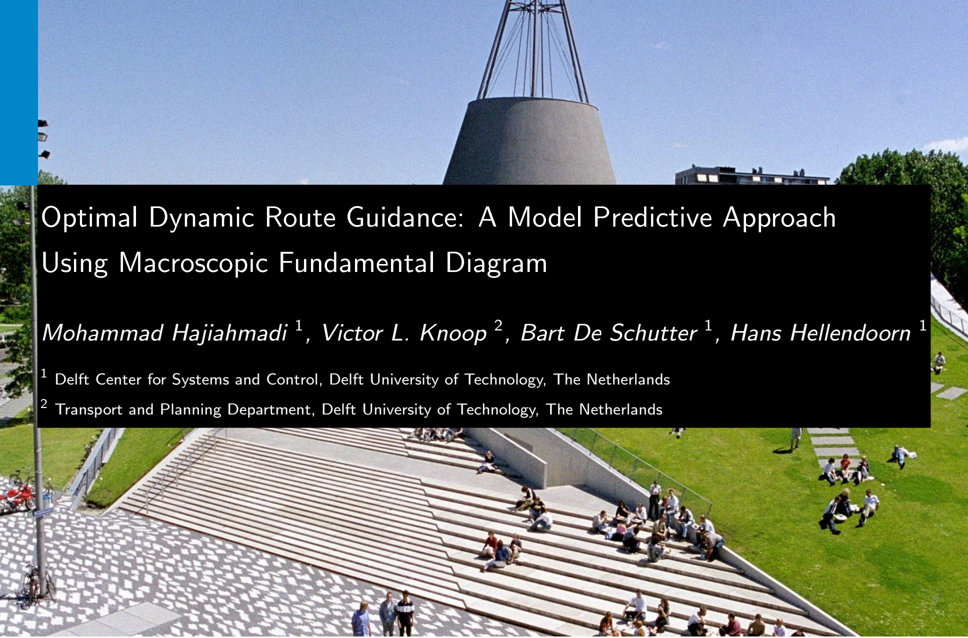


Optimal Dynamic Route Guidance: A Model Predictive Approach Using Macroscopic Fundamental Diagram

Mohammad Hajiahmadi [1], *Victor L. Knoop* [2], *Bart De Schutter* [1], *Hans Hellendoorn* [1]

[1] Delft Center for Systems and Control, Delft University of Technology, The Netherlands

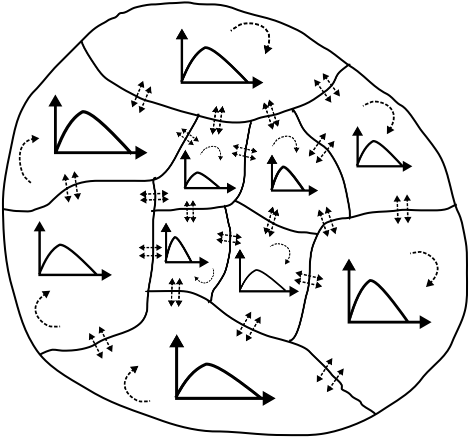
[2] Transport and Planning Department, Delft University of Technology, The Netherlands

TUDelft

Outline

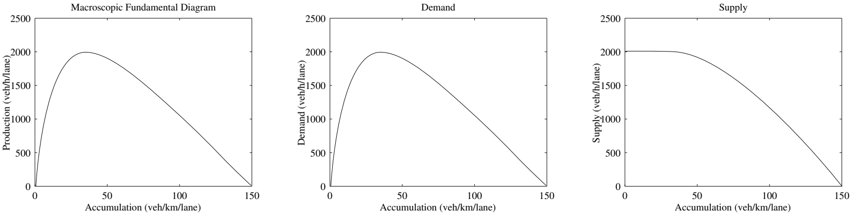
- Modeling large-scale urban networks would be a complex task if one wants to study and model dynamics of every single element
- Control using such detailed modeling approach would be a tedious task

$\Rightarrow$ Aggregate models



Urban network partitioned into multiple regions, each represented by an MFD

- $\mathcal{J}_i$: set of neighboring regions of region i
- Flow from region i to region $j \in \mathcal{J}_i$ is min. of 3 elements:
    1. Demand from region i to region j, $D_{i,j}$
    2. Supply in region j, S_j
    3. Capacity of boundary between region i & region j, $C_{i,j}$


Macroscopic Fundamental Diagram
Production (veh/h/lane)
Accumulation (veh/km/lane)
0 500 1000 1500 2000 2500
0 50 100 150
Demand
Demand (veh/h/lane)
Accumulation (veh/km/lane)
0 500 1000 1500 2000 2500
0 50 100 150
Supply
Supply (veh/h/lane)
Accumulation (veh/km/lane)
0 500 1000 1500 2000 2500
0 50 100 150

(a) (b) (c)

Supply function

$$S_j(k) = \begin{cases} P_{j,\mathrm{crit}} & \text{if } n_j(k) \leq n_{j,\mathrm{crit}} \\ P_j(n_j(k)) & \text{if } n_j(k) > n_{j,\mathrm{crit}} \end{cases}$$

$P_j(n_j(k))$: production determined from MFD

Demand function

$$D_{i,j}(k) = \sum_{d \in \mathcal{D}} \left(\alpha_{i,j,d}(k) \cdot \frac{n_{i,d}(k)}{n_i(k)} \cdot P_i(n_i(k)) \right)$$

$\mathcal{D}$: set of all destinations

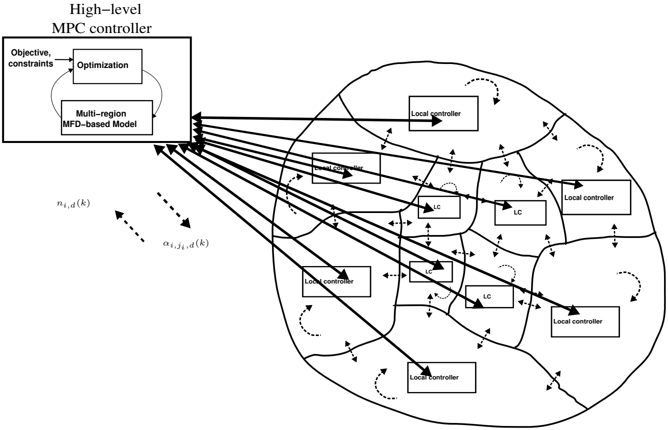
Update equations

$$n_{i,d}(k+1) = n_{i,d}(k) + \frac{T_\mathrm{s}}{\sum_{\lambda \in \Lambda_i} \kappa_\lambda L_\lambda} \Big(\sum_{j \in \mathcal{J}_i} q_{j,i,d}(k) - \sum_{j \in \mathcal{J}_i} q_{i,j,d}(k) \Big)$$

Total accumulation in region i:

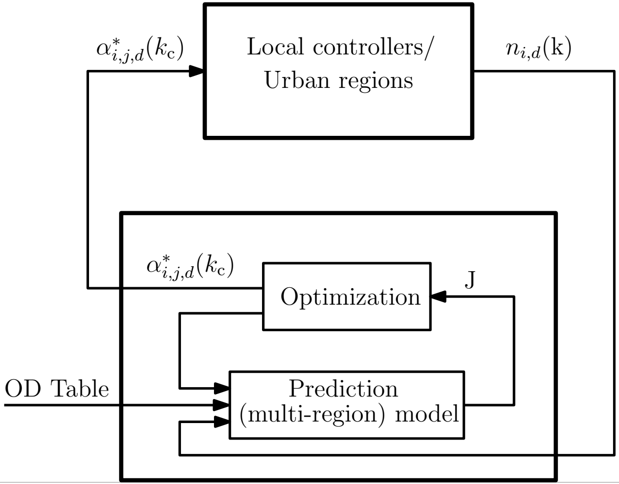
$$n_i(k+1) = \sum_{d \in \mathcal{D}} n_{i,d}(k+1)$$

- Regional destinations
- Optimal splitting traffic towards neighboring regions
- Aims:
  - avoid congestion in intermediate regions
  - decrease the overall travel time


High–level
MPC controller
Objective, constraints
Optimization
Multi–region MFD–based Model
$n_{i,d}(k)$
$\alpha_{i,j_i,d}(k)$
Local controller
Local controller
LC
LC
Local controller
LC
LC
Local controller
Local controller
Local controller

- Minimizing total travel delay:

$$J_{\mathrm{TD}} = T_{\mathrm{s}} \cdot \sum_{i \in \mathcal{R}} \sum_{k=0}^{K-1} \left(\Big(\sum_{\lambda \in \Lambda_i} \kappa_\lambda L_\lambda \Big) \cdot n_i(k) \right)$$


$\alpha^*_{i,j,d}(k_c)$
Local controllers/
Urban regions
$n_{i,d}$(k)
$\alpha^*_{i,j,d}(k_c)$
Optimization
J
OD Table
Prediction
(multi-region) model

$$J_{\mathrm{TD}}^{\mathrm{MPC}} = T_{\mathrm{s}} \cdot \sum_{i=1}^{R} \sum_{k=M\cdot k_{\mathrm{c}}}^{M\cdot(k_{\mathrm{c}}+N_{\mathrm{p}})-1} \left(\Big(\sum_{\lambda\in\Lambda_i} \kappa_\lambda L_\lambda \Big) \cdot n_i(k) \right)$$

– Overall optimization problem:

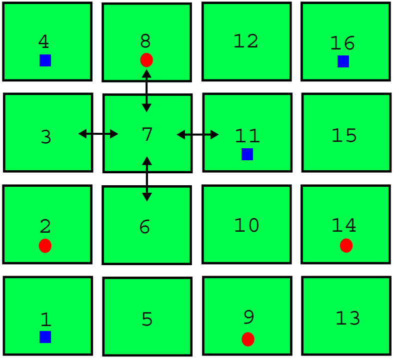
$$\min_{\tilde{\alpha}_{i,j,d}(k_{\mathrm{c}})} J_{\mathrm{TD}}^{\mathrm{MPC}}$$

subject to:

$$\text{model equations},$$

$$0 \leq \alpha_{i,j,d}(k) \leq 1,$$

$$\alpha_{i,j,d}(k) = \alpha_{i,j,d}^{\mathrm{c}}(k_{\mathrm{c}}), \qquad \text{if } k \in \{M\cdot k_{\mathrm{c}}, \ldots, M\cdot(k_{\mathrm{c}}+1)-1\}$$



Blue squares: origins

Red circles: destinations

For each region, the MFD is approximated by:

$$P_i = n_i \cdot V_{\text{free}} \cdot \exp\left(-\frac{1}{2}\left(\frac{n_i}{n_{\text{crit}}}\right)^2\right)$$

Table : Origin-destination demands* (veh/h)

	Region 2	Region 8	Region 9	Region 14
Region 1	1000	1800	1750	3000
Region 4	1900	1400	1000	1400
Region 11	1700	1200	1300	1300
Region 16	2000	1000	1000	1800

*: noise corrupted in the simulation model (network)

Determining splitting rates

- Static shortest-path (in time), Floyd-Warshall algorithm based on average speed of regions
- Shortest-path algorithm, updated every 60 seconds
- Dynamic, MPC algorithm using multi-region MFD model

Results for 4x4 network: (a) Uncontrolled (fixed routes), (b) Shortest-path algorithm, (c) Optimal dynamic routing using MPC

- High-level scheme for optimal dynamic route guidance using MFD-based multi-region model
- Optimal splitting rates towards neighboring *regions*
- Avoiding detailed modeling and hence decreasing computational complexity of route guidance

- Lower level control should be properly designed & connected to the high-level scheme
- Multi-level scheme needs to be validated using real networks' layouts and empirical data