Two-Variable Macropscopic Fundamental Diagrams for Traffic Networks

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Abstract:
Traffic processes can be described at different levels. Traditionally the most common are the vehicle level (microscopic), or the level of road sections (macroscopic). The last few years, another level has been studied, the level of an area. It has been shown that a strict relationship exists between the number of vehicles in an area (accumulation) and the average flow in that area (production). This relationship only hold in case congestion is spread of homogeneously over the network. Mazloumian et al. (2010) note using simulation that spatial variance of density within an area also plays a role.

This paper continues this research. We show that the performance of a network is a smooth function of the average network density (accumulation) and the spatial variation of the density. This way, we introduce a two-variable macroscopic fundamental diagram (TV-MFD) which holds not only for networks with homogeneous congestion, but also for networks which are in a transition state, for instance in queue build up.

We show the shape of the TV-MFD using simulation. Analyses are carried out with a grid network with periodic boundary conditions, with randomly chosen destinations. The underlying simulation program is a macroscopic traffic simulator, implementing a cell-transmission model. Traffic takes the shortest route to the destination, and once arrived, it is rerouted to the next destination. We restrict the capacity of the nodes to the capacity of the connecting links, to prevent crossing flows at the same time.

From the results of the simulation program we calculate the accumulation (constant throughout one simulation) and the spatial distribution of density (changing due to the traffic dynamics). This is repeated for different network loadings. This way, we construct the TV-MFD.

The cross-section of the TV-MFD for constant spatial variation of congestion is the macroscopic fundamental diagram. The cross-section in the other direction is the network performance as function of the spatial distribution of congestion. This function is decreasing, and decrease is steeper for higher accumulation levels. The full paper will describe and explain the shape. The TV-MFD can be used for network control, for instance at ramp metering installations or at a more aggregate level perimeter control.

Keywords: Macroscopic Fundamental Diagram, Inhomogeneous congestion, Spatial distribution of congestion

1 Introduction

To control traffic, its processes needs to be understood, or at least need to be predictable. Nowadays, research projects aim at collecting detailed data of driving processes, which reveals even more differences between drivers. These data will need much aggregation to come to the understanding of general traffic patterns.

As opposed to the movement of collecting more detailed data, Daganzo (2007) and Geroliminis and Daganzo (2008) started a simplified description of traffic. The traffic state is only based on the accumulation, being the number of vehicles in an area. Drawback of this description is that it is only valid for homogeneously loaded networks (Cassidy et al., 2011).

In this paper we analyse the onset of congestion in a simulated grid network. The focus for the paper is the effect of the spatial variation of density on the network performance. The next section summarises the recent developments in describing traffic with the macroscopic fundamental diagram. Section 3 then presents the setup of the simulation study. The 2-dimensional macroscopic fundamental diagram, performance as a function of accumulation and of spread of the accumulation, is presented in section 4. Finally, section 5 presents the conclusions and the further outlook.

Table 1: Overview of the papers discussing the macroscopic fundamental diagram

Paper	data	network	insight
Daganzo (2007)	theory	none	Overcrowded networks lead to a performance degradation – the start of the MFD
Geroliminis and Daganzo (2008)	real	Yokohama	MFDs work in practice, and there is a relation between the average flow (production) and the arrival rate (performance)
Daganzo and Geroliminis (2008)	data & simulation	Yokohama & San Francisco	The shape of MFDs can be theoretically explained
Buisson and Lavier (2009)	real	urban + urban motorway	There is scatter on the FD if the detectors are not ideally located or if there is inhomogeneous congestion
Ji et al. (2010)	simulation	urban + motorway	Hybrid networks give a scattered MFD; inhomogeneous congestion reduces flow, and should therefore be considered in network control
Cassidy et al. (2011)	real	3 km motorway	MFDs on motorways only hold if stretch is completely congested or not; otherwise, there are points within the diagram
Mazloumian et al. (2010)	simulation	urban grid – periodic boundary	spatial variability of density is important in deriving the production
Geroliminis and Ji (2011)	real	Yokohama	Spatial variability of density is important in deriving the production
Wu et al. (2011)	real	900m artirial	There is an arterial fundamental diagram, influenced by traffic light settings
Daganzo et al. (2011)	simulated	grid	Equilibrium states in a network are either free flow, or heavily congested. Rerouting increases the critical density for the congested states considerably.
Gayah and Daganzo (2011)	simulated	grid/bin	Hysteresis loops exist in MFDs due to a quicker recovery of the uncongested parts; this is reduced with rerouting.

2 Literature overview of Macroscopic Fundamental Diagrams

In the past five years the theory of a macroscopic fundamental diagram (MFD) has been developed. Concepts were already proposed by Godfrey (1969), but only when Daganzo (2007) reintroduced the concept, more studies started. An overview of the most important ones are given in 1.

The best-known studies are the ones by Daganzo (2007) and Geroliminis and Daganzo (2008). Geroliminis and Daganzo (2008) show the relationship between the number of completed trips and the production function which is defined as a weighted average of the flow on all links. This means that the network production can be used as a good approximation of the utility of the users for the network, i.e., it is related to their estimated travel time. Furthermore, after some theoretical work, Geroliminis and Daganzo (2008) were the first to show that MFDs work in practice. With pioneering work using data from the Yokohama metropolitan area, an MFD was constructed with showed a crisp relationship between the network production and the accumulation.

Also, theoretical insights have be gained over the past years. Daganzo and Geroliminis (2008) have shown that rather than to find the shape of the MFD in practice or by simulation, one can theoretically predict its shape. This gives a tool to calculate the highest production of the network, which then can be compared with the actual network production.

One of the requirements for the crisp relationship is that the congestion should be homogeneous over the network. Buisson and Lavier (2009) were the first to test the how the MFDs change if the congestion is not homogeneously distributed over the network. They showed a reasonably good MFD for the French town Toulouse in normal conditions. However, one day there were strikes of truck drivers, driving slowly on the motorways, leading to traffic jams. The researchers concluded that that leads to a serious deviation from the MFD for normal conditions. The inhomogeneous conditions were recreated

by Ji et al. (2010) in a traffic simulation of a urban motorway with several on-ramps (several kilometers). They found that inhomogeneous congestion leads to a reduction of flow. Moreover, they advised on the control strategy to be followed, using ramp metering to create homogeneous traffic states. Cassidy et al. (2011) studied the MFD for a motorway road stretch. They conclude, based on real data, that the MFD only holds in case the whole stretch is either congested or in free flow. In case there is a mix of these conditions on the studied stretch the production is lower than the production which would be predicted by the MFD.

The effect of variability is further discussed by Mazloumian et al. (2010) and Geroliminis and Ji (2011). Contrary to Ji et al. (2010), both papers focus on urban networks. First, Mazloumian et al. (2010) show with simulation that the variance of density over different locations (spatial variance) of density (or accumulation) is an important aspect to determine the total network production. So not only too many vehicles in the network in total, but also if they are located at some shorter jams at parts of the networks. The reasoning they provide is that "an inhomogeneity in the spatial distribution of car density increases the probability of spillover, which substantially decreases the network flow." This finding from simulation and reasoning is confirmed by an empirical analysis by Geroliminis and Ji (2011), using the data from the Yokohama metropolitan area. The main cause for this effect is claimed to be the turning movement of the individual vehicles.

A theoretical explanation for the phenomenon of the influence of the spatial variance of the accumulation is given by Daganzo et al. (2011). He shows that turning at intersections is the key reason for the drop in production with unevenly spread congestion. Gayah and Daganzo (2011) then use this information by adding dynamics to the MFD. If congestion solves, it will not solve instantaneous over all locations. Rather, it will solve completely from one side of the queue. Therefore, reducing congestion will increase the spatial variance of the accumulation and thus (relatively) decrease the production. This means that the production for a system of dissolving traffic jams is under the equilibrium state, thus under the MFD. This way, there are hysteresis loops in the MFD, as also noted by Ji et al. (2010). Note that these loops are an effect by themselves and are different from for instance the capacity drop (Hall and Agyemang-Duah, 1991; Cassidy and Bertini, 1999).

This paper will again look into this phenomenon by performing a macroscopic traffic simulation, but without individual turning movements. By means of this simulation we aim to reveal the importance of the microscopic (i.e., vehicle based) turning movement.

3 Experiment setup

This section describes the traffic simulation used for this research. The section first describes what will be simulated in terms of network and demands. Then, section 3.2 describes the model used for this simulation. Section 3.3 describes the output of the simulator that is used later in the paper.

3.1 Experimental settings

In the paper an urban network is simulated, since this is the main area where MFDs have been tested. We follow Geroliminis and Ji (2011) and choose a Manhattan network with periodic boundary conditions. This means that the nodes are located at a regular grid, for which we choose a 16x16 size. Then, one-way links connect these nodes. The direction of the links changes from block to block, i.e. if at $x = 2$ the traffic is allowed to drive in the positive y direction, at $x = 1$ and at $x = 3$ there are one-way roads for traffic to drive in the negative y direction. We assume 2 lanes per link, a 1 km block length, a triangular fundamental diagram with a free speed of 60 km/h, a capacity of 1500 veh/h/lane and a jam density of 150 veh/km/lane.

Furthermore, periodic boundary conditions are used, meaning that a link will not end at the edge of the network. Instead, it will continue over the edge at the other side of the network. An example of such

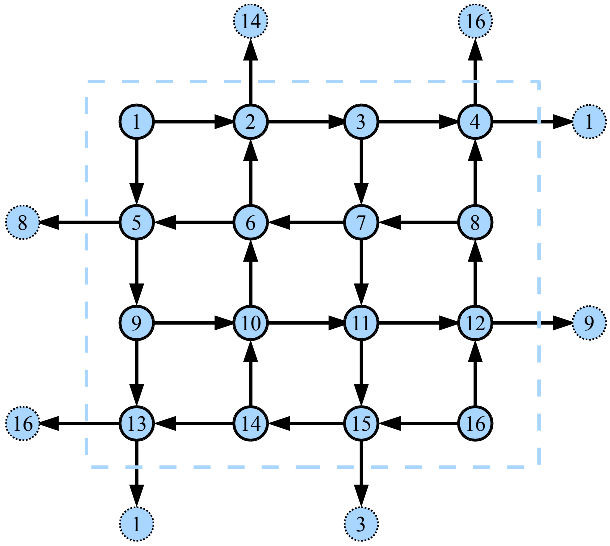


Figure 1: Illustration of a 4x4 grid network with periodic boundary conditions

a network is given in figure 1. Traffic can continue in a direct link from node 13 to node 1 or from node 5 to node 8. This way, all nodes have two incoming and to outgoing links and network boundaries have no effect

The destinations are randomly chosen from all points in the network. In the network, there are 15 nodes chosen as destination nodes. There are no origin nodes. Instead, at the beginning of the simulation, traffic is put on the links. Vehicles are assigned to a destination, and for this distribution is equal over all destinations.

When the cars have reached their destination, they will not leave the network, but instead they are assigned a new destination. We use a macroscopic model (see section 3.2), hence we can split the flow of arriving traffic equally over the 18 other destinations. The number of cars in the network is hence constant. This number will be a parameter setting for the simulations, but throughout one simulation, it is constant. The demand level is expressed as the density on all links at the start of the simulation, as fraction of the critical density. Figure 2a shows the network used under initial conditions.

3.2 Traffic flow simulation

This section describes the traffic flow model. The variables used in this section and further in the paper are listed in table 2. For the traffic flow modelling we use a first order traffic model. Links are split into cells with a length of 250 meters (i.e., 4 cells per link). We use the continuum LWR-model proposed by Lighthill and Whitham (1955) and Richards (1956) that we solve with a Godunov scheme (Godunov, 1959). Lebacque (1996) showed how this is used for traffic flows, yielding a deterministic continuum traffic flow simulation model. The flux from one node to the next is basically restricted by either the demand from the upstream node (free flow) or by the supply from the downstream node (congestion):

$$\phi_{c,c+1} = \min \{D_c, S_{c+1}\}; \tag{1}$$

At a node r we have inlinks, denoted by i which lead the traffic towards node r and outlinks, denoted by j which lead the traffic away from r. At each node r, the demand D to each of the outlinks of the nodes is calculated, and all demand to one link from all inlinks is added. This is compared with the supply S of the cell in the outlink. In case this is insufficient, a factor, α, is calculated which show

Table 2: The variables used

Symbol	meaning
r	Node
c	Cell in the discretised traffic flow simulation
L_c	Length of the road in cell c
q_c	Flow in cell c
k_c	Density in cell c
ϕ_{ij}	Flux from link i to link j
S	The supply of cell c
D	The demand from cell c
i	The links towards node r
j	The links from node r
C	The capacity of node r in veh/unit time
α	The fraction of traffic that can flow according to the supply and demand
β	The fraction of traffic that can flow according to the demand and the node capacity
γ	The fraction of the demand that can flow over node r
X	An area
N_X	Accumulation of vehicles in area X
P_X	Production in area X
σ	Standard deviation

which part of the demand can continue.

$$\alpha_r = \underset{[j \text{ leading away from } r]}{\operatorname{argmin}} \left\{ \frac{S_j}{D_j} \right\} \tag{2}$$

This is the model developed by Jin and Zhang (2003). They propose that all demands towards the node are multiplied with the factor α, which gives the flow over the node.

This node model is slightly adapted for the case at hand here. Also the node itself can restrict the capacity. In our case, there are two links with a capacity of 3000 veh/h as inlinks and two links with a capacity of 3000 veh/h as outlinks. Since there are crossing flows, it is not possible to have a flow of 3000 veh/h in one direction *and* a flow of 3000 veh/h in the other direction. To overcome this problem, we introduce a node capacity (see also for instance Tampère et al. (2011)). The node capacity is the maximum of the capacities of the outgoing links. This means that in our network, at maximum 3000 veh/h can travel over a node. Again, the fraction of the traffic which can continue over node r is calculated, indicated by β:

$$\beta_r = \frac{C_r}{\sum_{\forall i \text{to } r} D_i} \tag{3}$$

The demand factor γ is now the minimum of the demand factor calculated by the nodes and the demand factor due to the supply:

$$\gamma = \min \{\alpha_r, \beta_r, 1\} \tag{4}$$

Similar to Jin and Zhang (2003), we take this as multiplicative factor for all demands to get to the flux ϕ_{ij}, i.e. the number of cars from one cell to the next over the node:

$$\phi_{ij} = \gamma D_{ij} \tag{5}$$

The path choice is static, and determined based on distance to the destination. Traffic will take the shortest path towards the destination. For intersections where both directions will give the same path length towards a destination, the split of traffic to that direction is 50-50.

3.3 Variables

In this paper, several traffic flow variables will be used. In this section we will explain them and show the way to calculate them

Standard traffic flow variables are flow, q, being the vehicle distance couvered in a unit of time, and density, k, the number of vehicles per unit road length. The network is divided into cells, which we denote by c, which have a length L_c. Flow and density in cells are denoted by q_c and k_c.

Furthermore, the accumulation N in an area X is the weighted average density:

$$N_X = \sum_{c \in X} \frac{k_c * L_c}{L_c} \tag{6}$$

Similarly, the production P in an area X is the weighted average flow:

$$P_X = \sum_{c \in X} \frac{q_c * L_c}{L_c} \tag{7}$$

Since the cell length are the same for all links in the network, the accumulation and production are average densities and flows. Recall that there is a strong relationship between the production and the number of completed trips, as shown by Geroliminis and Daganzo (2008).

This paper also studies the variations in densities. The standard deviation of the cell density is found by considering all cell densities for one moment in time, and calculate the standard deviation of these numbers.

4 Simulation outcomes

This section describes the evolution of traffic over the time. First, the traffic flow phenomena are qualitatively described, then in section 4.2 the performance and variation are quantified.

4.1 Traffic flow phenomena

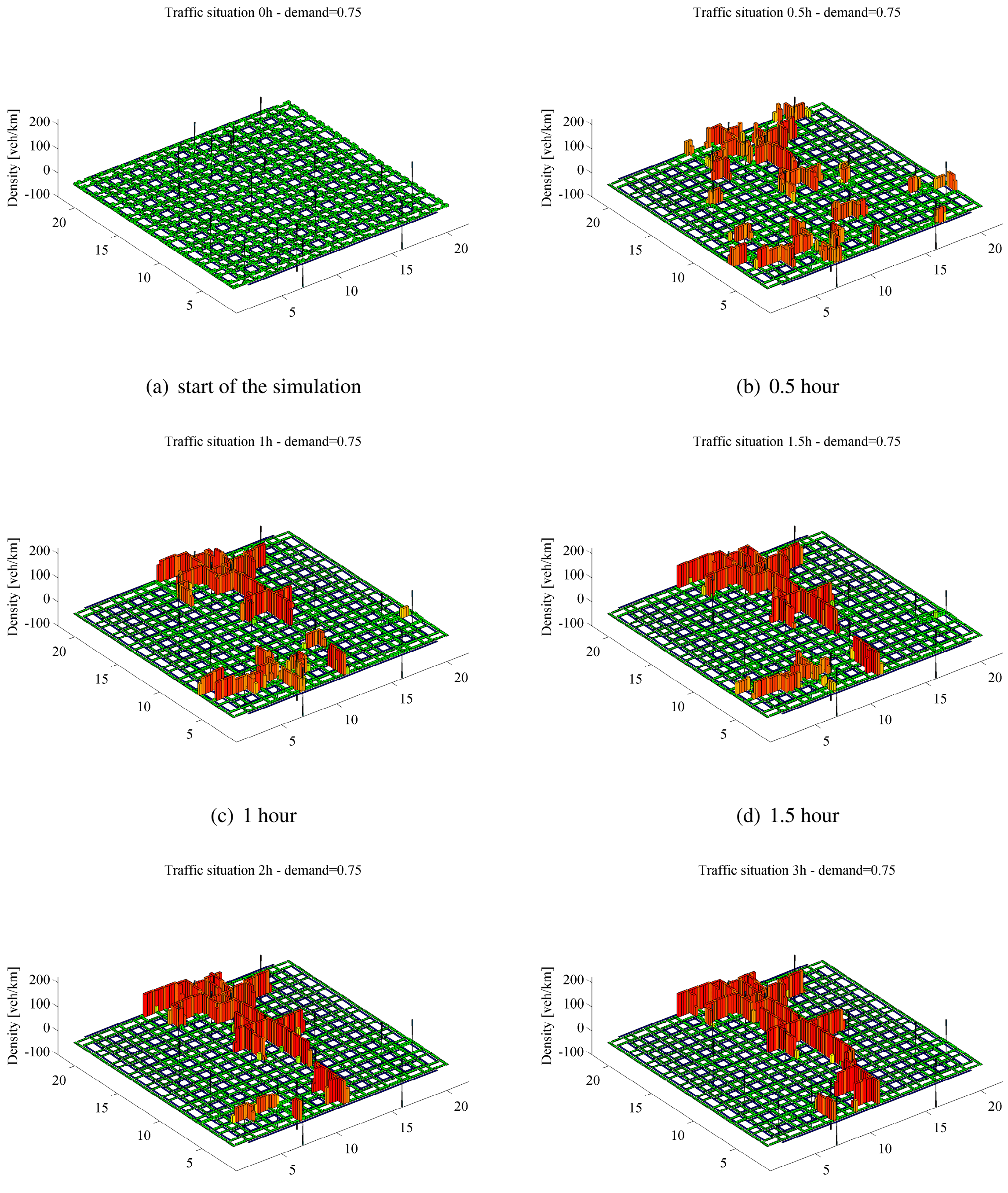
This section first describes the traffic flow over time. Figure 2 shows the outcomes of the simulation, in snapshots of the density and speed over time. At the start of the simulation (see figure 2a), traffic is evenly distributed over all links, since this was the initial situation as it was regulated externally. The destinations of the network are indicated by the vertical lines.

When the traffic starts to run, various distributed bottlenecks become active. This is shown in figure 2b. After some time (figure 2d-f), traffic problems concentrate more and more around one location. The number of vehicles in the rest of the network reduces, ensuring free flow conditions there. This complete evolution can be found in figure 2a-f. The network has periodic boundary conditions, which means that the network edges do not have any effect. Any deviations from a symmetry are due to random effects and thus to the location of the destinations, since the traffic simulation is deterministic.

At the end, the situation seems to have stabilised. From 2.5 to 3 hours (figure 2e-f) there have been little change, and the changes in the traffic state get smaller and smaller: an equilibrium has formed. Now, the number of vehicles passing the most restricting bottleneck equals the number of vehicles arriving at the end of the queue.

4.2 Influence of variation in density

Most articles describing macroscopic fundamental diagrams emphasize that the relationship is only valid as long as traffic states are similar for all links in the network (e.g. Geroliminis and Daganzo (2008)). This requirement of homogeneous distributed congestion clearly does not hold for our situation (see



(a) start of the simulation

(b) 0.5 hour

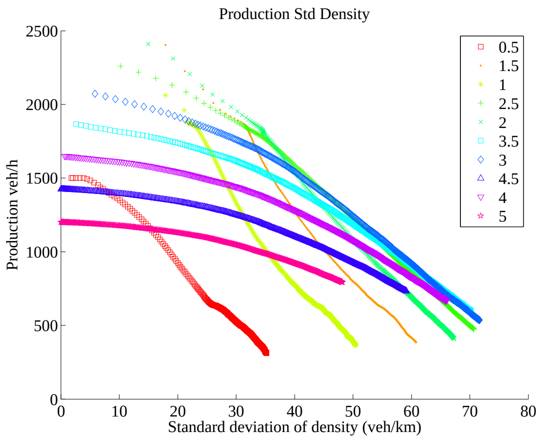
(c) 1 hour

(d) 1.5 hour

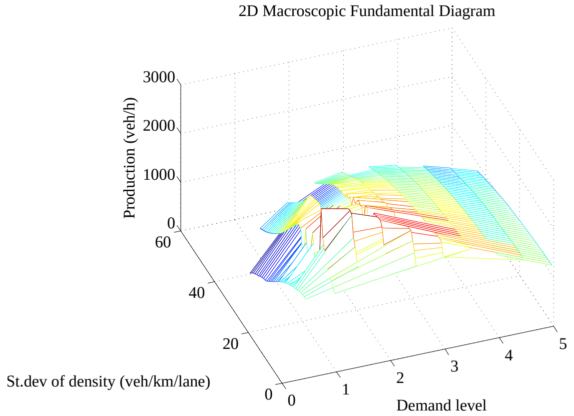
(e) 2 hour

(f) 3 hour

Figure 2: Evolution of the densities (bar heights) and speeds (colours) in the network



(a) Different lines for different demands



(b) The two-dimensional macroscopic fundamental diagram

Figure 3: The effect of demand and spatial variation on the network performance

figure 2f). Furthermore, a standard macroscopic fundamental diagram would relate the network performance to the network accumulation, being proportional to the number of vehicles in the network. In this case, however, the number of vehicles in the network is constant: vehicles cannot drive out of the network and at the destination they are simply given another destination. The accumulation is thus fixed, but there are various values for the performance, depending on the congestion, hence the macroscopic fundamental diagram would result in a vertical line.

Instead of linking the performance only to the accumulation, we also link it to the variation of the densities in the network. This relation is shown graphically in figure 3. Figure 3a shows production lines for the different network loadings. They decrease as function of the density variability, as also found by Mazloumian et al. (2010). Contrary to their setup, the simulation in our paper does not have a flow discretisation in vehicles. So the effect of the decreasing production with increasing variability is not mainly caused by the individual vehicle movements since it can found in a simulation modelled with aggregated flows.

It is remarkable that the slope of the lines in figure 3a is different. If the average accumulation is undercritical, the demand rapidly decreases with the increase of variability. This is caused by queues, and spillover of queues. With a demand higher than the critical level (demand>1), the decrease is less steep. In this case the temporal spillovers are less important, since vehicles will find a queue later on in the network anyway.

Figure 3b shows how these lines can be transformed into a two-dimensional macroscopic fundamental diagram. Note that the impact of the variation on the production is of a similar magnitude as the impact of the total accumulation. It is therefore essential that the variation of the density is also used in the state estimation, and thus the prediction of the production or performance. Note furthermore that we did not exclude any points from the observations. This two-dimensional macroscopic fundamental diagram can also be used if congestion is homogeneously distributed over the network.

5 Conclusions and outlook

This paper presented a simulation study to the effect of variation in the macroscopic fundamental diagram. Even with very simple simulation tools, being a first-order traffic simuation, a proportional node model, and a node capacity, we find a clear influence of the variation of density on the network performance. In fact, the variation has an influence on the performance which is is similar in magnitude to the influence of the accumulation. We therefore propose to use a two-variable macroscopic fundamental

diagram (TV-MFD) instead of the one-variable macroscopic fundamental diagram used up to now.

The traffic processes themselves cause a the variation of the congestion. This depends on the OD matrix, as well as on traffic dynamics and the route choice. Future work therefore includes how the shape of the two-dimensional fundamental diagram depends on the network internal structure, and route choice. Also the effect of route choice is topic of future study.

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