Delay of Incidents Consequences of Stochastic Incident Duration

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Summary. The delay caused by an incident depends on many variables. This paper introduces an analytical expression for the delay, describing the location and length of the queue by shockwave theory. As long as the congestion remains on the same link, delay is proportional to the square of the duration, even in case the outflow is reduced by a junction downstream. This gives an elegant expression for the expected delay. Once the queue grows to other links (spillback or blocking back), the influence of duration becomes even larger. Therefore, it is useful to avoid spillback by network design or reduce incident times as much as possible.

1 Introduction

It is important to compute the cost of the delay of an incident accurately. Using a mean value for the duration a stochastic incident duration on the delay leads to an error in the delay[2]. Olmstead[5] discusses the delay on the road as a result of an incident. He derives an equation for the expected total delay. A similar expression is found for the delay per delayed traveller[4]. All articles mention explicitly the proposed solution only holds for the vertical queuing model[6]. We will show how these equations change if the queues have a spacial extent.

2 Theory for mathematical formulation of queue lengths

The traffic flow modeling using shock wave wave theory assumes a "fundamental diagram" posing the relationship between the density k and the flow q. When describing the traffic situation with shock wave theory, traffic states A and B are separated by a boundary, referred to as shock, which propagates with speed $\omega_{AB} = (q_B - q_A) / (k_B - k_A)$ [1].

A driver in traffic state A, travelling with speed v_A instead of the free speed v_f, encounters delay only in congested states, due to the choice of a triangular fundamental diagram [1]. Assuming that in other time periods he travels without delay, contributes dl to the delay: $dl = \frac{v_f - v_A}{v_f} dt$. In total, there are $N_A(t)$ drivers in traffic state A at moment t. The total delay D is the integral over t of this product:

$$D = \int N_A(t) \frac{v_f - v_A}{v_f} dt. \quad (1)$$

3 Applying the model

This section shows how nodes influence the traffic states computed by shock wave theory. We consider 3 incident scenarios where the road capacity temporary reduces to a fraction r of the original capacity, shown in fig. 1. The remainder of this section will show the resulting delays. However, due to the limited length of the paper, it is not possible to show the complete mathematical derivation (for this, we refer to [3]) but the mathematical principles, the qualitative graphs and the results will be presented.

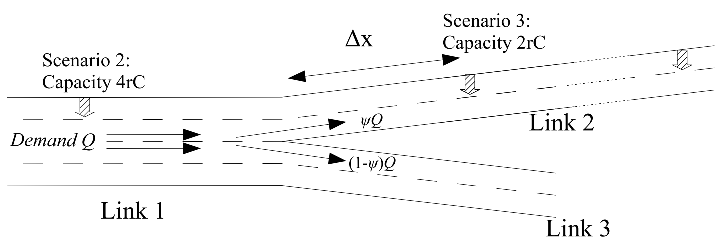


Fig. 1. The used road layout - incident locations indicated by a shaded arrow

3.1 Scenario 1: no influence of junctions

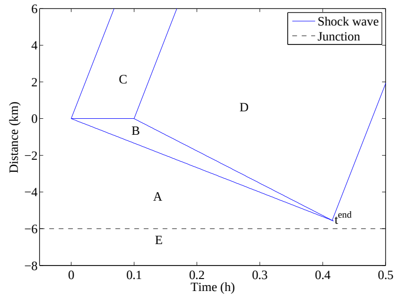
A typical pattern of traffic states for a short incident ($r = 0$ and $\Delta T = 0.1h$) is shown in fig. 2a. During a time ΔT the road is blocked at $x = 0$. Using the theory of section 2, we compute the speed at which the boundary between traffic state A and B travels backwards, ω_{AB}. After some mathematical derivation [3], we find the total delay:

$$D = \frac{1}{2} \frac{C_2 \Delta T^2 (r-1)(rC_2 - \psi Q)}{C_2 - \psi Q} \quad (2)$$

This is exactly the same function as Olmstead [5] finds for vertical queues.

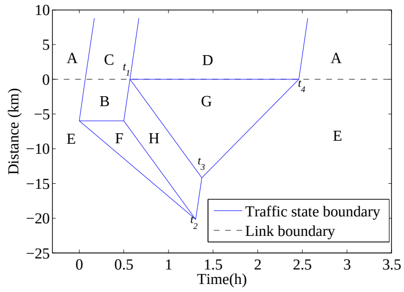
State	Lanes	Flow	Congested
A	2	ψQ	Free
B	2	$2rC$	Congested
C	2	$2rC$	Free
D	2	$2C$	Free
E	4	Q	Free

a) Scenario 1



State	Lanes	Flow	Congested
A	2	ψQ	Free
B	4	$4rC$	Free
C	2	$4rC\psi$	Free
D	2	$2C$	Free
E	4	Q	Free
F	4	$4rC$	Congested
G	4	$\frac{2C}{\psi}$	Congested
H	4	4C	Free

b) Scenario 2



State	Lanes	Flow	Congested
A	2	ψQ	Free
B	2	$2rC$	Congested
C	2	$2rC$	Free
D	2	$2C$	Free
E	4	Q	Free
F	4	$\frac{2rC}{\psi}$	Congested
G	4	$\frac{2C}{\psi}$	Congested

c) Scenario 3

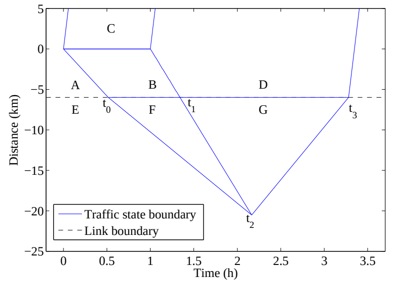


Fig. 2. Traffic states for the three scenarios

The delay is proportional to ΔT^2 and the number of drivers is proportional to ΔT. The average delay per delayed driver, A, is then proportional to ΔT. Some further analysis shows:

$$A = \Delta T \frac{1}{2} \frac{C_2(r-1)(rC_2 - \psi Q)}{C_2 - \psi Q} \Big/ k_A(v_A - \omega_{AB}) \frac{\omega_{BD}}{\omega_{BD} - \omega_{AB}} t^{\text{end}} \quad (3)$$

3.2 Scenario 2: incident upstream of a junction

When an incident happens upstream of a junction, the junction can have an influence. Suppose that an accident happens upstream of the junction (on link 1) which reduces the capacity temporarily to rC_1.

Fig. 2b shows a pattern of a traffic situation that one would typically find ($r = 0$ and $\Delta T = 0.5h$, x^{inc}=6 km). Areas A and E are the states in the non-incident situation, for links 2 and 1 respectively. There is an incident upstream of the bottleneck. Upstream of the incident a queue builds up (area F), and during the incident the outflow is lower (area B). This lower outflow reduces the demand to link 2 from t_0 to t_1. After the capacity is restored, the traffic flows out of the traffic jam at the capacity of link 1 (area H). This flow is larger than the original demand. If the capacity of any of the downstream links is lower than the new demand to that link (the splitfraction remains the same), a new area of congestion arises (area G). In this case, link 2 forms a bottleneck in case $\psi C_1 > C_2$. Since the demand is high, the flow on the downstream link equals capacity, and on link 1 the flow is maximized by the outflow to the downstream links.

Using shockwave theory one can compute the total delay in areas F and G. For the derivation we again refer to [3].

$$D = \frac{1}{2}\frac{\Delta T^2(r^2\psi C_1^2 - C_1\psi rQ - rC_1C_2 + C_2Q)}{C_2 - \psi Q} \tag{4}$$

Note that, like in the case without the influence of a junction, the delay is proportional to ΔT^2.

The number of travellers encountering delay is proportional to ΔT , as fig. 2b shows. Therefore, the average delay per traveler A, is proportional to the duration:

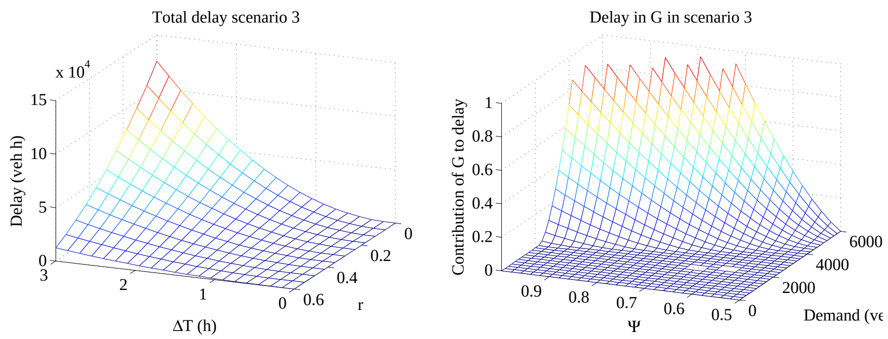
$$A = \Delta T\left(\frac{1}{2}\frac{(r^2\psi C_1^2 - C_1\psi rQ - rC_IC_2 + C_{l2}Q)}{C_2 - \psi Q} \middle/ \cdots \right.$$
$$\left. \frac{k_E(v_E - \omega_{EF})\omega_{FH}}{\omega_{FH} - \omega_{EF}} + k_E(v_E - \omega_{EG})\frac{-\omega_{GH}}{\omega_{EG}}\frac{\omega_{EF} * \Delta T}{\omega_{EF} - \omega_{FH}}\right) \tag{5}$$

3.3 Scenario 3: queues longer than the distance to the junction

This section explains what happens when the tail of a queue caused by an incident *downstream* of a junction, on link 2, reaches the upstream junction. Fig. 2c gives a typical pattern ($r = 0.3$, $x^{\text{inc}} = 6km$, and $\Delta T = 1h$).

The congestion that the incident causes, spills back to the more upstream link. From the moment that congestion reaches the junction, traffic that would like to turn to link 2 is reduced to rC_2. Consequently, the flow on link 1 is $\frac{rC_2}{\psi}$.

After the recovery of the incident the flow (in G) is at maximum $\min\left\{\frac{C_2}{\psi}, \frac{C_3}{1-\psi}, C_1\right\}$. Using shock wave theory, the dynamics of the boundaries can be computed, and the traffic delay in each traffic state (using equation 1). The resulting equation is quite long, and therefore we show the result as graph (fig. 3). The delay increases more than proportional with the duration ΔT due to spillback effects. Fig. 3b shows which part of this delay is caused by spillback as function of the split fraction and the demand. This fraction raises to 1 in case that the demand approaches a critical demand where the inflow in G is the same as the outflow.



(a) The delay caused by the incident, $x^{\text{inc}} = 6km$.

(b) The contribution of the total delay that is encountered in G

Fig. 3. Delay for scenario with spillback – scenario 3

4 Implications

We will now derive an equation for the total delay and the average delay. First, we find an expression for the expected value of general attribute Y. So suppose: $Y = cx^2$. In this equation, c is a constant. The variation of x, $\text{Var}(x)$, is expressed as: $\text{Var}(x) = \langle x^2 \rangle - \langle x \rangle^2$. In these equations, the angle brackets mean the expectation value. The expectation value of Y can be written as: $\langle Y \rangle = \langle cx^2 \rangle = c \langle x^2 \rangle$. Combining the last two expressions gives: $\langle Y \rangle = c \langle x \rangle^2 + c\text{Var}(x)$. This shows that if attribute Y is proportional to x^2 the expectation value of Y can be derived from the expectation value of x and the variation of x.

In the situations without spillback, scenarios 1 and 2, the delay is proportional to the square of the duration. Then, using the above theory, the following holds:

$$\langle D \rangle = c\langle \Delta T \rangle^2 + c\text{Var}(\Delta T) = D(\langle \Delta T \rangle) + c\text{Var}\Delta T \tag{6}$$

with c the proportionality constants of eqs. 2 and 4. Note that these equations are derived for the case that a traffic jam occupies space. However, the resulting formulation in case there is no influence of a junction is the same as [5] derives, even with less strict assumptions.

In cases without spillback, the average delay per driver that encounters congestion is proportional to the duration eqs. 3 and 5). The variation of this average delay is: $\text{Var}(A) = \langle A^2 \rangle - \langle A \rangle^2$. We now substitute A by $c\Delta T$, in which c is the proportionality constant expressed in equation 3 or 5:

$$\begin{aligned}\text{Var}(A) &= \left\langle (c\Delta T)^2 \right\rangle - \langle c\Delta T \rangle^2 \\ &= c^2 \left(\left\langle (\Delta T)^2 \right\rangle - \langle \Delta T \rangle^2 \right) = c^2 \text{Var}(\Delta T)\end{aligned} \tag{7}$$

This expresses the variation of the delay for individual drivers, but only in case of no spillback.

5 Conclusions

This paper analyses the traffic states after the occurrence of an incident. Using shockwave theory, the traffic states that result from the incident are calculated. As opposed to a point queue model, the head and the tail of the queue are modeled separately and in this way the spatial extent of the queue is described properly.

We found that in the scenarios without the tail of the queue blocking the flow on other links (the first two scenarios), the delay is proportional to the square of the duration of the blocking. Because this is not linear in the duration, the expectation value of the delay is not the delay of the incident with the expectation value of the duration. An expression for the expected delay as a function of the variation of the duration of the blocking was formulated. We also formulated the variation of the average delay per involved traveler as function of the variation of the delay. The delay in case that the queue spills back to an upstream link can be expressed analytically but the result is more complicated and even stronger non-linear with respect to the duration. We therefore conclude that in order to analyse the delays in a network, it is needed to assess incidents with various durations.

References

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