
Empirical Differences between Time Mean Speed and Space Mean Speed

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Abstract Insight into traffic flow characteristics is often gained using local measurements. To determine macroscopic flow characteristics, time aggregation of microscopic information is required.

Usually, a data collection system stores values averaged over time. However, it is well known that a time mean average overestimates the influence of faster vehicles, and consequently overestimates the mean speed. As a direct result, densities, computed from flow and speed, are underestimated.

This paper compares the time mean speed and space mean speed, using data of individual car passages on a motorway road stretch. We show that the differences between time mean and space mean averages are substantial, up to a factor four. In particular in the lower speed regions the error is big. We indicate the considerable consequences for the jam density and shock wave speed. Finally, a fundamental diagram based on correctly averaged microscopic data can be fitted much better.

1 Introduction

The most common data available are aggregated (dual) loop detector counts. The loop detects the passage of a vehicle and its speed. The data are usually presented in aggregated values for a time period, for instance, 1 or 15 minutes.

It has been known that there is a difference between the space mean speeds and the time mean speeds [1, 2]. In studies, usually mean speeds are used, as opposite from space mean speeds. Helbing discusses the use and meaning of both mean speeds [3]. It is well known that traffic density is better computed from a space mean speed (e.g., [4]).

Rakha and Zhang [5] discuss the difference and possible conversions between the two mean speeds. Under the assumption of stationary road way conditions, the space mean speed equals the harmonic average of the time mean speed [6], which always is higher than the arrhythmic mean. This raises

the question how large the differences are between these two averages in reality. Van Lint [7] discusses the topic and finds:

$$v_T = \frac{\sigma_M^2}{v_M} + v_M \quad (1)$$

In this equation, v_T is the time mean speed, v_M the space mean speed and σ_M^2 the variance of the space mean speed. Note that it does not give a rule to compute space mean speeds out of time-mean measurements: σ_M^2 is unknown if one has local measurements.

The next section explains how space mean speeds can be approximated using speeds of individual vehicles. We apply this technique on a dataset, thus getting space mean speeds and time mean speeds. Then, we analyze the differences of applying both averaging techniques. We found that the averaging method has a big impact on the mean speeds, up to a factor 4. Using this in calculations to find the density, the difference becomes more important; we also show the consequences for the speed of the propagation of a traffic jam, and the shock wave speed.

2 From dual loop counts to density

There are many possibilities to measure road traffic. One of the most common methods uses induction loops that are situated in the road surface. Generally, the output of the detectors state the flow and speed aggregated per time period. The flow q and the vehicle density ρ are calculated from the number of passages n and the time interval t_{agg} using the equations:

$$q = \frac{n}{t_{agg}} \quad (2)$$

$$\rho = \frac{q}{v} \quad (3)$$

Eq. 3 will yield the correct, space averaged, density if the space mean speed is used. However, averaging the speed of vehicles $i = 1..n$ at one location will give the local mean speed v_L :

$$v_L = \langle v \rangle_L = \frac{1}{n} \sum_{i=1}^n v_i \quad (4)$$

Assuming stationary conditions, we can compute the space mean speed from local measurements, using a harmonic mean of the measurements [4]:

$$v_M = \langle v \rangle_M = \frac{1}{\frac{1}{n} \sum_{i=1}^n \frac{1}{v_i}} = \frac{1}{\langle \frac{1}{v} \rangle_L} \quad (5)$$

In this paragraph, we will indicate why this formula will give the space mean speed under stationary assumptions. A detector lies at location x_{det} . Now, let us reconstruct which vehicles will pass in the time of one aggregation period. For this, the vehicle must be closer to the detector than the distance it travels in the aggregation time t_{agg} :

$$x_{det} - x_i \leq t_{agg} \cdot v_i \quad (6)$$

In this formula, x is the vehicle position on the road. For faster vehicles, this distance is larger. Therefore, Eq. 4 overestimates the influence of the faster vehicles. This holds for every property of traffic, including speed. The computed average speed v_L , being biased to the property of the faster cars, is therefore higher than the space mean speed v_M . To compensate, one could attach a weight of $1/v_i$ to each measurement. This is stated below. After rewriting, it states the same as Eq. 5.

$$v_L = \langle v \rangle_L = \frac{\sum_{i=1}^n \frac{1}{v_i} v_i}{\sum_{i=1}^n \frac{1}{v_i}} = \frac{\sum_{i=1}^n 1}{\sum_{i=1}^n \frac{1}{v_i}} = \frac{n}{\sum_{i=1}^n \frac{1}{v_i}} = \frac{1}{\frac{1}{n} \sum_{i=1}^n \frac{1}{v_i}} \quad (7)$$

3 Data collection and analysis

This paper shows the differences in magnitude between the local mean speed and the space mean speed. It also gives the error for the density, which is derived from the speed. Finally, it discusses the consequences of taking the wrong mean for jam density and shock wave speed

For this research, a dataset of individual vehicle passages was used. The loop detectors are placed at the ring road of Amsterdam, a three lane motorway with a 100 km/h speed limit. Time and speed of each individual vehicle were recorded in the period 16 June 2005 to 11 July 2005. From these data we compute both the local mean speed, v_L , using Eq.4 and the space mean speed v_M using Eq. 5. The speeds are aggregated over the lanes.

We used different aggregation times (10 and 20 seconds and 1, 2, 5 and 15 minutes) to see the influence of the aggregation time. The necessary assumption that the speed profile does not change over time is more likely to hold over a shorter time. In order not to be influenced by changing traffic states, the aggregation periods both before and after a change were removed. We used 2 possible states (congested, uncongested), which were distinguished by the mean speed (under and over 70 km/h). When within 5 aggregation intervals the traffic state would change again, all intermediate aggregation periods were discarded.

3.1 Computed densities for the roadway

Fig. 1 reads the quotient of the time mean speed compared and the space mean speed for different speeds. It thus shows for different speeds how large the difference is between the two averages. We see for lower speeds, the two averages are more distinct. Using Eq. 1, this can only be explained by a low variation of speeds in the higher speed regions. For the same reason, the speed variation in congestion must be higher. Higher aggregation times make the differences grow, so the speed variation grows.

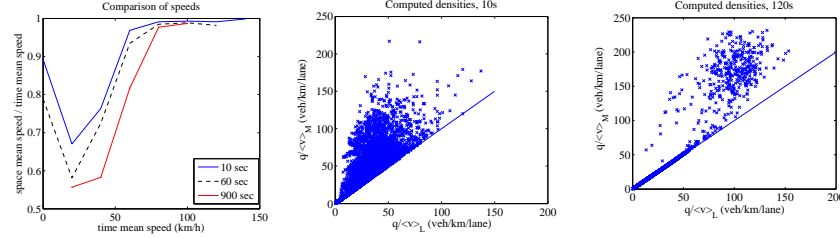


Fig. 1. (left) average speed difference (middle) densities, 10 s aggregation time (right) densities, measurements

We use flow q and sequentially speeds, v_L and v_M , in Eq. 3 to compute two different densities: $\rho_L = q/v_L$ and $\rho_M = q/v_M$. Fig. 1 shows both densities (ρ_M and ρ_L). The deviations from the line $x=y$ show the differences between the averaging methods.

The flows (Eq. 2) and densities given here are summed over the three lanes. When the aggregation time equals 10 seconds, the density q/v_S can be much (up to fourfold) higher than the estimation q/v_M . This is much higher than the results stated by Rakha and Zhang [5]. They already stated that the results differ per location. Since their measurements were performed in the USA, lower speed differences between trucks and cars are expected, and a lower variation causes a lower difference between time mean speed and space mean speed. Besides, in Europe overtaking is allowed only at the left, so vehicles in the left lane are faster than the right lane. Aggregation over the lanes causes than larger variations. Finally, the number of measurements (4 weeks in our case) results in more points, and therefore more extreme points.

In Fig. 2 we plot the flow versus the density (as illustration the one-minute data), both calculated using the time mean speed and calculated using the space mean speed. The figures show the cloud of measured points. The red line is fit to that cloud (the red line connecting them), according to the shape Wu proposed [8]. Each of the measured points is assigned to one of the branches [9]. It sometimes is unclear to which branch a point belongs, especially for the

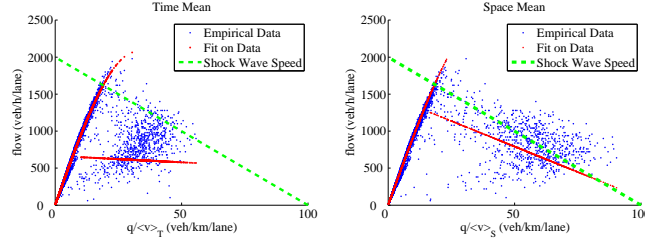


Fig. 2. (top) The fundamental diagrams constructed using time mean speed and space mean speed

lower time intervals. This confusion causes a bad fit for the time mean speed. The green line, the shock wave speed, will be discussed in section 3.2.

For each time period, we determine the flow that would be related to the computed density, according to the fit parameters. The errors between these flows and the measured flows are squared and averaged. The errors, stated in Tab. 1, show that using the space mean speed improves the fit.

Table 1. Results of fits of the fundamental diagrams for different aggregation times

Aggregation time	Error on fit	Error on fit	Jam Density	Jam Density
(s)	time mean ((km/h) ²)	space mean ((km/h) ²)	time mean (veh/km/lane)	space mean (veh/km/lane)
10	112	85	118	166
20	90	66	145	525
60	70	48	158	107
120	65	41	113	249
300	56	39	99	113
900	40	37	87	131

3.2 Consequences for macroscopic traffic models

Two different average speeds lead to two different densities, but the flows are equal. Consequently, the speed of the propagation of traffic jams, the shock wave speed (ω), is calculated differently, since Stokes law states:

$$\omega = \frac{\Delta q}{\Delta \rho} \quad (8)$$

The shock wave speed can also be derived directly from the measurements. The speed of the shockwave is approximately 20 km/h. According to Eq. 8, this speed should equal the slope of the congested branch of the fundamental diagram. The green line in the left plots of Fig. 2 illustrates this speed; note

that the line can be shifted and still indicate the same shockwave speed. It fits much better with the fundamental diagram using the space mean speed.

Furthermore, the buffer capacity of a road stretch is estimated completely different, see Table (1). Due to fit a wide cloud of points, outliers can occur (e.g., 525 veh/km, 249 veh/km). When disregarding the outliers, we see an underestimate of the jam density in almost all cases if a time-mean average is made. Using the space mean speed also gives more consistent results.

3.3 Conclusions and future research

In this contribution, we discussed two ways to average the speeds of passing vehicles: a local time mean average and a harmonic average, approximating the space-mean averages. Theory says that the space mean speed is to be used in the fundamental relation. Using empirical data, we showed from that the two different mean speeds might differ up to a factor 4.

We show that the space mean speed gives a better fit for a fundamental diagram. The estimates for both jam density and shock wave speed differ considerably between the two mean speeds. A better knowledge of these values can improve traffic models.

This paper assumes stationary traffic conditions. Using reliable trajectory data, one could compare the two mean speeds presented in this paper to a directly observed space mean speed. Aggregation times and aggregation lengths then are an interesting aspect.

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