Automatic fitting procedure for the fundamental diagram

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ABSTRACT
The fundamental diagram of a road, including free flow capacity and queue discharge rate, is very important for traffic engineering purposes. In the real word, most traffic measurements come from stationary loop detectors. This paper proposes a method to fit Wu's fundamental diagram to loop detector data. Wu's fundamental diagram is characterised by five parameters, being the free flow speed, wave speed, free flow capacity, queue discharge rate and jam density. The fits appear to be not very sensitive to the right value for the wave speed or the free flow speed. The proposed method therefore entails fixing these two parameters. The method consists of two steps. We first use a triangular fundamental diagram to separate the congested branch from the free flow branch. Then, the remaining three parameters of Wu's fundamental diagram are fitted on each of the branches using a least square fit. This method is shown to be robust for cases tested in real life, and hence very noisy, data.

INTRODUCTION

In traffic flow theory the fundamental diagram is an essential concept. The fundamental diagram relates two of the three variables average speed (v), flow (q) and density (k) to each other. If two of these variables are known, the third can be derived using the relation $q = kv$. Therefore, if only one variable is known, and the fundamental diagram is known, the traffic state can be determined.

Since Greenshields (*1*) showed the relation between speed and density, many theories have been developed to work with the fundamental diagram, including the often used LWR model (*2, 3*).

Although many papers are written on the shape of the fundamental diagram (see section 3.1), it is remarkable how much spread is present if one plots observed flows versus observed densities. This spread can have different causes, for instance:

1. The traffic is not stationary during the aggregation interval.
2. The traffic itself is heterogeneous.
3. There are detector failures, or other measurement errors.
4. Within the aggregation interval an integer number of vehicles is measured.
5. The speed is calculated as time mean speed instead of space mean speed.
6. Intrinsic measurement errors occur, for instance loops do not detect vehicles standing still (speed 0 km/h).

Although solutions have been found for several of these problems, most often traffic engineers still use loop detector data from freeways, with the abovementioned problems. If they want to apply theoretical concepts such as shock wave theory or the method of characteristics, they need to have a fundamental diagram. The parameters of the fundamental diagram should be calibrated for the observed road, as each road has its own characteristics, leading to a unique fundamental diagram.

Traffic control measures, such as ramp metering (*4*) or main lane metering (*5*) are exploiting the capacity drop, i.e. the difference between free flow capacity and the queue discharge rate. In order to find the right settings for these measures, it hence is required to find a reliable value for both values. For consistency of the results and calculation speed (if various locations are involved), an automatic fitting procedure is useful. However, due to errors in the data automatic fitting procedures often lead to extreme values for capacity or jam density, as will be shown later in the paper. This paper presents a method which can be applied in an automated way, preventing extreme values on noisy data – a property we will call robustness in the remainder of the paper. The key of the method proposed here (see also figure 4) is that first a triangular fundamental diagram is fitted to find the critical density, used to separate the free flow and the congested branch. Then, a more realistic fundamental diagram is fitted on the congested data and the free flow data seperately. Variables which are known are fixed, thereby impoving the robustness of the fit.

This paper first gives an overview of the literature on the fundamental diagram and the fitting procedures. Then, the experimental setup is introduced, showing the different choices in the research approach. This setup also includes the simulation setup and a description of the empirical data used to test the methodology. Then, the methodology is introduced and the simulation results are discussed, followed by a section presenting the fitting results on empirical data. Finally, we discuss the results and present the conclusions.

LITERATURE REVIEW

This section first describes the general shapes of the flow-density relationship, followed by a detailed discussion of the shape proposed by Wu . Then, the existing literature of fitting a functional form to this relationship is discussed.

Shapes of the fundamental diagram

Many shapes have been proposed for the fundamental diagram, ranging from very simple functions to functions with mathematically useful properties, functions including the microscopic traffic characteristics or relations based on empirical findings.

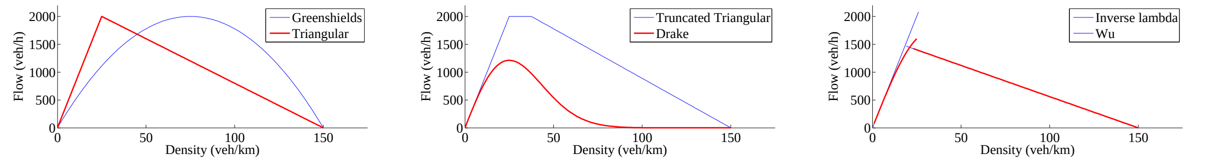
Greenshields (*1*) was the first to observe traffic flows and to hypothesize a linear relationship between speed and density. Applying the relation

$$q = kv \tag{1}$$

the linear relation between speed v and density k corresponds to a parabolic relation between speed v and flow q. The fundamental diagram defines a number of characteristics of the traffic flow. First of all, it determines the capacity, or the maximum flow that can be maintained for a short period of time. The corresponding density is the so-called critical density. This capacity distinguishes two states or regimes: for densities lower than the critical density, traffic is in an uncongested state, while for higher densities, traffic is in a congested state. The Greenshields fundamental diagram is called a univariate model, since both the uncongested regime and the congested regime are described using the same formula. Another well known univariate model is Drake's model (*6*), where the speed is an exponentially decreasing function of the density.

Two-variate models use distinct formulas to describe the congested and the uncongested regime. The simplest two-variate model is a triangular fundamental diagram, that describes both regimes with a straight relationship. The slope of the uncongested regime is equal to the free speed, while the slope of the congested regime equals the shock wave speed (describing the speed with which the congestion front is moving upstream). The latter shows a direct relation between the fundamental diagram and shockwave theory (*2*). Daganzo (*7*) introduced the truncated triangular fundamental diagram. Here, the capacity is not only reached at the critical density, but for a certain range of densities the flow is constant, and maximal. Edie (*8*) was among the first researchers to show that a discontinuous relation might be more appropriate to describe traffic dynamics. He distinguished the regime of the free traffic, with the so-called free flow capacity as maximum flow, and the congested traffic, having the so-called queue discharge rate as maximum flow. As the curve is discontinuous, free flow capacity is higher than the queue discharge rate. This is called the capacity drop (*9*). Koshi et al. (*10*) introduced an inverse lambda shaped fundamental diagram, which included this capacity drop. In this diagram, traffic has a free speed up to a capacity point. However, the congested branch does not start at capacity, but connects below the capacity point to the free flow branch. An addition to the inverse lambda shaped fundamental diagram is made by Wu (*11*). He assumes that the speed in the free flow branch is not constant, but decreasing with increasing density. The shape of the free flow branch is determined by the overtaking opportunities, which depend on the number of lanes. Figure 1 visulises the density-flow and density-speed diagram mentioned above.

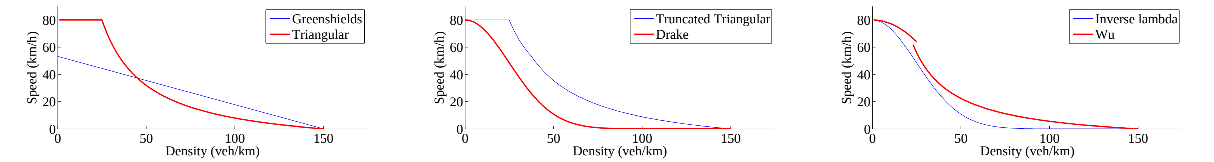
Kerner (*12*) proposed a different traffic flow theory, the so-called three phase traffic flow theory. Instead of distinguishing an uncongested regime and a congested regime, Kerner describes three phases of traffic. In addition to the uncongested regime, the congested regime has been split



(a) Density-flow Greenshields and triangular

(b) Density-flow truncated triangular and Drake

(c) Density-flow Inverse Lambda and Wu



(d) Density-speed Greenshields and triangular

(e) Density-speed truncated triangular and Drake

(f) Density-speed Inverse Lambda and Wu

FIGURE 1 Different shapes of the fundamental diagram

into two distinct phases: synchronised flow and wide moving jam. In a wide moving jam traffic stands still and the wave moves upstream (similar to stop-and-go waves); in synchronised flow, the traffic is moving. Contrary to other researchers, Kerner claims that in this state several equlibrium distances exist for the same speed, leading to an area rather than a line in the flow-density plane.

So far, the described fundamental diagrams are all deterministic. Recent research (*13*) has tried to explicitly include stochastic aspects in the fundamental diagram.

Each of the fundamental diagrams mentioned above has its advantages and disadvantages. The existence of a capacity drop requires the fundamental diagram to have at least two regimes: a free flow regime and a congested regime. In free flow, the most realistic model is that speed decreases with increasing densities, implying a non-linear free flow branch. Taking these two requirements as a starting point, we have chosen to fit Wu's fundamental diagram to our data.

Wu's fundamental diagram

The fundamental diagram by Wu (*11*) is based on the overtaking possibilities. Those differ for roads with a variable number of lanes. Wu (*11*) defines a fundamental diagram by prescribing the speed as function of density. He argues that the speed changes gradually from the free flow speed v_{free} to the critical speed v_{crit}. The way it decreases depends on the number of lanes, N. In an equation, this gives:

$$v_{\text{free flow branch}}(k) = v_{\text{free}}(v_{\text{free}}v_{\text{crit}})\left(\frac{k}{k_{\text{crit}}}\right)^{N-1} \tag{2}$$

Multiplying both sides with the density gives the relation between the flow (speed times flow is density: equation 1) and the density. For the congested branch, Wu proposes a straight line in the flow-density relationship, starting at a lower flow than the free flow, hence incorporating a capacity drop. It can be easily verified that these equations can be rewritten, and different parameters can be chosen to represent the same shape. For this paper, we rewrote the equations such that the parameters of the fundamental diagram were as meaningful as possible. Note that rewriting the equations or changing variables does not change the relationship, nor the parameter values. Table 1 shows the variables and their default values being used for this paper.

TABLE 1 The parameters of Wu's fundamental diagram and their default values

name	symbol	default value
Free flow speed	v_{free}	115 km/h
Wave speed	w	-18 km/h
Free flow capacity	C_f	2400 veh/h
Queue discharge rate	C_q	1800 veh/h
Jam density	k_{jam}	150 veh/km/lane

Algorithms to fit a fundamental diagram

Although the shape of the fundamental diagram is under discussion, not much literature can be found on how to estimate a fundamental diagram. More researchers discuss which data to use for such an estimation.

Typically, fundamental diagrams are estimated from aggregated data at a specific location (*14, 15*). Depending on the type of detectors used to derive these data, occupancy or densities are observed. In addition, speeds can be averaged over time or harmonically averaged. One of the drawbacks is the existence of transient phases, which are difficult to distinguish from equilibrium phases. Only the latter should be used to derive the fundamental diagram, as this describes traffic in an equilibrium state. When using the fundamental diagram to predict how traffic states propagate (both in time and in space), the estimation should be based on spatial measurements (in accordance to Edie's definition of equilibrium). As detectors only collect data at a single cross-section, probe vehicles can be used (*16*). Chiabaut et al. (*17*) use the passing rate to estimate a fundamental diagram, as this is independent of the traffic flow state. Using NGsim data, they prove that a linear fundamental diagram can be used to describe the traffic and that both the congested wave speed and the jam density can be accurately predicted.

Although literature clearly has shown drawbacks of detector data, this is the type of data typically available. Therefore, we have developed a method to estimate the fundamental diagram based on detector data.

EXPERIMENTAL SETUP

This section introduces the different parameters to be fit and which combinations of these parameters we compare. Also, the setup of the simulation to be used to assess the methodology is elaborated upon, and some details are given on the empirical data we use for evaluation.

Influencing elements in fitting a fundamental diagram

In the process of the fitting of the fundamental diagram several aspects play a role. We can fix parameters , that is, not all the parameter values need to be estimated. Also, different speed averages might lead to different results. Moreover, the stochasiticy of real traffic might be different from a traffic simulation. This section comments on these variables , while in the next section it is indicated which combinations are tested.

Fixing parameters

Fixing one or more parameters will increase the robustness of a fit, simply because one can set the parameter to a value which must be approximately right. Thereby, it is avoided that a variable is set to an extreme value because another variable is wrong.

Wu's fundamental diagram has five parameters, of which 2 are the capacities which are to be estimated. The remaining 3 are the free flow speed, the wave speed and the jam density. The

TABLE 2 The analyses performed

Par fixed	Data source	Type of data	Results in
-	Simulation	Detector, arithmetic mean	figure 7(a)
v_{free} and w	Simulation	Detector, arithmetic mean	figure 7(c)
v_{free}	Simulation	Detector, arithmetic mean	figure 7(d)
w	Simulation	Detector, arithmetic mean	figure 7(e)
-	Simulation	Edie	table 3
v_{free} and w	Simulation	Edie	table 3
-	Simulation	Detector, harmonic mean	figure 7(f)
v_{free} and w	Simulation	Detector, harmonic mean	figure 7(g)
w	Simulation	Detector, harmonic mean	figure 7(h)
-	Real	Detector, arithmetic mean	figure 7(i)
v_{free} and w	Real	Detector, arithmetic mean	figure 7(j)
w	Real	Detector, arithmetic mean	figure 7(k)

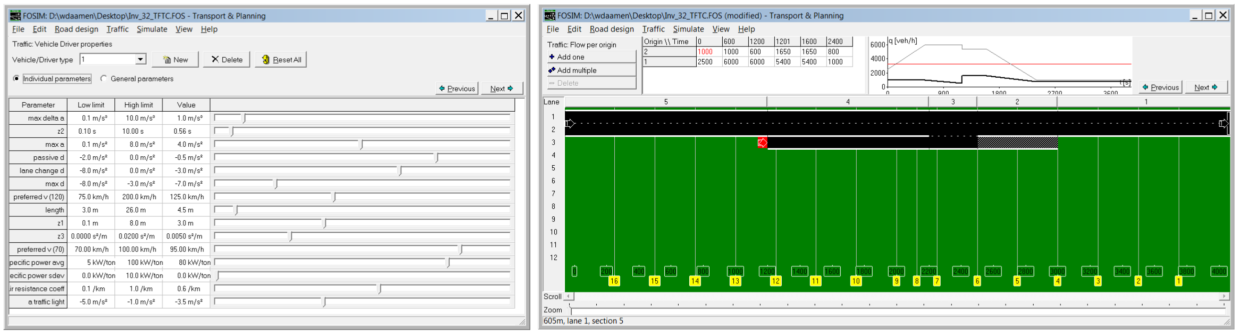
jam density is most constant, and can be determined by other methods as well (e.g., (*17, 18*)). Also the free flow speed can relatively easy be estimated based on the speed limit. The jam density is more difficult to estimate on beforehand, and this will not be considered in the tests here. Fixed wave speed and/or free flow speed will be considered.

Type of data in relation to speed averaging
Some of the problems when determining the fundamental diagram are caused by the fact that the reported mean speeds are time mean speeds. For the density calculation by $k = q/v$ (eq. 1) one needs the space mean speed, or as approximation the harmonically averaged speed. Figure 6(a) shows the differences of these different density calculations for the simulations. Still, high density areas hardly occur, but the congested conditions are better represented in the diagram. This has to do with the fact that the congested states are more affected by the speed averaging procedure (*19*).

Another cause of problems when determining the fundamental diagram is the fact that the measurements are taken at one location. From the simulation, also the full trajectories are known. That means that we can apply Edie's definitions (*8*). In particular, we divide the road into segments of 100 meters and consider time intervals of $10s$, leading to an area in space and time $A = 100 * 10$=1000 ms. For these rectangles in space and time, the total distance travelled D by all vehicles and the total time spent in the area T can be calculated. The flow and density then are calculated by $q = D/A$ and $k = T/A$. In this paper we will look at the consequences of using detector data or Edie's definitions based on the full trajectories.

Real world data or simulation data
In the paper, first the method is applied to simulation data. The advantage of using data from a microscopic simulation program is that the ground truth (i.e., the underlying fundamental diagram) is known, and there are no measurement errors. Moreover, all type of data (time mean speeds, harmonically mean speeds, Edie's generalised density using trajectories) can be obtained from the simulation. The simulation setup is chosen simple in order to test the properties of the proposed methodology. Later, the same method is applied to real world data.

Analyses
The section above indicated several combinations of influencing elements in the fitting procedure. Not all combinations are tested in this paper. Table 2 shows the analyses we chose for this paper.



(a) Parameters (b) Road layout

FIGURE 2 Fosim simulation setup

The selection we made is based on a combination of simulation data and real data, but the emphasis is on simulation data. The main reason is that the ground truth (theoretical fundamental diagram) is known for these simulation data Moreover, we can get arithmetic mean speeds and harmonic mean speeds at (virtual) detectors as well as Edie's data from the trajectories. So, in short, all information is available to perform extensive tests on the proposed method.

As said, fixing of parameters might increase the robustness of the fitting results, so we will consider fixing the free flow speed and/or the wave speed. As will turn out, especially fitting the congested part is difficult, particularly if only the arithmetic mean speed is known. Therefore, for real data only fixed wave speed is considered.

Simulation

In this section we discuss the various elements of the simulation, such as the simulation model and the setup of the simulation.

Simulation program

The methodology is tested using simulation data. The advantage of traffic simulation is that the ground truth is known (see figure 3(a)), although lane changing effects are not taken into account. To generate simulation data we use Fosim (*20*). Fosim is the only simulation model that has been validated for Dutch freeways, and it is used by the Dutch road authority for ex-ante capacity assessments for new infrastructure (*21*).

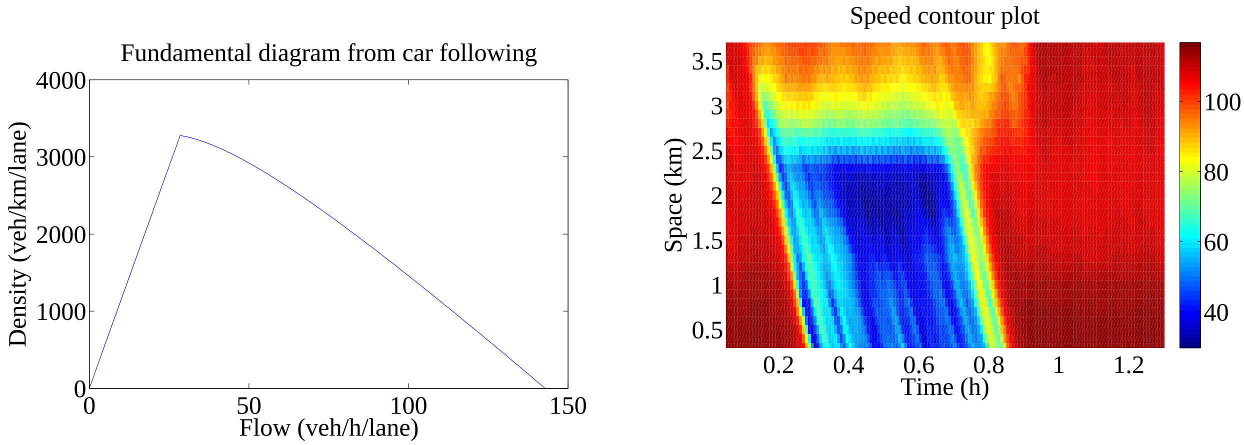
Fosim implements a Wiedeman principle (*22*) in the car-following model. The equilibrium spacing drivers prefer is a quadratic function of their speed:

$$s = z_3 v^2 + z_2 v + z_1 \tag{3}$$

Fosim implements by default two user classes: passenger cars and Heavy Good Vehicles (HGV). For passenger cars, three different types of drivers are distinguished: aggressive drivers, more timid drivers and drivers driving vans. For each of these driver-vehicle combinations, input parameters are determined (see the screenshot in figure 2(a)).

Simulation setup

For the sake of simplicity and traceability, we only consider a homogeneous flow of timid drivers in passenger cars in the simulation. We have used the default (thus validated) parameters for this



(a) Flow density diagram

(b) Speed contour plot

FIGURE 3 The traffic characteristics in Fosim simulation model

vehicle class. For our driver population, this means that the parameter values for z_1, z_2, and z_3 are respectively 3m, 0.72s and 0.0050s^2/m. This, together with the vehicle length of $4m$ gives the speed spacing diagram, which can be translated into a flow-density diagram (see figure 3(a)). Note that this theoretical calculation does not take stochasticity into account. The resulting capacity from this relationship seems to be rather high, at a value of approximately 3275 veh/h/lane, equalling a headway of 1.1s. This is the theoretical capacity in case all vehicles are following (in both lanes). These flow values are being observed in the Netherlands for passenger cars only in the left lane. For instance, on the A4 section between The Hague and Leiden (see the section on fitting results on empirical data) we find a maximum flow in the left lane of 56 vehicles in one minute, equalling 3360 veh/h.

The case study we consider for estimating the fundamental diagram is as simple as possible. We consider a two lane freeway with an onramp, see figure 2(b). The top of this figure shows the demands, both for the main road (origin 2), and for the onramp (origin 1). With these demands the capacity of the road is temporarily exceeded. First, a stop-and-go wave is created by increasing the flow on the main road, where the head of the congestion is just downstream of the end of the onramp. Secondly, bottleneck congestion is created by a higher inflow from the ramp and a lower inflow from the main road.

We have collected speeds, flows and density at detectors, located 250m apart, see the yellow boxes in figure 2(b). As these detectors observe passing moments of vehicles per lane, only data from the two main lanes are used for the estimation of the fundamental diagram. As merging occurs at the merging lane, causing disturbance of the process, we have excluded detectors 6 and 7.

In Fosim, simulations often lead to congestion where vehicles come to a complete stop. Also, passenger cars accelerate at a high rate, thus not passing detectors at a low speed. As mentioned, when traffic is completely stationary (standing still), no vehicles pass the detector, and no traffic is observed. So the only traffic that is being observed is the relatively fast moving – because quickly accelerating – traffic. To overcome these problems , we create other types of bottlenecks as well. Instead of creating an infrastructure bottleneck, we halve the free speeds. This way, we

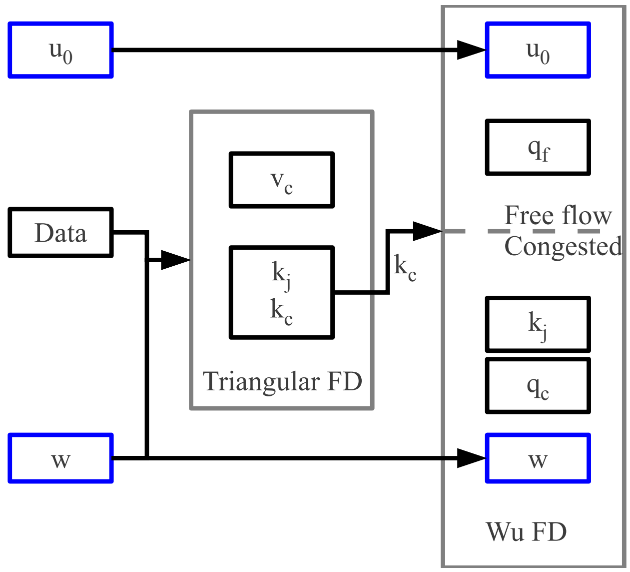


FIGURE 4 The overview of the fitting procedure

prevent vehicles from standing still, and thus we prevent the introduction of model bias into our data set.

Experimental data

The method proposed in this paper is also applied on data from the A4 freeway in the Netherlands between Amsterdam and Rotterdam. The stretch we consider is 10km long (from Leiden to The Hague). On this freeway only loop detector data are being collected, consisting of flows and time mean speeds.

FITTING METHODOLOGY

The main innovation of the method proposed in this paper is that we first use a triangular fundamental diagram to separate the congested branch from the free flow branch. Then, Wu's fundamental diagram is fitted on each of the branches. A graphical overview of the method is presented in figure 4. In this fitting process, some variables can be fixed. In this figure, it is indicated that the wave speed and the free flow speed are taken fixed, which will turn out to give the most robust fitting results.

Use of data points

We assign a weight p to each of the data points based on their reliability. This weight is not constant, as data points that can be trusted should contribute more to the quality of the fit than unreliable points and outliers. The following weights are assigned:

- Speeds under 10 m/s cannot be trusted. It would be very rare to have situations where the speed is constantly 10 km/h. Therefore, this will be a mixture of states, in which only the moving vehicles are measured. Due to the nature of local measurements, vehicles standing still will not be measured at all. Moreover, the approximation of the space mean speed by time mean speed will be worse when the spread of (measured) speeds is higher. Therefore, we give measurements with these low speeds a lower weight. In order to get a

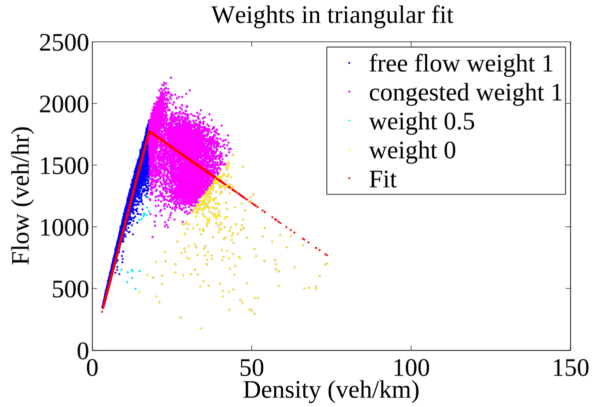


FIGURE 5 The triangular fit

robust fit, we do not want to pre-specify the speed which is too low, but we would like to relate that to the speeds resulting from the fundamental diagram. If the weight would be zero, the fitting procedure benefits from choosing a very high free speed, thereby ignoring all aggregation intervals with a lower free speed. Since this is undesirable, a weight of 0.5 is given to these points.

- The measurements are taken at a freeway. Therefore, in low density conditions we expect high speeds. Consequently, low speeds at low densities are a measurement error. This could be due to the fact that speeds are time mean speeds, which distorts the speed and hence the estimation of density. This means these data points are unreliable. If the speeds are below 20 m/s in free flow, the points are neglected.

Performance measures and optimization

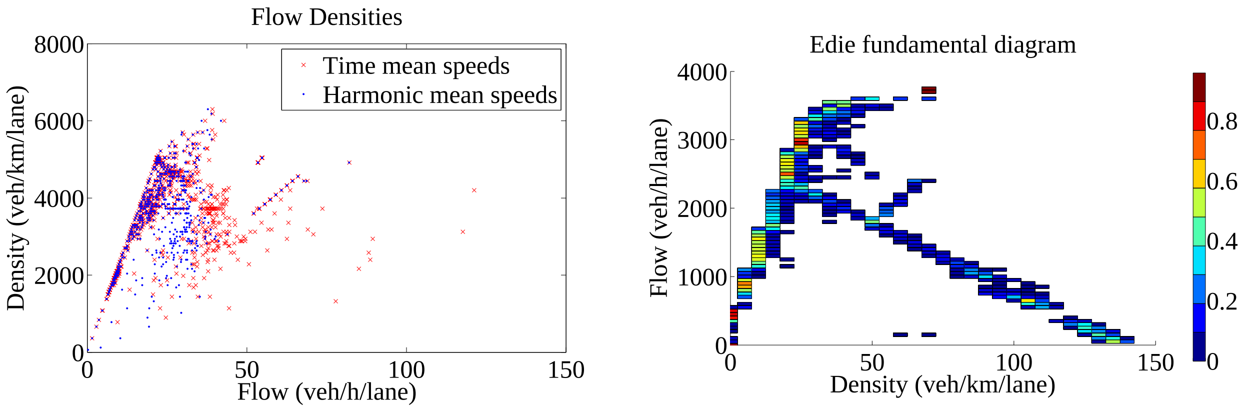
The fit will give a predicted flow ($\tilde{q}$) based on the measured density k_i. The error for a data point (one observation of flow and density) can be calculated as the difference between the observed flow q and the predicted flow $\tilde{q}$. As error of the fit we consider the weighted root mean square error of all data points, indicated by ϵ:

$$\epsilon = \sqrt{\frac{\sum_I p_i \left(\tilde{q}_i - q_i\right)^2}{\sum_I p_i}} \tag{4}$$

The goal of the optimization is to minimize this error.

Fitting a triangular fundamental diagram

First, we will estimate the triangular fundamental diagram. This will give a robust estimation of the critical density. That critical density will be used later on to separate the free flow branch from the congested branch. The separation of the two branches is made based on density. A separation on speed is less reliable. The fitting procedure could end up fitting a high free flow speed, which fit data points with the high speeds very well. The other points, which one would assign to the free flow branch, would be moved to the congested branch, which has a higher spread anyway, and therefore would fit in the line of expectation.



(a) The points in the fundamental diagram for time mean speeds and harmonically averaged speed

(b) Fundamental diagram according to Edie's definitions

FIGURE 6 Observations of combinations of speed and density using different (speed) definitions

This fundamental diagram has three degrees of freedom: the free flow speed, the critical density and the jam density. Note that the capacity is determined by the free flow speed times the critical density. In our approach the shock wave speed is fixed, so two degrees of freedom remain: the free flow speed and either the critical density or the jam density, which is for fitting purposes equivalent.

The result of this fitting procedure is a triangular fundamental diagram. This triangular fundamental diagram will give the critical density which is input to the fitting procedure of the Wu fundamental diagram (below).

Fitting Wu's fundamental diagram

For the same reasons as above, we assume a fixed shock wave speed w. Moreover, for freeway driving, we can assume a fixed free speed. This is because it does hardly influence capacity, and it is relatively constant for freeways. In this paper, we set the free flow speed u_0 to 30 m/s.

The critical density is used to split the data into free flow points and congested data points. There might be an error in the estimation of the critical density. Therefore, data points with a density between 90% and 120% of the estimated critical density from the triangular fit are not taken into account. If one considers the fundamental diagram, this is generally not an issue: the fitting of the diagram can be done on the "slopes" of the fundamental diagram. The points near the top of the diagram can belong to both branches, thus introducing an error in the fit, but these points are not necessary to find the slopes of the congested branch and the free flow branch respectively.

The remaining parameters of the fundamental diagram which need to be estimated are: (1) the free flow capacity, (2) the queue discharge rate and (3) the jam density (see figure 4).

SIMULATION RESULTS

This section presents the results of the fitting procedure. We start showing the fundamental diagram using Edie's definitions. Then, the estimation results when different parameter values are fixed in order to see the sensitivity of each parameter are shown. Finally, the improvement of the fit when using harmonic mean speeds is identified.

TABLE 3 The value of the parameter estimates

	Detector (arithm. mean speed)				Edie			Harmonic mean		
Fixed parameters	-	v_{free}&w	v_{free}	w	readout	-	v_{free}&w	-	v_{free}&w	w
v_{free} (km/h)	126	[115]	[115]	121	100	116	[115]	126	[115]	
w (km/h)	-4.6	[-18]	-8.2	[-18]	-22	-3.6	[-18]	-5.7	[-18]	[-18]
C_f (veh/h/lane)	2789	4015	3880	3311	3400	4919	5942	2949	4126	3264
C_q (veh/h/lane)	2039	2149	2069	2098	2250	2460	2593	2169	2286	2258
k_{jam} (veh/km/lane)	477	144	276	144	135	709	171	410	153	153

Edie's definitions

The fundamental diagram using the points obtained by Edie's definitions can be visualized by the proportion of observations for a combination of density and flow. This is done in figure 6(b). The line of the fundamental diagram agrees to a large extent with the theoretical lines; however, the diagram constructed from the trajectories shows a large capacity drop, while this is not present in the theoretical curve. This is because the theoretical curve is only based on car-following behavior, and does not take into account that drivers might leave a larger gap than desired when leaving the queue.

Fix various parameters

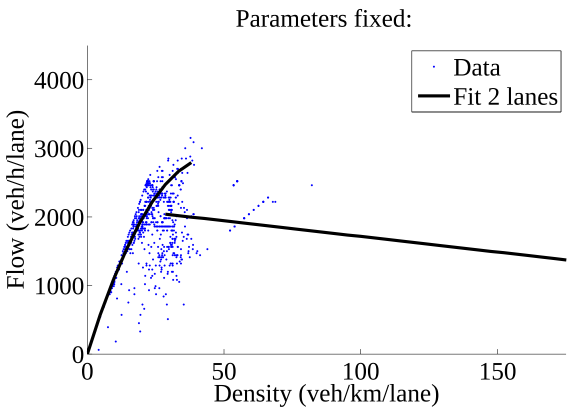
This section describes what happens to the fit of the fundamental diagram if some of the parameters are fixed, while applying the proposed methodology. Table 3 presents the results for the different parameters values, as well as the values for these parameters as derived from the graph using Edie's definitions. For Edie's definitions, we read the parameter values for the ground truth from the figure; this is possible because the figure give very clear lines.

The table shows large differences between the different fits. The graphical representation is also shown in figure 7. Figure 7(a) shows the curve if no parameters are fixed. The jam density is estimated at 477 veh/km/lane, which is much higher than the approximately 145 veh/km/lane which follows from theory and the analysis of trajectories using Edie's definitions. Figure 7(b) shows the fundamental diagram with the free flow speed and the wave speed being fixed. The shape and parameters matches relatively well with the shape of fundamental diagram from Edie's definitions, although the free flow capacity is higher. In fact, it seems that in the fundamental diagram according to Edie's definitions the speed decreases strongly just before the free flow capacity point is reached. Therefore, a comparable fit of the shape of Wu leads to a higher capacity point Given these intrinsic shortcomings, the fit is reasonably well.

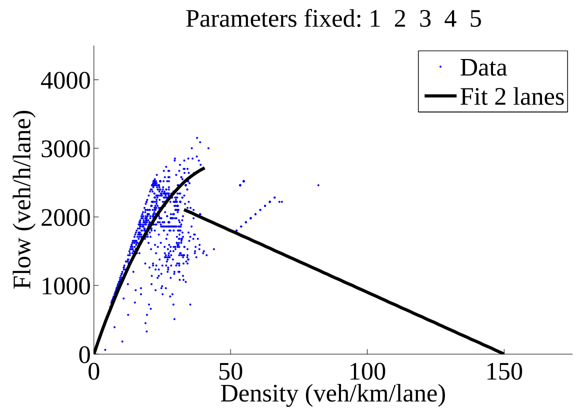
It is of course a disadvantage to fix both the free flow speed and the wave speed. Therefore, it is also checked whether the same results can be obtained by fixing one of these. Figure 7(d) shows that fixing only the free flow speed does not alleviate this problem, but figure 7(e) shows that fixing only the wave speed does.

Effect of the speed averaging

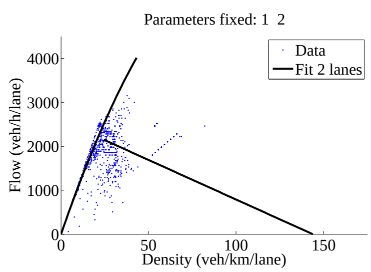
As mentioned before, speed averaging has an effect on the points in the fundamental diagram, particularly on the congested branch. Figure 7(f) shows the fit of the flow-density diagram where the density is computed using the harmonic mean speed and all parameters are estimated. In figure 7(g) the free flow speed and the wave speed have been fixed in the fit. Overall, the density can be better estimated when the density is based on the harmonic mean speed. However, as the amount of points along the congested branch is very limited, not much information is known on



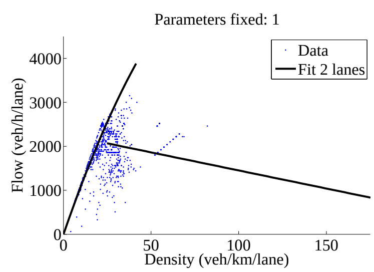
(a) All free



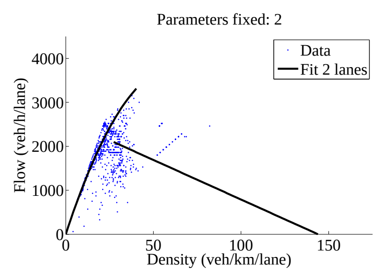
(b) All fixed



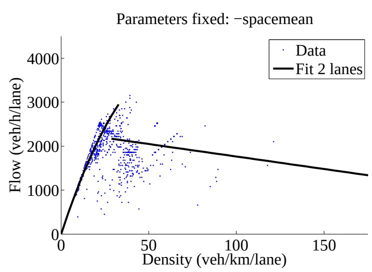
(c) The proposed method: fix the free flow speed and the wave speed



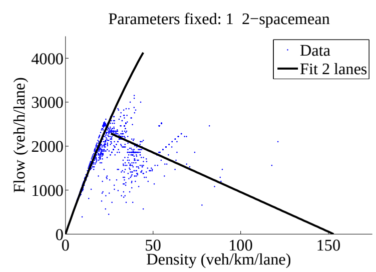
(d) Fix the free flow speed



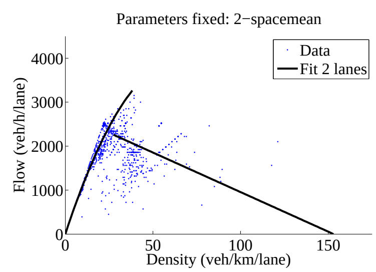
(e) Fix the wave speed



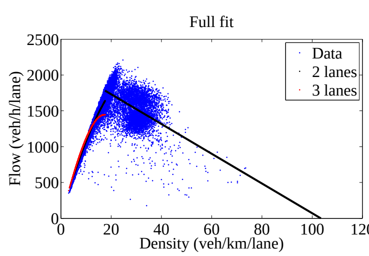
(f) All parameters have been estimated on harmonically averaged speeds



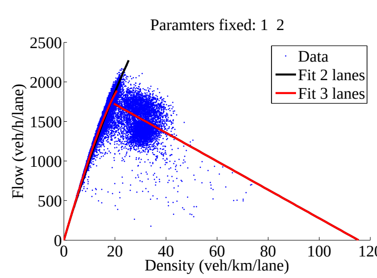
(g) The proposed method: free flow speed and wave speed have been fixed, using harmonocally averaged speed



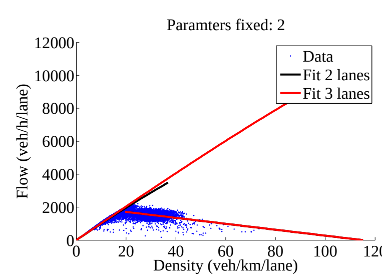
(h) Only the wave speed fixed, using harmonocally averaged speed



(i) All fixed



(j) The proposed method: fix the free flow speed and the wave speed



(k) Fix the wave speed

FIGURE 7 Fits of the fundamental diagrams on simulation data (a-h) and real data (i-k)

the shape of the congested branch. This is due to the very low speeds at which these points are located. The fit with all 5 parameters estimated still has a very large estimated jam density of 410 veh/km/lane, hence this method is unfeasible for practice. This can be improved by fixing the wave speed, and possibly also the free flow speed. Especially the method only fixing the wave speed yields good results (see table 3), with a closely matching free flow capacity and queue discharge rate.

APPLYING THE METHOD TO EMPIRICAL DATA

Figure 7(j) shows the fitting results on the real data. First, figure 7(i) shows what happens if all five parameters of Wu's fundamental diagram are fitted simultaneously. This leads to a fit which visually does not seem to follow the data points. Moreover, the free flow capacity is estimated to be(much) lower than the queue discharge rate. If we fix the wave speed and the free flow speed (see figure 7(j)), the fit follows the expected shape of the fundamental diagram better. However, the capacity values are slightly lower than the capacities indicated in the Dutch version of the Highway Capacity Manual (*23*). The fitted capacities are 1890 veh/h/lane for the three lane section and 2270 veh/h/lane for the two lane section, where the Dutch Highway Capacity Manual indicates values of 2100 veh/h/lane for both situations. Also the data points seem to point at a slightly higher capacity. However, for design purposes, a slight underestimation of the capacity is better than an overestimation. The queue discharge rate is estimated at 1725 veh/h/lane.

Fixing only the wave speed, which gives a good fit with simulation results, is not working for the case with real data, since the free flow capacity will be given an extreme value (see figure 7(k)).

We can thus conclude that the method fixing the wave speed and the free flow speed can also be used to fit a fundamental diagram on empirical data.

CONCLUSIONS

This paper presents a method to fit a fundamental diagram to traffic data, thereby estimating the free flow capacity and queue discharge rate. Difficulty is that points near the capacity can be assigned to either of the two branches, which makes the fitting process difficult, leading to extreme values. We propose to first separate the free flow branch from the congested branch by a triangular fit. Then, the free flow speed and the wave speed are fixed, thereby limiting extreme values. The method proved to yield reasonable, though not perfect, results on simulation data. Also for more noisy real world data, the method gives realistic capacity estimates.

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