

Capacity Drops at Merges: New Analytical Investigations

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Abstract— This paper focuses on the derivation of analytical formulae to estimate the effective capacity at freeway merges. It extends previous works by proposing a generic framework able to account for a refined description of the physical interactions between upstream waves and downstream voids created by inserting vehicles within the merge area. The provided analytical formulae permits to directly and accurately compute the capacity values when the merge is self-active, i.e. when both upstream roads are congested while downstream traffic conditions are free-flow.

Index Terms—Active Bottleneck, Capacity Drop, Freeway Merge, Kinematic Wave, Heterogeneous Vehicles.

I. INTRODUCTION

Determining the effective merge capacity, i.e. the maximal flow that can be observed just downstream of freeway merges, is crucial for traffic operations. This is not only important for simulation purpose but also to develop better control strategies. Effective capacity is referred in some papers as the queue discharge rate. Experimental findings show that capacity drops are often observed at merges even if downstream traffic conditions are in free-flow, e.g. [1]–[5]. The magnitude of the capacity drops is mentioned to be between 10 to 30% of the maximal observed flow. The main physical explanations for such a phenomenon are lower speeds for merging vehicles combined with bounded acceleration, e.g. [6]–[9], and the impacts of driver behaviors, e.g. [10]–[12]. In a nutshell, slower vehicles create voids in front of them that locally reduce the available capacity and lead to temporal flow restrictions.

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Except for direct experimental observations, the most common way to determine the effective merge capacity is to use a traffic model able to reproduce the underlying physical mechanisms, e.g. [9], [13]. This requires running a simulation for every new set of parameters and is not really convenient when looking for a first and quick approximation of how a merge behaves or to determine which parameters are the most influential, e.g. for sensitivity analysis. To the authors’ knowledge, [14] is the only attempt to derive an analytical expression that explicitly relates the effective capacity to the different parameters. This expression is derived by considering that inserting vehicles act as moving bottlenecks [15], [16] with bounded acceleration while mainstream vehicles behave according to the kinematic wave model [17], [18] with a triangular fundamental diagram. The central point of this contribution is to handle the interactions between moving bottlenecks when vehicles insert at different location along the on-ramp.

This first attempt has two main shortcomings. First, vehicle characteristics are supposed homogeneous, i.e. same acceleration, same jam spacing... Second, interactions of traffic waves with voids created downstream of moving bottlenecks are neglected. This paper proposes new analytical investigations able to tackle these two shortcomings. As a major result an updated expression for the effective capacity, (5) in [14], will be established. In this paper, we will assume that both the on-ramp and the freeway are congested. Reference [14] provides all the materials to extend the results to situations when the on-ramp is in free-flow. Furthermore, we will consider that the inserting flow q_0 is given when calculating the merge effective capacity C . One more time, the major challenge is to update (5) in [14]. Then, all methodology already presented in [14] can be directly applied. Notably, when the merge ratio α is given [19], q_0 can be derived by solving $(1+1/\alpha)q_0=C(q_0)$. This provides both equilibrium traffic states upstream of a self-active merge, i.e. when the congestion is not coming from downstream. Finally, note that we will restrict our investigations here to a one-lane freeway. Extensions to multi-lane freeways have already been discussed in [14].

This paper is organized as follow: section II proposes a generic expression for the effective capacity. Section III deals with proper consideration of voids downstream of moving bottlenecks. The question of heterogeneous vehicle characteristics can be addressed in the same framework but will only be described in an extended version of this paper

that will be submitted for consideration in a special issue of the *IEEE Transactions on Intelligent Transportation Systems* related the ITSC14 conference. The main work in section III is to derive the characteristics of the statistical distributions that appear in the generic expression. The main challenge is to maintain full analytical tractability. Analytical expressions will be compared to numerical simulations to test the relevance of the required approximations. Finally, section IV presents a brief discussion.

II. GENERIC EXPRESSION FOR THE EFFECTIVE CAPACITY

Consider a merge with two one-lane roads. Vehicle i inserts from the on-ramp at time t_i and location x_i ($0 \leq x_i \leq L$), where L is the length of the insertion lane, see Fig. 1a. The time headway $h_i = t_{i+1} - t_i$ between two successive insertions follows an unknown distribution $H(h_0, s_H)$ with mean $h_0 = 1/q_0$ and standard deviation s_H . Inserting vehicles are considered as moving bottlenecks [15], [16] on the freeway with initial speed $v_{0,i}$ and bounded acceleration a_i . The distributions of these parameters are respectively described by $V_0(v_0, s_{V_0})$ and $A(a, s_A)$. Note that capital letters will be used for defining the distributions associated to random variables labeled with lower case letters. Platoons of vehicles upstream of each moving bottleneck on the main road are described by the kinematic wave model [17], [18] and a triangular fundamental diagram with wave speed w and jam density κ_i . Free-flow speed has no influence here and it seems reasonable for freeway traffic to assume same wave speeds for all platoons [20]. A different jam density value is assigned to each inserting vehicles following $K(\kappa, s_K)$. In this paper, we will assume that this value also characterize the mean jam density of the platoons led by the inserting vehicle.

To establish the generic expression for the effective capacity C , vehicles are first assumed to all insert at $x=0$, i.e. $L=0$, see Fig. 1b. Let δ_i be the cumulative number of vehicles that have crossed $x=0$ between time t_i and t_{i+1} . Variational theory [21] states that δ_i can be equally calculated on the paths A→B or A→C→B, see Fig. 1b. No vehicle can pass the bottleneck between A and C, so δ_i is equal to $w\kappa_i(h_i - \tau_i)$, where τ_i is the time duration between points A and C. The effective capacity C corresponds to the ratio between the sum of δ_i and the total duration of the process, i.e. the sum of h_i , when the number of insertion tends to infinity. It is then given by:

$$C = \frac{\sum_{i=1}^{n \rightarrow \infty} \delta_i}{\sum_{i=1}^{n \rightarrow \infty} h_i} = w \frac{\sum_{i=1}^{n \rightarrow \infty} \kappa_i (h_i - \tau_i)}{\sum_{i=1}^{n \rightarrow \infty} h_i} \quad (1)$$

$$\tau = \frac{1}{a_i} (-w - v_{0,i} + v); \quad v(h_i, v_{0,i}, a_i) = \sqrt{(w + v_{0,i})^2 + 2wa_i h_i}$$

The law of large numbers tells us that $(1/n)\sum \delta_i$ and $(1/n)\sum h_i$ respectively converge to Δ and h_0 , i.e. the mean of the corresponding distributions. To determine Δ , we can apply the multivariate generalization of the Delta method [22]. This method consists in performing a second-order Taylor expansion of δ_i around the mean values h_0 , v_0 , a and κ

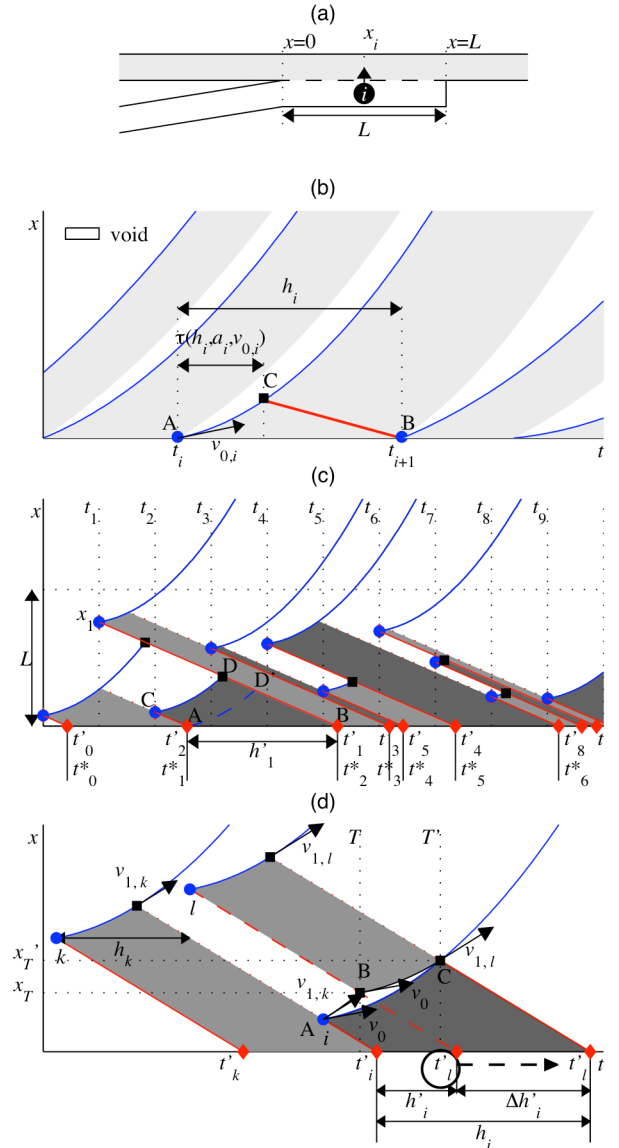


Fig. 1. (a) Sketch of the merge (b) Inserting process when $L=0$ (c) Inserting process when $L>0$, no interaction between waves and voids (d) Accounting for interactions between waves and voids.

before applying the sum operator. First order terms disappear because the sum of each parameters divided by n converges to the mean. Only second order terms remain and are weighted by either the standard deviation s_X of each distribution X or the covariance $\theta_{X,Y}$ between all X and Y . Thus, Δ is given by:

$$\Delta/w = \delta(h_0, v_0, a, \kappa) + \frac{1}{2} \sum_{X \in \{h_0, v_0, a, \kappa\}} s_X^2 \frac{\partial^2 \delta_i}{\partial X^2} + \sum_{(X,Y) \in \{h_0, v_0, a, \kappa\}, X \neq Y} \theta_{X,Y} \frac{\partial^2 \delta_i}{\partial X \partial Y} \quad (2)$$

We first assume that H and V_0 are respectively independent from A and K because they depend on the on-ramp traffic conditions and not on the vehicle characteristics. The covariance between these distributions is then null. It can easily be verified that the second derivative of δ_i with respect to K is null. Interestingly, this means that the K -

distribution has no influence on C , only the correlation between K and A does. Finally, all the derivatives of δ_i can be expressed using the derivatives of τ . We then obtain the following generic expression for C :

$$C = (w\kappa/h_0)(h_0 - \tau(h_0, v_0, a)) - \frac{1}{2}s_H^2 \frac{\partial^2 \tau}{\partial H^2} - \frac{1}{2}s_{V_0}^2 \frac{\partial^2 \tau}{\partial V_0^2} - \frac{1}{2}s_A^2 \frac{\partial^2 \tau}{\partial A^2} - \theta_{H,V_0} \frac{\partial^2 \tau}{\partial H \partial V_0} - \frac{\theta_{A,K}}{\kappa} \frac{\partial \tau}{\partial A} \quad (3)$$

The derivatives of τ are provided in (4). We will show in section III and IV that introducing more relevant physical hypothesis like $L>0$, interactions between voids and waves or random vehicle characteristics is “just” a question of properly calculating the moments of all distributions.

$$\begin{aligned} \frac{\partial^2 \tau}{\partial H^2} &= \frac{-aw^2}{v^3(h_0, v_0, a)}; \quad \frac{\partial^2 \tau}{\partial V_0^2} = \frac{2wh_0}{v^3(h_0, v_0, a)} \\ \frac{\partial^2 \tau}{\partial H \partial V_0} &= \frac{-w(w+v_0)}{v^3(h_0, v_0, a)}; \quad \frac{\partial \tau}{\partial A} = -\frac{\tau(h_0, v_0, a)}{a} + \frac{wh_0}{v(h_0, v_0, a)} \\ \frac{\partial^2 \tau}{\partial A^2} &= \frac{2}{a^2} \left(\tau(h_0, v_0, a) - \frac{wh_0}{v(h_0, v_0, a)} - \frac{aw^2 h_0^2}{2v^3(h_0, v_0, a)} \right) \end{aligned} \quad (4)$$

III. CONSIDERING INTERACTIONS BETWEEN WAVES AND VOIDS

In this section, we now consider that insertions can happen anywhere between 0 and L ($L>0$). Vehicle characteristics are still homogeneous ($s_A=0$ and $s_K=0$). We first show that the physical process with random inserting position can still be described with the generic expression. Second, we look for the analytical expression for the relevant moments in (3). Finally, we will derive the full analytical expression.

A. Applying the generic expression

The general principle for considering insertions at different locations between $x=0$ and $x=L$ has already been presented in [14]. When vehicle i is inserting at time t_i and location x_i , it generates a wave whose speed is w and which carries the speed $v_{0,i}$. This wave reaches $x=0$ at time t'_i , see Fig. 1c. Reference [14] explains in details what clearly appears in Fig. 1c: the process for determining the effective capacity at $x=0$ is the same when considering $L>0$ or $L=0$. Indeed, the cumulative number of vehicles can be calculated on either on paths $A \rightarrow C \rightarrow D \rightarrow B$ or $A \rightarrow D' \rightarrow B$. Calculation on path $A \rightarrow D' \rightarrow B$ is exactly the same for the path $A \rightarrow C \rightarrow B$ when $L=0$, see Fig. 1b&a. To determine C , we only have to replace the distribution H by the distribution H' where $h'_i = t_{i+1}^* - t_i^*$ and t^* is the ordered series gained from the realizations of t' , see Fig. 1c. This result resorts to a restrictive assumption: waves coming from moving bottlenecks are not influenced by voids created upstream by other bottlenecks, e.g. wave coming from vehicle 1 propagates until $x=0$ without considering the void created downstream of vehicle 2, see Fig. 1c. The authors of [14] mentioned that this assumption helps to keep the analytical

calculation simple but they do not investigate how it influences the effective capacity values. This will now be done. Note that [14] provides the analytical expression for $s_{H'}$ when the time between two insertions is set to h_0 , i.e. $s_{H'}=0$, and when the distribution of inserting positions in congestion is uniform as suggested by experimental evidence [23]:

$$s_{H'} = \begin{cases} L/\sqrt{6}w & \text{if } L \leq wh_0 \\ h_0(L - wh_0/\sqrt{6}) / (L + (\sqrt{6} - 2)wh_0) & \text{if } L > wh_0 \end{cases} \quad (5)$$

Fig. 1d shows what happens when considering the interactions between voids and waves. The wave coming from vehicle l meets the void created downstream of vehicle i . The void progressively vanishes and the wave can only propagate further downstream when the void has disappeared, i.e. at time T in Fig. 1d. This changes the time t'_l when the wave reaches $x=0$ and potentially influences the H' distribution that is now simply relabeled H . This also modifies the V_0 distribution. In fact, the initial speed when calculating the cumulative vehicle number between t'_l and the time when the next wave arrives at $x=0$ is no longer equal to $v_{0,l}$ but is now equal to $v_{1,l}$, see Fig. 1d. $v_{1,l}$ corresponds to the speed carried by the wave coming from vehicle l and that goes through the point C where the void created by i disappears. The position of this point depends on the initial speed $v_{1,k}$ of the void downstream boundary. This last speed can be determined by identifying the vehicle k that determines the speed profile when i is inserting, see Fig. 1d.

All the challenge is to maintain analytical tractability when calculating the new moments of the H and V_0 distributions considering the extended physical process with voids. In order to validate the analytical simplifications that we will make, we have developed a numerical code that, for a given $h_0=1/q_0$, (i) randomly affects the inserting positions for a set of 5000 vehicles, (ii) matches each vehicle l with the corresponding vehicles i and k , (iii) makes the proper calculation for the vehicle i void boundaries, (iv) determines the modified values for t'_l and $v_{0,l}$. Note that the wave coming from l not necessarily meets a void and is then unaffected. Such numerical simulations provide samples for H and V_0 distributions and also allow us to directly estimate C . We perform extensive simulation runs but for illustration purposes, most figures of this article are drawn with the following parameters: $w=19.4$ km/h, $\kappa=130$ veh/km, $a=1.8$ m/s², $q_0=0.174$ veh/s.

B. Determining the moments of the different distributions

We further assume that all vehicles have the same speed v_0 when inserting. This speed is associated to q_0 through the fundamental diagram. This assumption is reasonable because the on-ramp is congested. Note that mathematical expectation (mean) of a distribution X is further labeled $E(X)$.

H-distribution. Considering interactions between voids and waves does not change the number of waves created.

Thus $E(H)$ remains unchanged and is equal to h_0 . The ordering processing when switching from t' to t^* makes inaccessible the analytical derivation of the H -distribution from the distributions of the inserting position and time. In [14], the analytical expression of s_H (5) has been obtained by considering extreme case when L is very small and very high and by fitting the global expression using extensive numerical simulations. Notably, it appeared that H' follow an exponential distribution when L tends to infinity. Here, we apply the same approach. Fig. 2a shows an example for the evolution of s_H with respect to L when interactions between voids and wave are considered or not. This figure highlights that the standard deviation of H is unaffected by the modification of t'_l when waves encounter interactions with upstream voids. The analytical expression (5) remains fully accurate. This has been confirmed by multiple simulation runs using a wide range of parameter values.

Probability for interacting p_{int} . Not all waves meet voids before reaching $x=0$. Before going further in the calculation of the moments, we need to establish an analytical formulation for the probability p_{int} that a wave starting from a moving bottleneck experiments interactions. Consider a vehicle i that is inserting at time t_i and location x_i . The wave starting from i can interact with a void created by a vehicle that inserted in the close past or that will insert in the near

future, see diamond dots in Fig. 2c. To maintain the analytical tractability, we will only consider the closest neighbors, i.e. vehicle $i-1$ and $i+1$, see the shaded area in Fig. 2c. There will be no interactions if the starting point of vehicle $i-1$ makes it arrive above x_i at time t_i and if vehicle $i+1$ is created above the wave coming from i at time t_{i+1} . These two conditions should jointly be true and correspond to initial inserting positions for vehicle $i-1$ and $i+1$ along the two green lines in Fig. 2c. This means that $x_{i-1} > x_i - 0.5ah_0^2 - v_0h_0$ and $x_{i+1} > x_i - wh_0$. Let denote b_1 and b_2 respectively the min and the max between $x_i - 0.5ah_0^2 - v_0h_0$ and $x_i - wh_0$. As the inserting position for all vehicles obeys to a uniform distribution, it comes that the conditional probability $P(\text{no}|x_i)$ of no interaction given x_i is:

$$P(\text{no}|x_i) = \begin{cases} 1 & \text{if } x_i \leq b_1 \\ (L - x_i - b_1)/L & \text{if } b_1 < x_i \leq b_2 \\ (L - x_i - b_1)(L - x_i - b_2)/L^2 & \text{if } x_i > b_2 \end{cases} \quad (6)$$

The law of total probability makes it possible to determine p_{int} with respect to $P(\text{no}|x_i)$ noticing that the probability for vehicle i to insert at position x_i is $1/L$:

$$p_{\text{int}} = 1 - \frac{1}{L} \int_0^L P(\text{no}|x_i) dx_i \quad (7)$$

The black curve in Fig. 2b compares the numerical and analytical results for p_{int} and different L values in our example case. It appears that the analytical expression is close to the numerical results even if we only consider the two closest neighbors. p_{int} is underestimated because our approximation neglects interactions with farer neighbors. This result has also been confirmed by extensive simulation runs.

V_0 -distribution. The initial speed when wave i arrives at $x=0$ may either be equal to $v_{1,i}$ or v_0 depending on whether an interaction happens or not. We first only consider cases with an interaction. Let denote $A(t_i, x_i)$ the point where the void is created, $B(T', x_T')$ the point where the wave meet the downstream void boundary and $C(T'', x_T'')$ the point where the void disappears, see Fig. 1d. C is the intersection point of two parabolas corresponding to bounded acceleration trajectories (curves BC and AC). Solving the associated equations leads to $T'' = T' + (v_{1,k} - v_0)/a$. Equation of curve BC also tell us that $v_{1,i} = a(T' - T) + v_0$. It comes then a very simple result when v_0 is constant: $v_{1,i} = v_{1,k}$. Finally, $v_{1,i}$ is given by:

$$v_{1,i} = a\tau(h_k, a, v_0) + v_0 \quad (8)$$

Equation (8) means that the V_1 -distribution of only depends on the distribution of $\tau(H)$. Its two first moments are then given by:

$$\begin{cases} E(V_1) = aE(\tau(H)) + v_0 \\ E(V_1^2) = v_0^2 + 2aE(\tau(H)) + a^2E(\tau^2(H)) \end{cases} \quad (9)$$

The mean and standard deviation of the V_0 -distribution can then be derived by applying the law of total expectation with condition probability depending on whether an interaction appears or not:

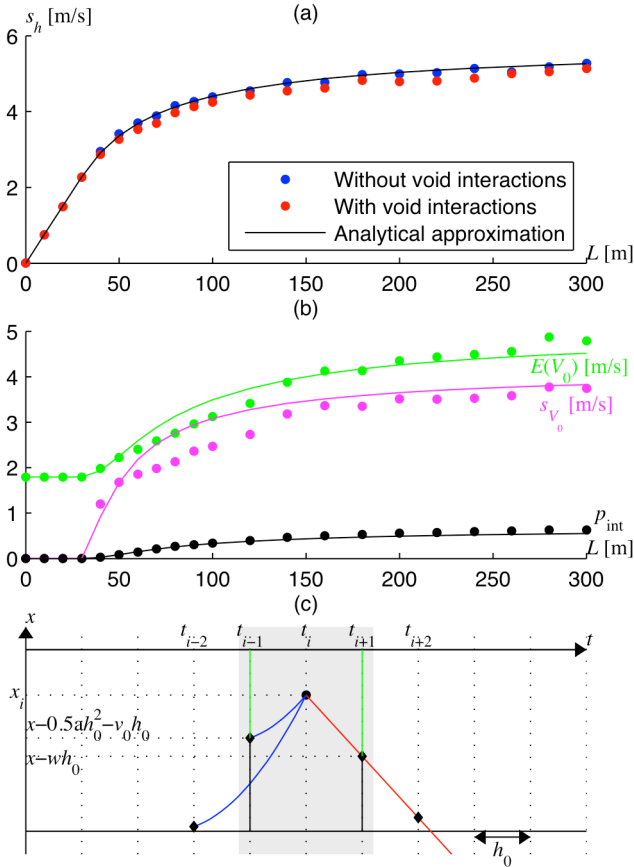


Fig. 2. Analytical approximations for (a) the standard deviation of H with and without considering void interactions (b) the mean and the standard deviation of V_0 and the probability p_{int} for waves to interact with voids (c) Process for approximating the probability p_{int} .

$$\begin{cases} E(V_0) = (1 - p_{\text{int}})v_0 + p_{\text{int}}E(V_1) \\ s^2_{V_0} = E(V_0^2) - E(V_0)^2; \quad E(V_0^2) = (1 - p_{\text{int}})v_0^2 + p_{\text{int}}E(V_1^2) \end{cases} \quad (10)$$

$$\Rightarrow \begin{cases} E(V_0) = v_0 + ap_{\text{int}}E(\tau(H)) \\ s^2_{V_0} = ap_{\text{int}}(2E(\tau(H)) + aE(\tau^2(H))) \end{cases}$$

The last thing we need to finalize is the calculation of the mathematical expectation of τ and τ^2 . This can be achieved by again applying the Delta method [22]:

$$\begin{cases} E(\tau) = \tau(h_0, v_0, a) + \frac{1}{2}s_H^2 \frac{\partial^2 \tau}{\partial H^2}; \quad \frac{\partial^2 \tau}{\partial H^2} \rightarrow (4) \\ E(\tau^2) = \tau^2(h_0, v_0, a) + \frac{1}{2}s_H^2 \frac{\partial^2 \tau^2}{\partial H^2}; \quad \frac{\partial^2 \tau^2}{\partial H^2} = \frac{2w^2(w + v_0)}{v^3(h_0, v_0, a)} \end{cases} \quad (11)$$

Fig. 2b shows the comparison between the analytical and the numerical calculations for $E(V_0)$ and s_{V_0} and the example case. The results are quite good except for some discrepancies for low L values (between 60 and 150 m). The reason is that $v_{1,l} = v_{1,k}$ and (8) hold only if vehicle l inserts outside the void created by vehicle i . Otherwise, the void disappears more quickly and $v_{1,l} < v_{1,k}$. Such situations are properly handled in the numerical code but can hardly be introduced in the analytical derivation. Of course, they happen more frequently if the insertion length is small. This explains why the analytical formulas overestimate $E(V_0)$ and

s_{V_0} when L is quite small. As usual, extensive simulation runs have been performed to verify that the errors remain in the same level of magnitude. Furthermore, we will see later that such discrepancies have few impacts when calculating C .

Covariance between H and V_0 . To apply (3) with homogeneous vehicle characteristics, the last missing term is θ_{H,V_0} . The analytical derivation of this term is almost impossible because multiple interactions occur. Indeed, when a wave is delayed due to a void this change the time headways of both neighboring waves and the initial speed for one of them, see Fig 1d. This speed depends on the time headway of another wave associated to vehicle k . Because of the ordering process from t' to t^* , it is very difficult to analytically determine the headway index associated to an initial speed modification. Fortunately, when performing the extensive numerical tests it appears that the value of θ_{H,V_0} remains very low compared to the variances of other distributions whatever the parameters, q_0 and L are. H and V_0 are clearly not independent but their covariance can be neglected. θ_{H,V_0} is then assumed equal to 0 for further analytical calculations.

C. Calculating the effective capacity for different inserting flows

Fig. 3 presents the analytical and numerical results for the effective capacity C . Three values for the inserting flow are tested. The blue curve and dots correspond to the case when interactions are neglected and so to the results already stated in [14]. The red curve and dots clearly show the importance of considering the interactions between voids and waves. The estimation of the effective capacity increases up to 15% when this phenomenon is taken into account. This is explained by the fact that voids created by upstream inserting vehicles reduce the impacts of other vehicles that insert downstream. This tends to increase the capacity.

The effect of interactions starts being noticeable when $L > 50$ m except for the lowest q_0 value. This is because when all vehicles insert on a short distance, waves quickly reaches $x=0$ and do not interact with voids. Another interesting result is that the effective capacity stops significantly increasing when L becomes higher than 150 m. However, the influence of L is important for the lowest values. For example, the effective capacity increases about 15 to 20% when L increases from 20 to 160 m. This may be interesting for road design.

The most important insight in Fig. 3 is that the extended analytical formula performed well whatever the q_0 and L values are. The discrepancies with the numerical results are always below 3%. This means that (3) provides a very good estimate for the effective capacity even if we resort to restrictive assumptions when determining the moment of some distributions. This is really appealing because this formula provides a direct estimate for the effective capacity without requiring any complex simulation runs.

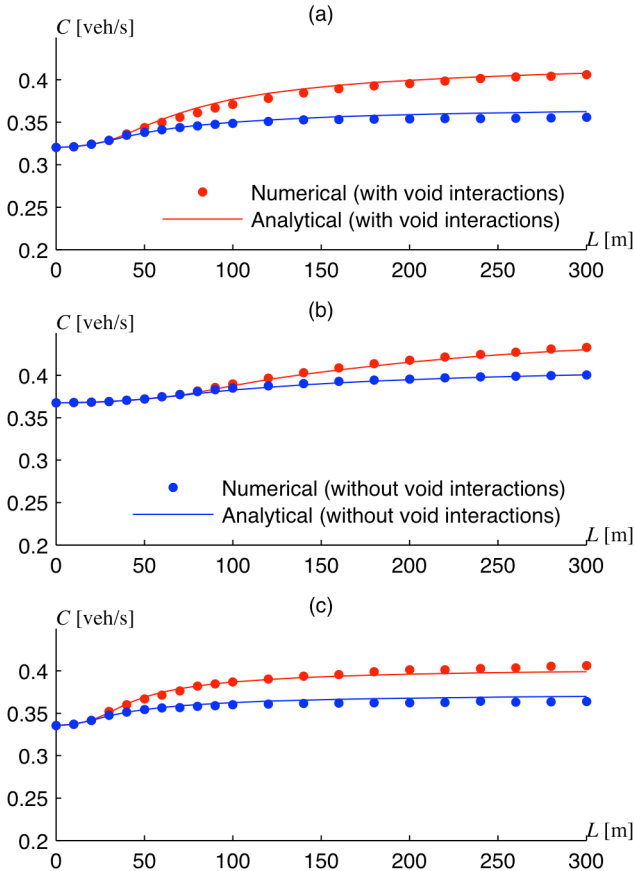


Fig. 3. Comparison between analytical and numerical results when considering or not the interactions between waves and voids. (a) regular inserting flow: $q_0=0.174$ veh/s (b) low inserting flow: $q_0=0.08$ veh/s (c) high inserting flow: $q_0=0.26$ veh/s.

IV. CONCLUSION

This paper provides a new analytical formulation for the effective capacity at active freeway merges. This formulation is able to account for interactions between voids that appear downstream of inserting vehicles. This phenomenon has a significant impact when calculating the capacity because voids leave spaces for downstream traffic to expand. This happens in reality when the on-ramp is not very short and should be taken into account to properly derive the capacity value.

The proposed framework can also be extended to account for heterogeneous driver characteristics, i.e. random accelerations and jam densities. The terms involving $\theta_{A,K}$ and s_A in (3) should then be added and the moments of the other distributions be further updated. The full process will be described in the extended version of this article. However, we can already mention the main conclusion of the full analysis: A proper estimation of their mean characteristics is sufficient to derive an accurate analytical estimation of the effective capacity. In fact, the results provided by the extended analytical formulae with heterogeneous vehicle characteristics are close to the one provided by the analytical formulae with homogeneous vehicle and the same mean values for parameters. This result is really appealing because it means that formulas presented in section III are sufficient. Such formulas are much more simpler and can easily be implemented for practical applications.

Reference [14] provides all the methodological background to implement such updated analytical formulations into a full merge model that also account for situations where the on-ramp is not congested.

Further research directions investigated by the authors are (i) analytical derivations of other indicators than the mean flow for the same physical process and (ii) a refined multilane extension to this framework compared to what is included in [14]. This refined framework will account for the effect discretionary lane-changings that occur on freeway lanes.

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