Applications of the Generalized Macroscopic Fundamental Diagram

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Abstract The generalised Macroscopic Fundamental Diagram (g-MFD) relates the network traffic density and the spatial variation of this density. Recent work of the authors show that by using both the average and the standard deviation in the density, a very crisp relation can be found, also in case the network conditions are inhomogeneous. This paper presents results for the g-MFD using empirical data collected for the freeway network around Amsterdam. Next to presenting the g-MFD, we will show how the dynamics in the network relate to the evolution of the network state in terms of average density and spatial density variation. The paper discusses regular dynamics, as well as the dynamics in case of incidents occurring in the network. The presented results justify using the g-MFD for a number of applications that will be detailed in the rest of the paper. First of all, the g-MFD can be used to determine the network-wide service-level, both for recurrent and non-recurrent situations. The results for incident situations motivate the second application, namely the analysis of the resilience of the network by studying the changes in the service level for specific network states. We will illustrate these applications using the aforementioned Amsterdam test case.

1 Generalised Macroscopic Fundamental Diagrams

The concept of the generalised Macroscopic Fundamental Diagram was first introduced by [1]. The g-MFD generalises the Macroscopic Fundamental Diagram (MFD) by introducing another independent variable next to the average density k, namely the spatial variation in the density. In this manuscript, we will use $\sigma = \sigma(t)$ to denote the standard deviation of the density at time instant t, i.e.:

$$\sigma(t) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(k_i(t) - k(t))^2} \tag{1}$$

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where $k_i(t)$ is the traffic density at segment i of a network consisting of n segments (or links) at time instant t and $k(t)$ denotes the average density at time instant t:

$$k(t) = \frac{1}{n}\sum_{i=1}^{n} k_i(t) \tag{2}$$

The general idea of the g-MFD is that the average network flow Q, and thus the average network speed V are functions of both the average density and spatial standard deviation of the density, that is $Q = Q(k, \sigma)$, and $V = V(k, \sigma)$.

Based on the estimation results in [1], it is shown that this generalisation provides a much better fit to traffic network data than the original MFD. In fact, the g-MFD relaxes the rather restrictive requirement to the applicability of the MFD of network traffic conditions needing to be homogeneous.

In the remainder of this section, we will present a novel intuitive functional form for the g-MFD that will be fitted on motorway traffic data.

2 Example g-MFD for a Freeway Ring Road

This section presents empirical results for the g-MFD using empirical data collected for the freeway network around Amsterdam (so-called A10 ring road, clockwise direction). The 33 km ring road has a general speed limit of 100 km/h, apart for a stretch of about 11 km for which the speed limit is 80 km/h.

Figure 1 shows average results for 2 months of data (November and December 2011), as well as the (k, σ) path of a single day (8th of November 2011). The figure

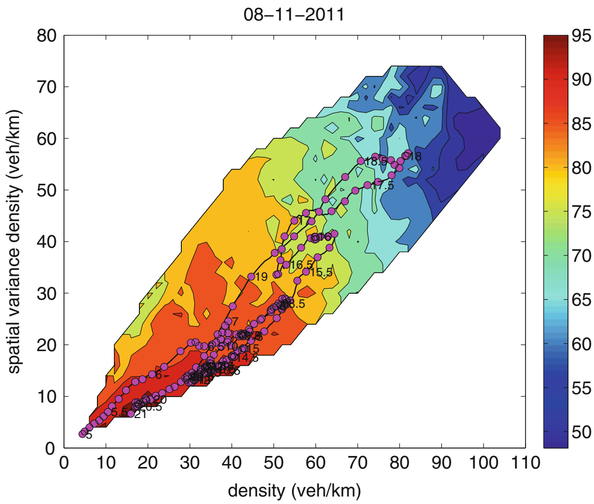

Fig. 1 Network state path $(k(t), \sigma(t))$ in relation to the g-MFD for a regular day

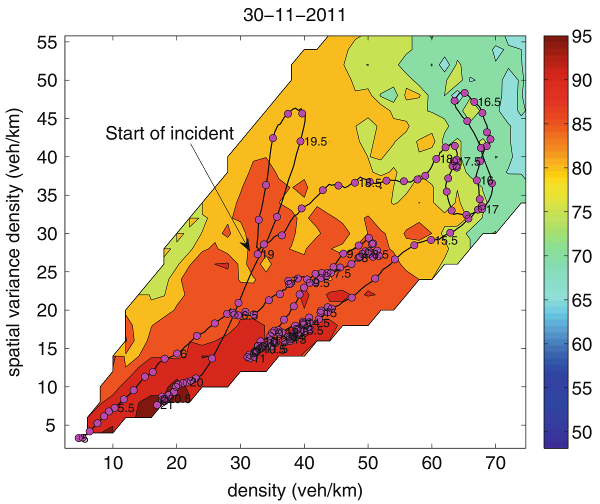

Fig. 2 Network state path $(k(t), \sigma(t))$ in relation to the g-MFD. The *arrow* indicates the starting time of the incident

shows how the morning peak is less congested than the evening peak, maintaining average speeds well above 85 km/h. In the evening however, more severe congestion brings the network speed to around 60 km/h. Note that the path approximately follows a straight line described by $\sigma = 2k/3$. This turns out to be the case for all regular days where no incidents occur. In all of these days, we see slight counterclockwise hysteresis showing that in general, the spatial variation is less when the density is increasing and higher when it is decreasing. Note that the hysteresis phenomena is discussed in detail by [2]

An example is given by Fig. 2, showing the network state path for an incident case. We see how the dynamics of the path are influenced by the incident: when the incident occurs at 19:00, the path breaks away from the congestion recuperation path it was moving along. From this point onwards, the path moves in the vertical direction showing that the average density stays about the same, while the spatial variation increases considerable. The reasons for this are clear: the incident causes an increase in the spatial variation, while the overall density is not necessarily increasing.

3 Functional Relation

A functional form for the g-MFD can be found by fitting a multivariate function to the data. For this function to be realistic, it needs to satisfy some requirements. First of all $V(k, \sigma)$ is a decreasing function of both k and σ, i.e.:

$$\frac{\partial V}{\partial k} < 0 \quad \text{and} \quad \frac{\partial V}{\partial \sigma} < 0 \tag{3}$$

Furthermore, we hypothesise the existence of a maximum density, possibly dependent on the spatial variation, for which the network speed is zero (full gridlock):

$$V(k_{jam}(\sigma), \sigma) = 0 \tag{4}$$

That is, we assume that for each σ we can find a density value for which the speed equals zero. Note that $k_{jam}(0)$ denotes the jam density for fully homogeneous conditions, i.e. when each network link has jam density. We expect:

$$\frac{\partial k_{jam}(\sigma)}{\partial \sigma} < 0 \tag{5}$$

The following function satisfies these conditions. Furthermore, we found that it provided a very reasonable fit to the data on the A10:

$$V(k, \sigma) = \min\left(v_0, \beta \cdot \left(\frac{1}{k} - \frac{1}{k_{jam}}\right)\right) \cdot \left(1 - \frac{\sigma}{\sigma_0}\right) \tag{6}$$

with $v_0 = 100.6$ km/h, $k_j = 166.6$ veh/km/lane, $\sigma_0 = 113.5$ veh/km/lane, and $\beta = 2{,}382$ veh/h; the resulting fit yields an adjusted rho-square of 0.9894. All parameters are statistically significant at 95 %. This function was chosen since the density-dependent part of the function is a often used model form for the (normal) fundamental diagram (i.e. congested branch stemming from a simple car-following model).

Most of the parameters in this equation have a nice interpretation: v_0 can be interpreted as the average free network speed, which for this situation in 100.6 km/h. The parameter k_{jam} can be seen as the average jam density in the network; the estimated value of 166.6 veh/km/lane seems very plausible. The parameter σ_0 denotes a scaling parameter for the spatial variation; note that when $\sigma = \sigma_0 = 113.5$, the speed is zero. The scale parameters β denotes the reduction in the average distances s between vehicles with decreasing speed. This can be seen by noticing that in the congested branch of the (g-) MFD, we have $V/\beta = 1/k - 1/k_{jam} = s - 1/k_{jam}$.

4 Application to Network Resilience Analysis

The g-MFD has many applications, including modelling and control. In [3], the g-MFD is used as a real-time of to determine the network service level (e.g. the average quality of the network operations in the network). In this case, both speed and flow are used to determine the level-of-service, which for example can be defined as an average speed of the network or the production of average speed and density. For more details, we refer to [3].

In this section, we propose a new approach to test the network resilience which uses the concept of using the g-MFD as a network service level indicator. The idea is that the *change in the level-of-service* due changes in the spatial variation is an indicator of resilience. In other words, if the spatial variation in the density increases, the extent in which this yields a change in the level of service (e.g. speed) provides information on how well the network can deal with such disturbances. In Sect. 2 we have illustrated the state dynamics in case of incident conditions. Here it was shown that as an incident occurs, the spatial density variation increases. If this yields only a limited reduction in the level-of-service, then the network is robust since it can deal with such disturbances.

4.1 Network Resilience Definition

The *resilience* of the network is defined by "the ability to provide and maintain an acceptable level of service in the face of faults and challenges to normal operation". For traffic networks, these "faults" could be interpreted as incidents or other events that (temporarily) reduce the capacity of a roadway segment.

Given the path dynamics discussed in Sect. 2, we know that in case of a (possibly temporary or partial) blockade, the (k, σ) path moves in the upward direction: the density remains constant, while the spatial variation in the density increases. The impact on the level-of-service λ – as shown in examples later on – can thus be defined logically by taking the partial derivative of the level-of-service to the spatial density variation, i.e.:

$$\xi = \xi(k, \sigma) = \frac{\partial}{\partial \sigma} \lambda(k, \sigma) \tag{7}$$

From this definition, we see that resilience is defined by *the rate in which the level-of-service drops when the spatial variation in the density increases*. If this rate ξ is high, then a small increase in the standard deviation of the density σ causes a large reduction in the level-of-service λ. On the contrary, when the rate ξ is small, the level-of-service in relatively insensitive to an increase in σ. Note that the resilience is determined by a number of factors, such as the availability of alternative routes in the network, but also the level of information provided to the road users allowing them to reroute in case of an incident.

In the remainder, we will take the average speed $\lambda = V(k, \sigma)$ as a proxy for the level-of-service λ, although other (continuous) definitions can be used as well, for instance the production (which may yield completely different results); see [3] for more information.

4.2 Functional Expression for Network Resilience of the A10 Ring Road

In illustration, let us revisit the A10 ring road example and take a look at the resilience definition. Recall that we used the formal expression Eq. (6). Using this functional form, we get:

$$\xi(k,\sigma) = -\frac{1}{\sigma_0}\min\left(v_0, \beta \cdot \left(\frac{1}{k} - \frac{1}{k_j}\right)\right) \tag{8}$$

This expression shows that ξ becomes smaller when the density becomes larger. That is, when the speed is already low, the extra speed reduction due to for instance an incident is less pronounced. Note that when taking the production as a measure for the level-of-service γ, this conclusion does not hold. In that case, we get:

$$\xi(k,\sigma) = -\frac{1}{\sigma_0}\min\left(v_0 \cdot k, \beta \cdot \left(1 - \frac{k}{k_j}\right)\right) \tag{9}$$

meaning that the maximum reduction rate in level-of-service occurs at some critical density value k_c.

5 Discussion

This paper has presented the generalised Macroscopic Fundamental Diagram (g-MFD) as a means to analyse the dynamics in a network (for recurrent and non-recurrent conditions) and to assess the level-of-service and network resilience. Using data from a Dutch freeway ring road, we showed that a good fit could be obtained using a pre-specified functional form of the g-MFD that satisfied key functional criteria. Using this fitted functional form, conclusions can be drawn about the network resilience, depending on the used level-of-service indicator (e.g. average speed or production).

Future research focuses on cross-comparing different network structures with different types on traffic information and management strategies. In doing so, insight will be gained into the applicability of the approach presented here to quantify network resilience.

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