Differentiated Pricing of Urban Transportation Networks with Vehicle-Tracking Technologies

Mahmood Zangui
Yafeng Yin*
Siriphong Lawphongpanich
Shigang Chen

Department of Civil and Coastal Engineering
University of Florida

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Price Differentiation

Economic concept

- Defined by Dupuit (1894)
- Identical products sold at different prices

Examples in transit fare

- One two-way ticket is cheaper than two one-way tickets
- Senior citizens pay lower bus fare

Literature of congestion pricing

- Differentiation with respect to vehicle type
- Differentiation with respect to value of time

We investigate a new differentiated congestion pricing scheme that differentiates travelers with respect to their *travel characteristics*.

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We investigate a new differentiated congestion pricing scheme that differentiates travelers with respect to their *travel characteristics*.

How does a differentiated pricing scheme work?

Definitions

Anonymous scheme: Everyone pays the same toll for the same link.
Differentiated scheme: Toll depends on travel characteristics.
Level of differentiation:

- L0: None $\rightarrow$ anonymous scheme
- L1: Origin $\rightarrow$ origin-specific scheme
- L2: Origin and destination $\rightarrow$ OD-specific scheme
- L3: Path $\rightarrow$ path-based scheme

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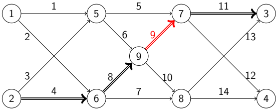
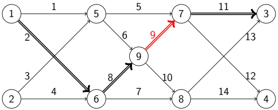
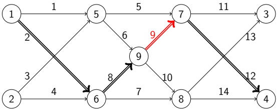
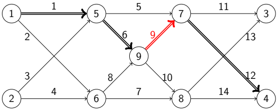
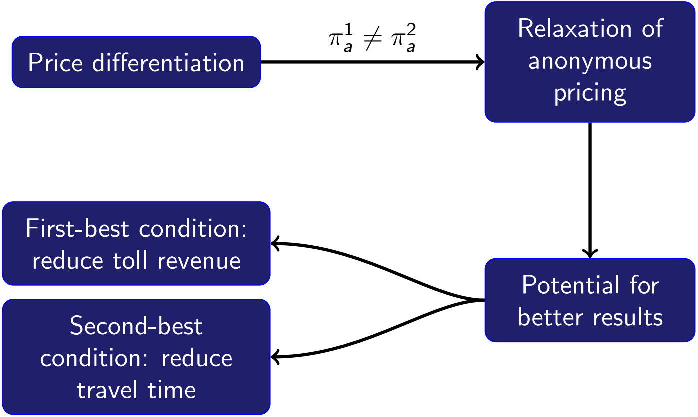
Level of Differentiation

Path	L0	L1	L2	L3
	π_9	π_9^2	$\pi_9^{2,3}$	$\pi_{4,8,9,11}$
	π_9	π_9^1	$\pi_9^{1,3}$	$\pi_{2,8,9,11}$
	π_9	π_9^1	$\pi_9^{1,4}$	$\pi_{2,8,9,12}$
	π_9	π_9^1	$\pi_9^{1,4}$	$\pi_{1,6,9,12}$

Benefits of Differentiation



Example: First-best Condition

- All links are tollable $\rightarrow$ System optimum is achievable with anonymous tolling
  - $x_a = \bar{x}_a \qquad \forall a \in A$
- Benefits of price differentiation can only be reflected on a secondary objective
- Toll revenue is a financial burden on travelers
- Higher toll revenue implies less public acceptance
- Choose the tolls with minimum revenue
  - min $\quad \sum_{w \in W} \sum_{p \in P_w} \pi_p f_p$

Example: First-best Condition

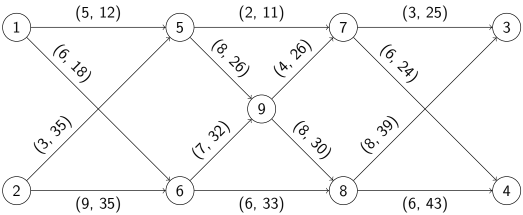


Table : First-best pricing for nine-node network

Tolling Scheme	Toll Revenue		OD Generalized Travel Cost			
	Amount	Reduction	[1, 3]	[1, 4]	[2, 3]	[2, 4]
Anonymous	887.6	0%	30.6	29.2	33.0	31.6
Origin-specific	311.6	65%	23.4	29.3	25.8	24.4
OD-specific	295.6	67%	23.4	22.0	25.8	27.6
Path-based	263.6	70%	23.4	22.0	29.0	24.4

Example: Second-best Condition

- Suppose links $a \in \bar{\Psi}$ are untollable
  - Origin-specific $\gamma_a^{o(w)} = 0 \quad \forall w \in W, a \in \bar{\Psi}$
  - OD-specific $\gamma_a^{w} = 0 \quad \forall w \in W, a \in \bar{\Psi}$
- Travel time as the performance measure
- Choose tolls that minimize total system travel time
  - min $\sum_{w \in W} \sum_{p \in P_w} t_p(f) f_p$

Example: Second-best Condition

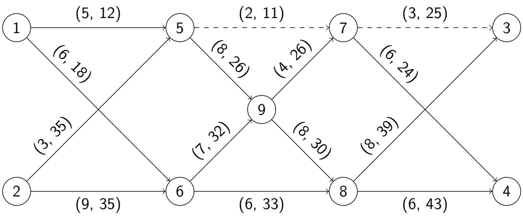


Table : Second-best pricing for nine-node network

Tolling Scheme	Total Travel Time		OD Generalized Travel Cost			
	Amount	Saving	[1, 3]	[1, 4]	[2, 3]	[2, 4]
UE	2455.9	0%	24.9	23.8	24.3	25.1
Anonymous	2361.2	46.9%	25.8	24.9	25.1	25.9
Origin-specific	2306.1	74.2%	24.3	24.2	27.1	25.7
OD-specific	2281.7	86.2%	24.4	22.9	26.8	25.3
SO	2253.9	100%	-	-	-	-

How to design optimal differentiated pricing schemes?

Finding Optimal Differentiated Schemes

Feasible region:

$$
\begin{aligned}
&\sum_{p \in P_w} f_p = d_w && \forall w \in W \\
&f_p \left(t_p(f) + \pi_p - \lambda_w \right) = 0 && \forall p \in P_w, w \in W \\
&t_p(f) + \pi_p - \lambda_w \geq 0 && \forall p \in P_w, w \in W \\
&f_p \geq 0 && \forall p \in P_w, w \in W \\
&\pi_p \geq 0 && \forall p \in P_w, w \in W \\
&x_a = \sum_{w \in W} \sum_{p \in P_w} \delta_{ap} f_p && \forall a \in A
\end{aligned}
$$

Finding Optimal Differentiated Schemes

Additional constraints:

- Origin-specific pricing

$$\pi_p = \sum_{a \in A} \delta_{ap} \gamma_a^{o(w)} \qquad \forall p \in P_w, w \in W$$

$$\gamma_a^{o(w)} \geq 0 \qquad \forall a \in A, w \in W$$

- OD-specific pricing

$$\pi_p = \sum_{a \in A} \delta_{ap} \gamma_a^{w} \qquad \forall p \in P_w, w \in W$$

$$\gamma_a^{w} \geq 0 \qquad \forall a \in A, w \in W$$

Finding Optimal Differentiated Schemes

Objective functions:

- First-best condition

$$\min \quad \sum_{w \in W} \sum_{p \in P_w} \pi_p f_p$$

- Second-best condition

$$\min \quad \sum_{w \in W} \sum_{p \in P_w} t_p(f) f_p$$

Example: Path-based Pricing Scheme (First-best Condition)

$$\min \quad \sum_{w \in W} \sum_{p \in P_w} \pi_p f_p$$

$$s.t.$$

$$\sum_{p \in P_w} f_p = d_w \qquad \forall w \in W$$

$$f_p\left(t_p\left(f\right) + \pi_p - \lambda_w\right) = 0 \qquad \forall p \in P_w, w \in W$$

$$t_p\left(f\right) + \pi_p - \lambda_w \geq 0 \qquad \forall p \in P_w, w \in W$$

$$f_p \geq 0 \qquad \forall p \in P_w, w \in W$$

$$\pi_p \geq 0 \qquad \forall p \in P_w, w \in W$$

$$\bar{x}_a = \sum_{w \in W} \sum_{p \in P_w} \delta_{ap} f_p \qquad \forall a \in A$$

Solution Algorithms

- The formulations presented above all belong to the class of mathematical programs with complementarity constraints (MPCC)
- These problems are non-convex and standard stationary conditions, i.e., KKT conditions, may not hold for them because they do not satisfy Mangasarian-Fromovitz constraint qualification (MFCQ)
- Efficient algorithms may be developed to solve the above formulations by exploring special properties or structures that they may possess.

Example: Path-based Pricing Scheme (First-best Condition)

$$\min \quad \sum_{w \in W} \lambda_w d_w$$

$s.t.$

$$\sum_{p \in P_w} f_p = d_w \qquad \forall w \in W$$

$$f_p \left(\bar{t}_p + \pi_p - \lambda_w \right) = 0 \qquad \forall p \in P_w, w \in W$$

$$\bar{t}_p + \pi_p - \lambda_w \geq 0 \qquad \forall p \in P_w, w \in W$$

$$f_p \geq 0 \qquad \forall p \in P_w, w \in W$$

$$\pi_p \geq 0 \qquad \forall p \in P_w, w \in W$$

$$\bar{x}_a = \sum_{w \in W} \sum_{p \in P_w} \delta_{ap} f_p \qquad \forall a \in A$$

Reformulation 1: MILP-1

$$
\begin{aligned}
\min \quad & \lambda^T d & \\
s.t. \quad & f \in \bar{F} & \\
& f_p \leq y_p d_w & \forall p \in P_w, w \in W \\
& \bar{t}_p + \pi_p - \lambda_w \leq (1 - y_p) M & \forall p \in P_w, w \in W \\
& \bar{t}_p + \pi_p - \lambda_w \geq 0 & \forall p \in P_w, w \in W \\
& \pi_p \geq 0, \quad y_p \in \{0, 1\} & \forall p \in P_w, w \in W
\end{aligned}
$$

where $\bar{F} = \{\sum_{p \in P_w} f_p = d_w, \forall w \in W; f_p \geq 0, \forall p \in P_w, w \in W; \bar{x}_a = \sum_{w \in W} \sum_{p \in P_w} \delta_{ap} f_p, \forall a \in A\}$

- One binary variable for each path, which is equal to 1 if the path is utilized

Reformulation 2: MILP-2

$$\min \quad \sum_{w \in W} \left(\sum_{p \in P^w} \bar{t}_p y_p \right) d_w$$

$$s.t. \quad f \in \bar{F}$$

$$\sum_{p \in P^w} y_p = 1 \qquad \forall w \in W$$

$$f_p \leq d_w \left(\sum_{k \in P^w \& \bar{t}_k \geq \bar{t}_p} y_k \right) \qquad \forall p \in P_w, w \in W$$

$$\pi_p \geq 0, \quad y_p \in \{0, 1\} \qquad \forall p \in P_w, w \in W$$

- One binary variable for each path, equal to 1 if it is the longest utilized path of the OD pair. The longest utilized path is toll free
- Linear mixed integer model, can be solved by CPLEX.

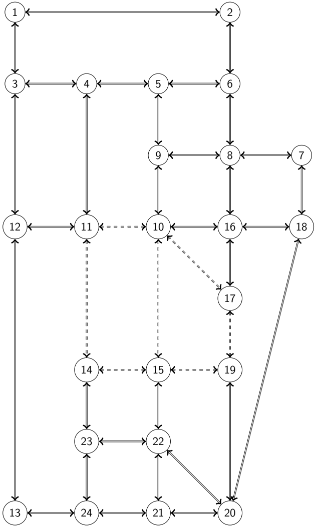
Computational Experiments

Sioux Falls Network

- 24 nodes
- 76 links
- 528 OD pairs

Anaheim Network

- 416 nodes
- 914 links
- 1406 OD pairs



Results

Table : Comparisons of toll revenues

	Toll Revenue		
	Anonymous	Path-based	Reduction
Anaheim			
Original Data	59766	2178	96.4%
Rand. Data 1	56381	1812	96.8%
Rand. Data 2	65959	6826	89.7%
Rand. Data 3	77598	5107	93.4%
Rand. Data 4	67984	3570	94.7%
Sioux Falls			
Original Data	20.666	0.183	99.1%
Rand. Data 1	19.462	0.088	99.5%
Rand. Data 2	59.019	0.085	99.9%
Rand. Data 3	27.102	0.108	99.6%
Rand. Data 4	32.862	0.069	99.8%

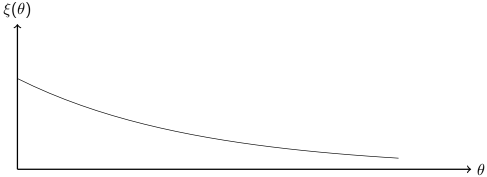
Is there any issue associated with these schemes?

Location Privacy

- Location privacy: the ability to prevent other parties from learning one's current or past location
- For anonymous tolling, it is possible to design a privacy-preserving electronic toll collection system.
- It is difficult, if not impossible, to design a privacy-preserving differentiated pricing system because differentiated pricing requires the location information to determine tolls

Value of Privacy

- Individuals value their privacy differently
- They can be grouped into categories of privacy unconcerneds, privacy pragmatists, and privacy fundamentalists
- Mathematically, we can use a distribution to represent different individual valuations of privacy across the population

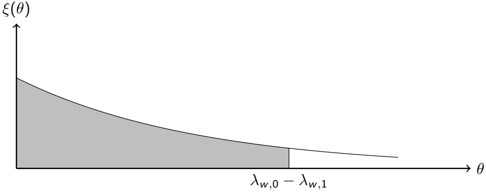


- If travelers value their privacy highly, the savings of travel cost that they may enjoy from differentiated schemes will be offset by the loss of their privacy

Who benefits from differentiated schemes?

We now use origin-specific pricing as an example for modeling privacy. For each OD-pair w:

- Those who value their privacy less will more likely benefit from price differentiation
- Travel cost saving: $\lambda_{w,0} - \lambda_{w,1}$
- The percentage of motorists who will be better off under origin-specific pricing: $\int_0^{\lambda_{w,0}-\lambda_{w,1}} \xi(z)dz$



Addressing Privacy Concerns

We propose an incentive program that allows each traveler to opt in to differentiated schemes

- Self-selection mechanism: travelers who choose to reveal their location information will pay differentiated tolls while those who remain anonymous will pay uniform tolls
- Anonymous scheme preserves location privacy
- Incentives, such as subsidies or credits, can be provided to encourage travelers to participate in differentiated scheme
- In the simplest setting, the travel cost difference between differentiated and anonymous schemes can be viewed as incentive

Design of Incentive Program

- Demand split constraints:

$$
\begin{aligned}
& d_{w,0} + d_{w,1} = d_w && \forall w \in W \\
& \sum_{p \in P_w} f_{p,c} = d_{w,c} && \forall w \in W, c \in \{0,1\} \\
& d_{w,1} = \Xi(\lambda_{w,0} - \lambda_{w,1}) d_w && \forall w \in W \\
& d_{w,0}, d_{w,1} \geq 0 && \forall w \in W
\end{aligned}
$$

Tolled User Equilibrium

- Tolled user equilibrium

$$
\begin{aligned}
& f_{p,c}\left(t_p\left(f\right)+\pi_{p,c}-\lambda_{w,c}\right)=0 && \forall p\in P_w, w\in W, c\in\{0,1\}\\
& t_p\left(f\right)+\pi_{p,c}-\lambda_{w,c}\geq 0 && \forall p\in P_w, w\in W, c\in\{0,1\}\\
& \pi_{p,c}\geq 0 && \forall p\in P_w, w\in W, c\in\{0,1\}\\
& \pi_{p,0}=\sum_{a\in A}\delta_{ap}\gamma_a && \forall p\in P_w, w\in W\\
& \gamma_a\geq 0 && \forall a\in A\\
& \pi_{p,1}=\sum_{a\in A}\delta_{ap}\gamma_a^{o(w)} && \forall p\in P_w, w\in W\\
& \gamma_a^{o(w)}\geq 0 && \forall a\in A, w\in W
\end{aligned}
$$

Objective functions

- First-best network conditions (all links are tollable)

$$\min \quad \sum_{w \in W} \left(\int_0^{\lambda_{w,0} - \lambda_{w,1}} d_w \xi(z) z dz + \sum_{p \in P_w} (\pi_{p,0} f_{p,0} + \pi_{p,1} f_{p,1}) \right)$$

- Second-best network conditions (some of the links are untollable)

$$\min \quad \sum_{w \in W} \left(\int_0^{\lambda_{w,0} - \lambda_{w,1}} d_w \xi(z) z dz + \sum_{p \in P_w} t_p(f) f_p \right)$$

Implementation on Nine-node Network

Table : Incentive program under first-best conditions

Pricing Scheme	Distribution of β	$E(\beta)$	Toll Rev.	Privacy Cost	Total User Cost
Anonymous	-	-	887.60	0.00	887.60
Origin-specific	-	2	311.60	200.00	511.60
	-	4	311.60	400.00	711.60
	-	8	311.60	800.00	1111.60
Hybrid	$U(0,4)$	2	247.82	28.46	276.28
	$U(0,8)$	4	235.25	58.47	293.72
	$U(0,16)$	8	220.76	116.96	337.72
	$EXP(0.500)$	2	249.84	17.49	267.33
	$EXP(0.250)$	4	237.43	35.52	272.95
	$EXP(0.125)$	8	213.08	71.02	284.10

- Anonymous scheme yields highest toll revenue, and origin-specific leads to highest privacy cost.
- The hybrid scheme offers an option for travelers of high value of privacy to remain anonymous. Such a self-selection mechanism leads to much less loss of privacy and subsequently lower total user cost.

Implementation on Nine-node Network

Table : Incentive program under second-best conditions

Pricing Scheme	Distribution of β	$E(\beta)$	Travel Time	Privacy Cost	Total System Cost
Anonymous	-	-	2361.16	0.00	2361.16
Origin-specific	-	2	2306.10	200.00	2506.10
	-	4	2306.10	400.00	2706.10
	-	8	2306.10	800.00	3106.10
Hybrid	$U(0,4)$	2	2291.79	9.13	2300.92
	$U(0,8)$	4	2296.76	13.08	2309.84
	$U(0,16)$	8	2304.63	17.57	2322.20
	$EXP(0.500)$	2	2291.45	5.82	2297.27
	$EXP(0.250)$	4	2293.47	9.56	2303.04
	$EXP(0.125)$	8	2299.10	13.30	2312.40

- Anonymous scheme yields highest travel time, and origin-specific leads to highest privacy cost.
- The hybrid scheme is able to reduce total system cost.

Implementation on Sioux Falls Network

Table : Incentive program under second-best conditions

Pricing Scheme	Distribution of β	$E(\beta)$	Travel Time	Privacy Cost	Total System Cost
Anonymous	-	-	74.043	0.000	74.043
Origin-specific	-	0.02	73.060	7.212	80.272
	-	0.04	73.060	14.424	87.474
	-	0.08	73.060	28.848	101.908
Hybrid	$U(0, 0.04)$	0.02	73.294	0.118	73.412
	$U(0, 0.08)$	0.04	73.421	0.138	73.421
	$U(0, 0.16)$	0.08	73.591	0.163	73.753
	$EXP(50.0)$	0.02	73.272	0.086	73.357
	$EXP(25.0)$	0.04	73.355	0.106	73.461
	$EXP(12.5)$	0.08	73.455	0.163	73.618

- Similar observations can be made.
- Hybrid schemes perform better than anonymous or differentiated pricing schemes

Summary

Contribution:

- Explored a new class of tolling schemes that charge different amount of toll for users with different origins, destinations, or paths.
- Developed an approach for modeling location privacy of travelers
- Proposed an incentive program that allows the tolling agency to take advantage of the potentials of differentiated pricing without doing harm to privacy rights of travelers.

Conclusion:

- Differentiated pricing shows great promise in optimizing system performance
- Differentiated pricing may not be appealing for everyone
- Distribution of value of privacy has a significant effect on the acceptability of differentiated schemes
- Incentive program may create a win-win situation for all travelers

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Questions? yafeng@ce.ufl.edu