The 20th International Symposium on Transportation and Traffic Theory
July, 17-19th, 2013
Noordwijk, Netherland

Estimating MFDs in Simple Networks with Route Choice

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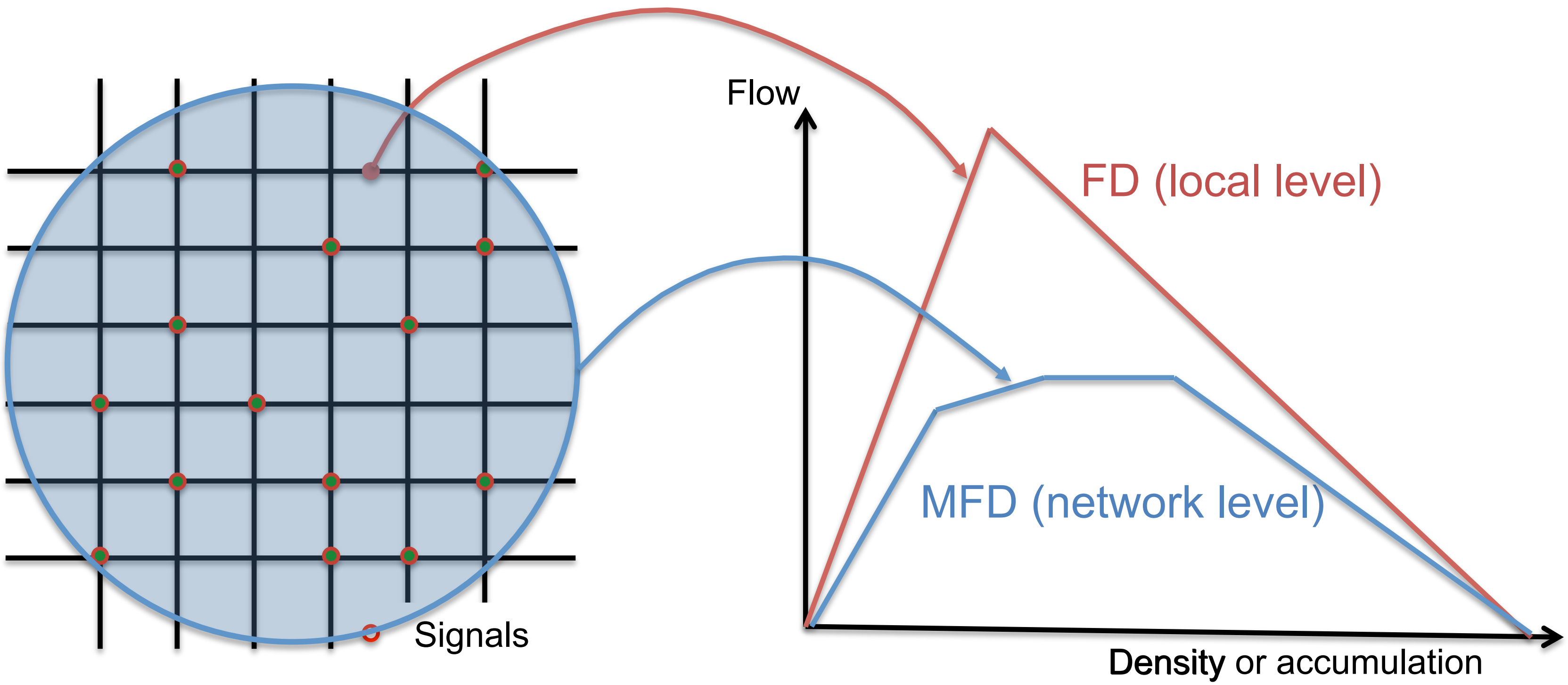
Outline

- Short recap of existing estimation methods for the Macroscopic Fundamental Diagram (MFD)
- Defining an advanced variational method for estimating MFD on a single hyperlink
- Testing the influence of route choices on the MFD for a parallel network
  - Static case
  - Dynamic case
- Conclusion

MFD definition



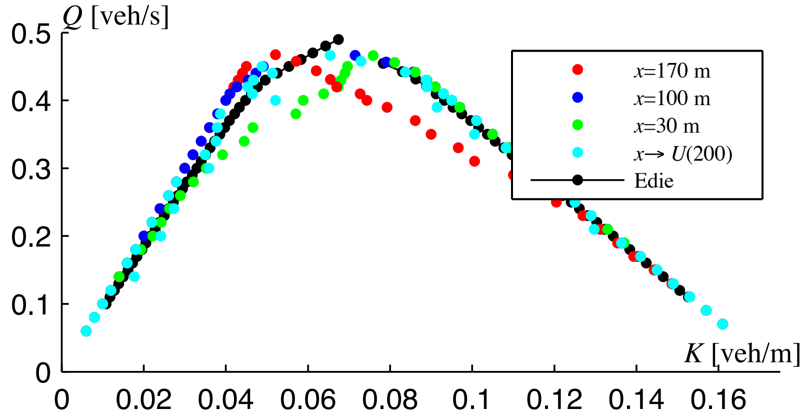
FD + Network structure (topology / signal timings) + Route choices = MFD

Existing estimation methods

- Experimental methods
  - Edie's definitions from trajectories
  - Aggregation of local information from loop detectors
  - Mean speed information gained from probe vehicles

Unbiased

Biased



(Leclercq *et al*, 2013)

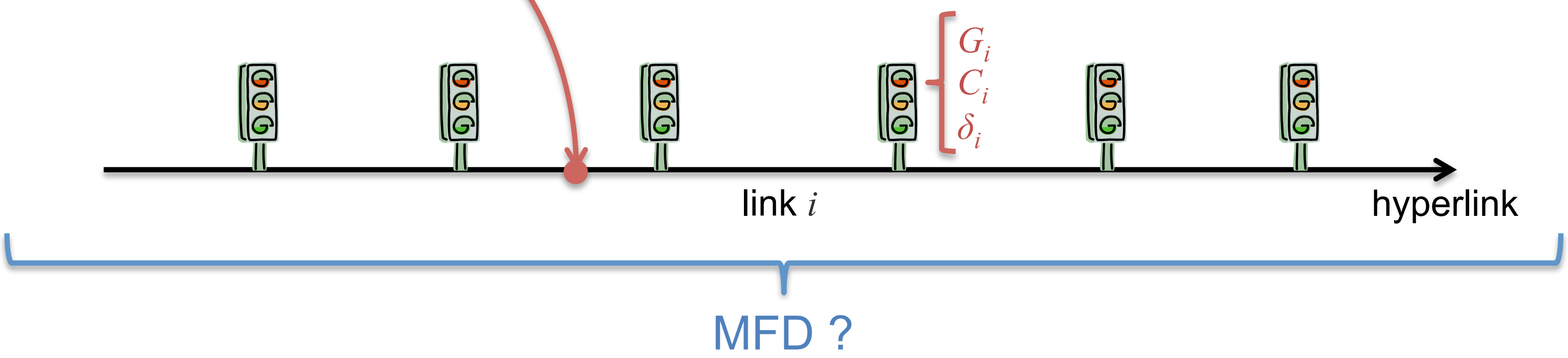
- Analytical methods
  - Variational Theory (VT) and practical cuts
    (Daganzo and Geroliminis, 2008)
    (Geroliminis and Boyacı, 2012)

Only provide a tight upper bound when the network is homogenous and regular

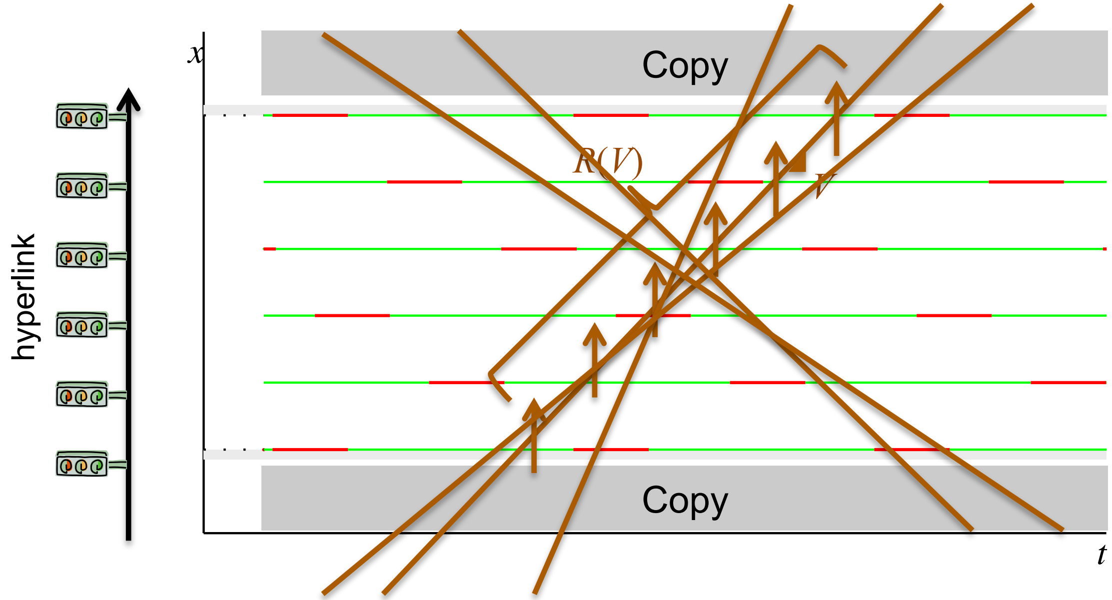
Advanced variational method for estimating MFD on single hyperlink

Estimating MFD on a hyperlink

- Definitions
  - Hyperlink: series of m successive links ended by traffic signals
  - Homogeneous traffic conditions, i.e. no or well-balanced turning flow
  - FD parameters: free-flow speed u, wave speed w, jam density κ



Analytical estimation of MFD

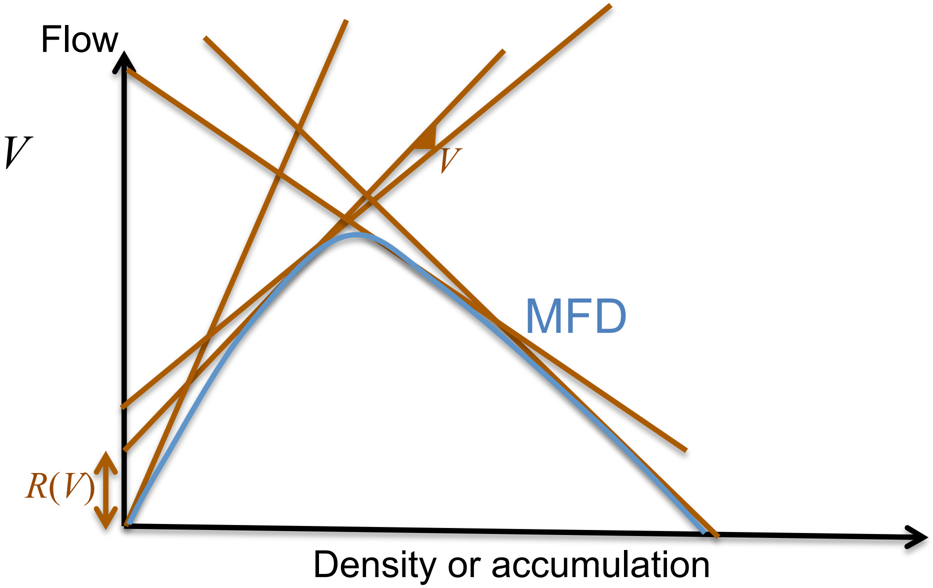


(Daganzo and Geroliminis, 2008)

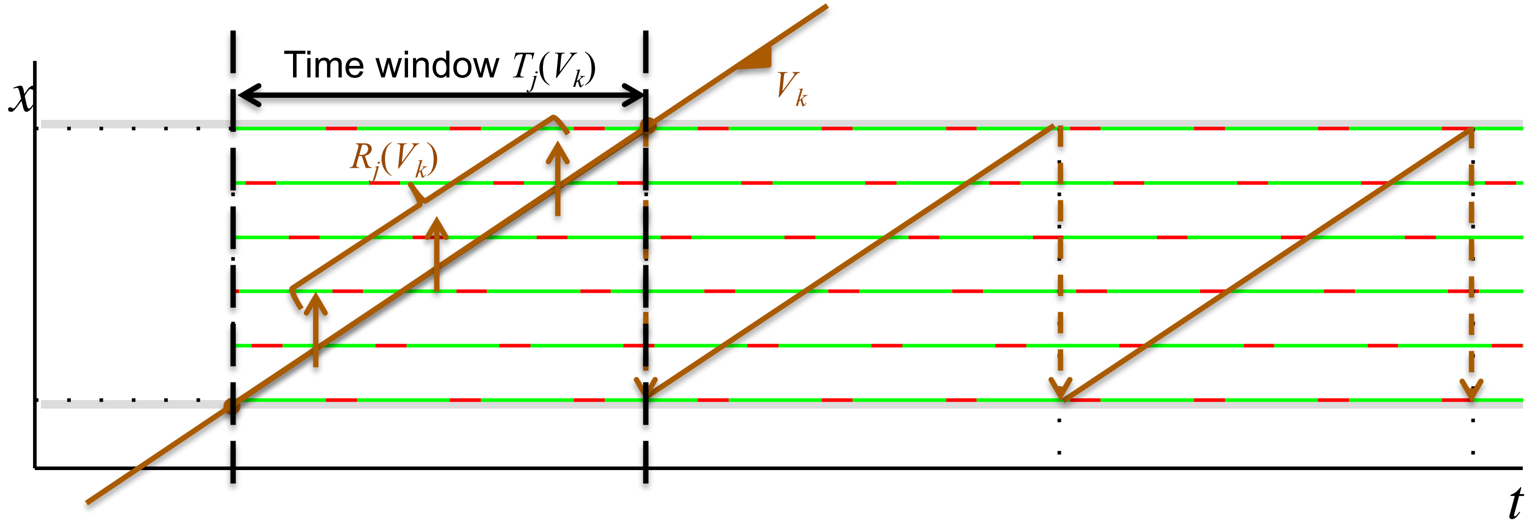
Moving observer with mean speed V

Maximum passing rate $R(V)$

Cut: $Q=\min_V(KV+R(V))$



Defining Time Windows



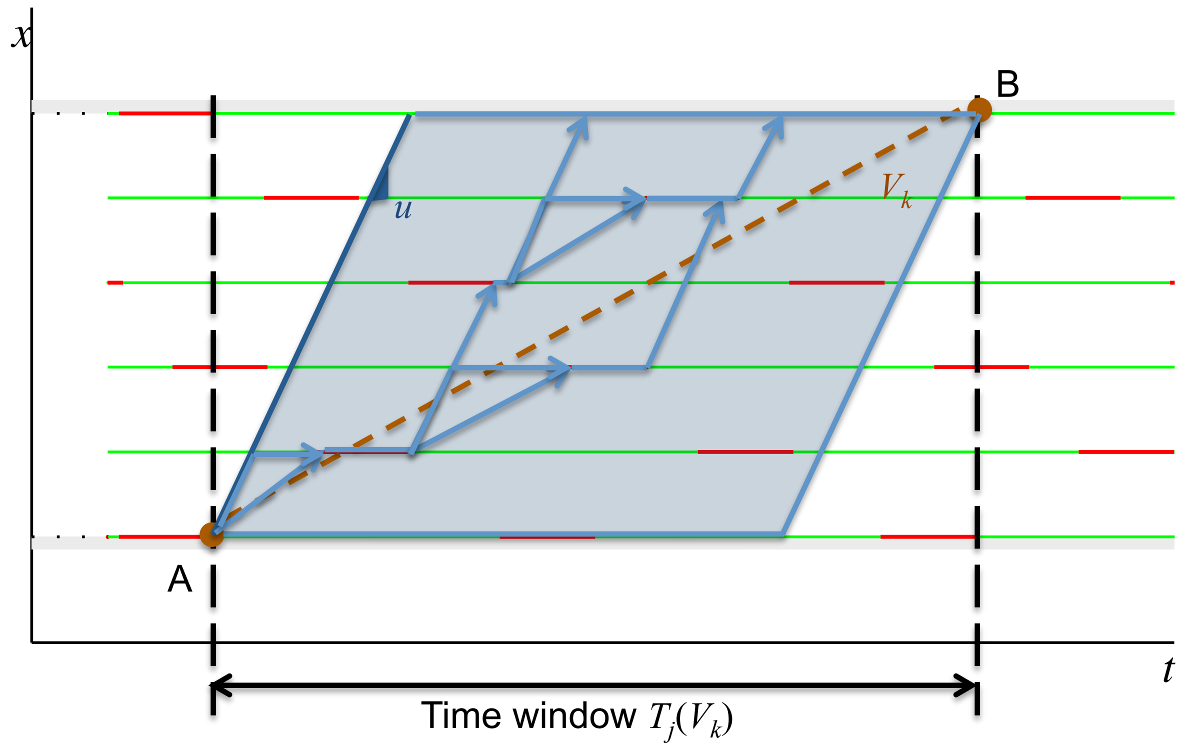
We only consider a discrete set of moving observers with speed $\{V_k\}$

One travel across the hyperlink at V_k defines the time window $T_j(V_k)$

Travelling across infinite copies of the hyperlink is equivalent to travelling across successive time windows: $R(V_k)=\text{mean}(R_\text{j}(V_k))$

One only has to calculate $R_j(V_k)$ over a sufficient number of T_j (law of large numbers)

Calculating $R_j(V_k)$ with VT



To minimize costs, the observer has to maximize the time spent on red phases – Shortcuts theory (Daganzo and Menendez, 2005)

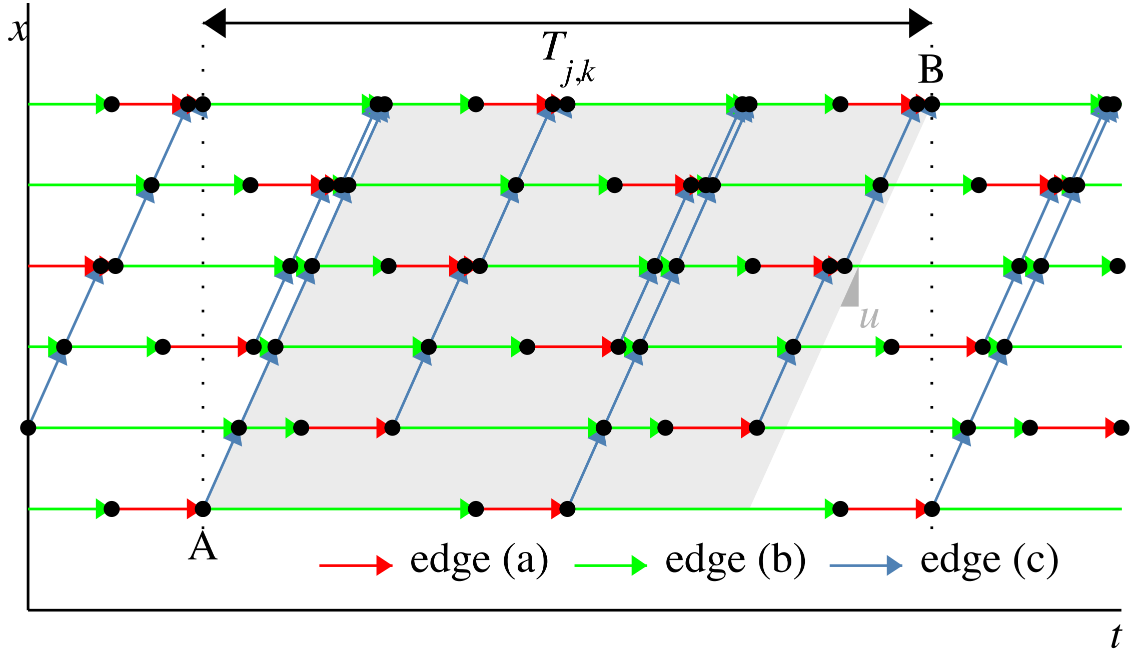
Internal subpaths with $v<u$ may be replaced at same costs.

A graph can simply be constructed to explore all the possible paths

Sufficient variational graph (V_k>0)

The graph is defined by three type of edges:

- edge (a): red phase (cost 0)
- edge (b): green phase (cost $s=uw\kappa/(u+w)$)
- edge (c): path with speed u that starts at the end of red phases (cost 0)



This graph is proved to be sufficient

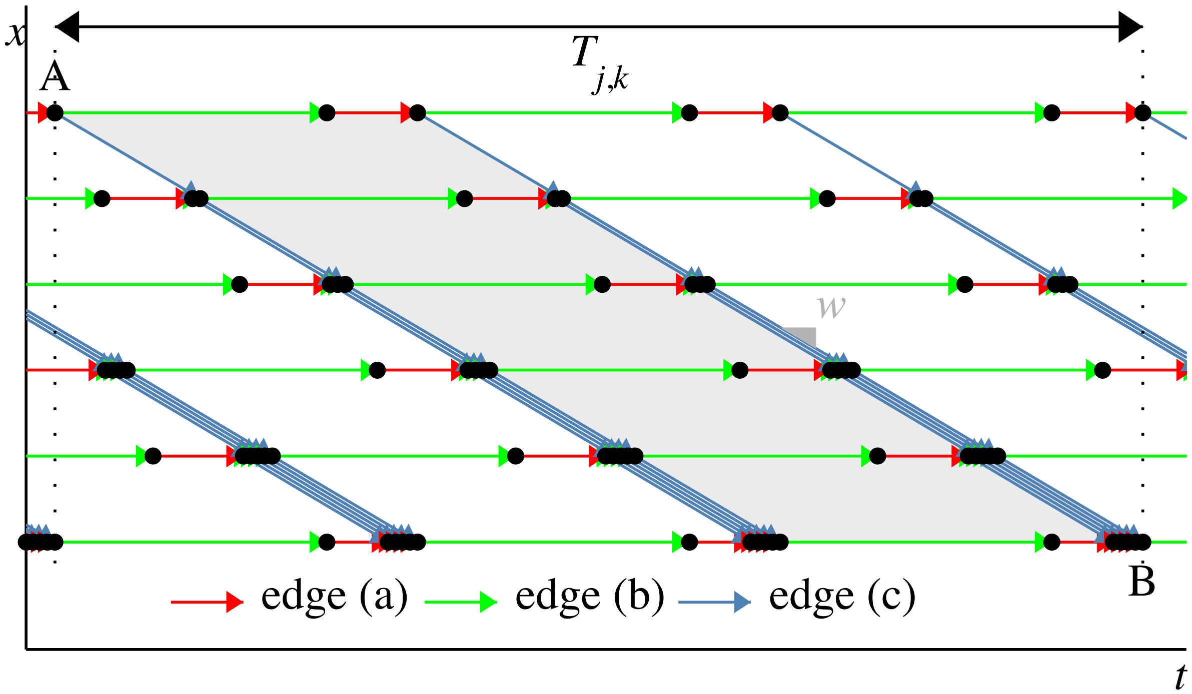
A classical shortest path algorithm provides $R_j(V_k)$

The same graph can be used for all $V_k>0$ (only the ending points change)

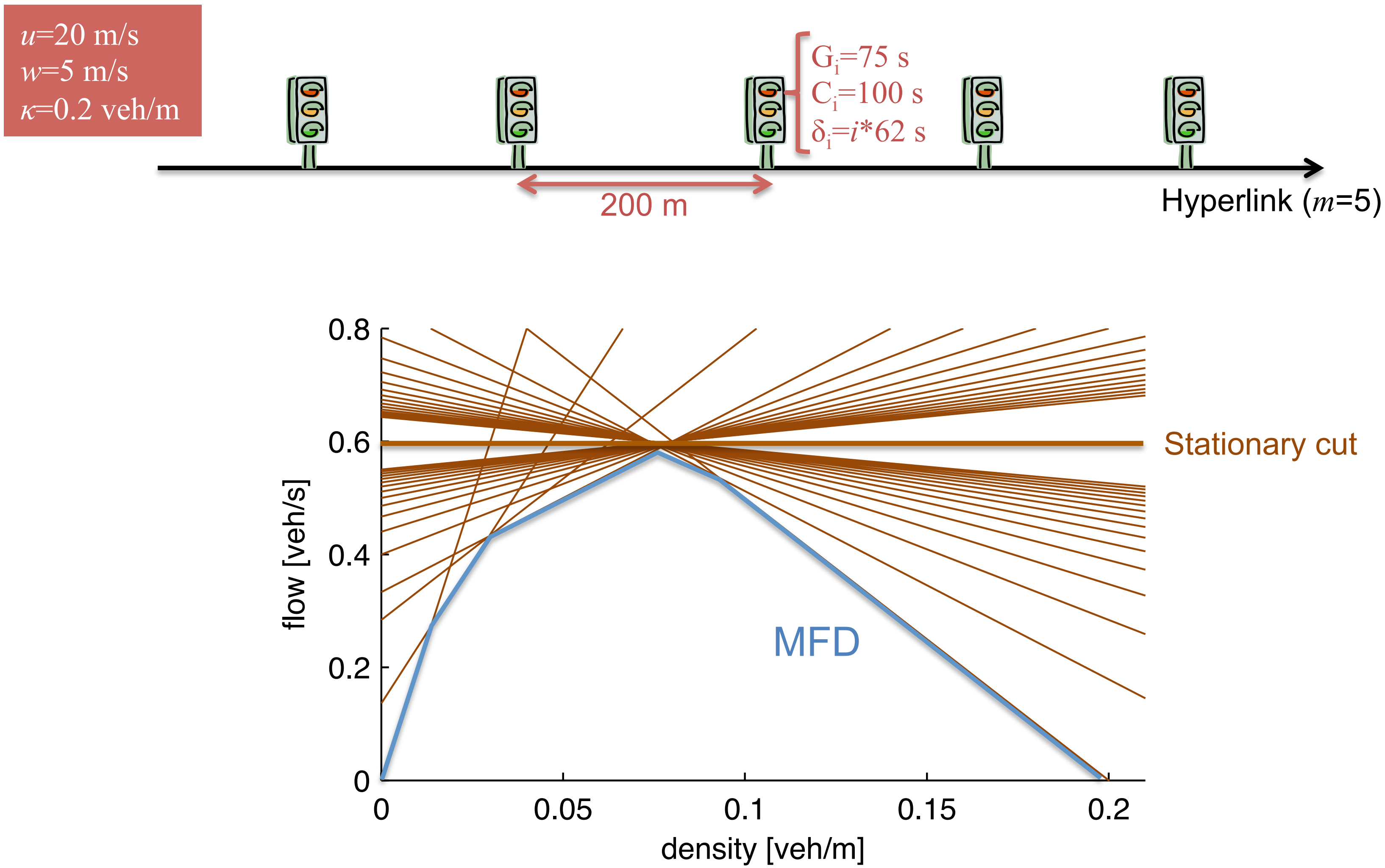
Sufficient variational graph (V_k<0)

A similar sufficient graph can be constructed when V_k<0.

Edges (c) have a slope $-w$ and a cost $w\kappa$ in that case.

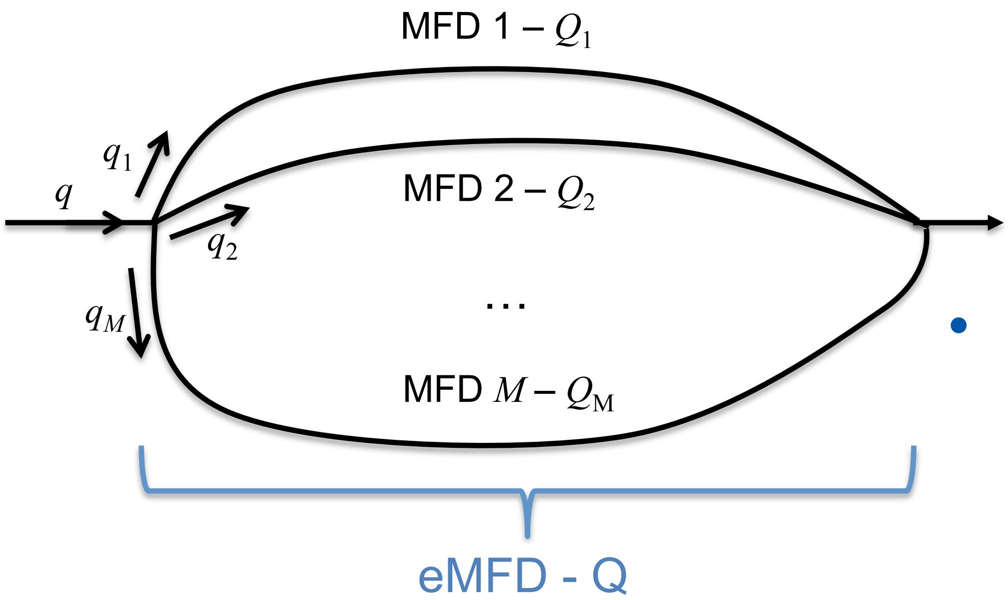


Resulting MFD for an example



Influence of route choices on the MFD

Estimating MFD on a parallel network



- Different route choice assumptions
  - User Equilibrium (UE) (Wardrop, Logit)
  - System Optimum (SO)
- Different traffic conditions
  - Static equilibrium state (free-flow and congestion)
  - Dynamic loading (trapezoidal time dependent demand)

UE and SO calculations

- Static conditions (constant upstream demand or downstream supply)
  - UE: we first derive $v_i=F_i(v_1)$ and then calculate the eMFD

$$\begin{cases} q = \sum_i q_i = \sum_i Q_i(v_i) = \sum_i Q_i(F_i(v_1)) \\ n = \sum_i n_i = \sum_i \frac{q_i L_i}{v_i} = \sum_i \frac{Q_i(F_i(v_1)) L_i}{F_i(v_1)} \end{cases}$$

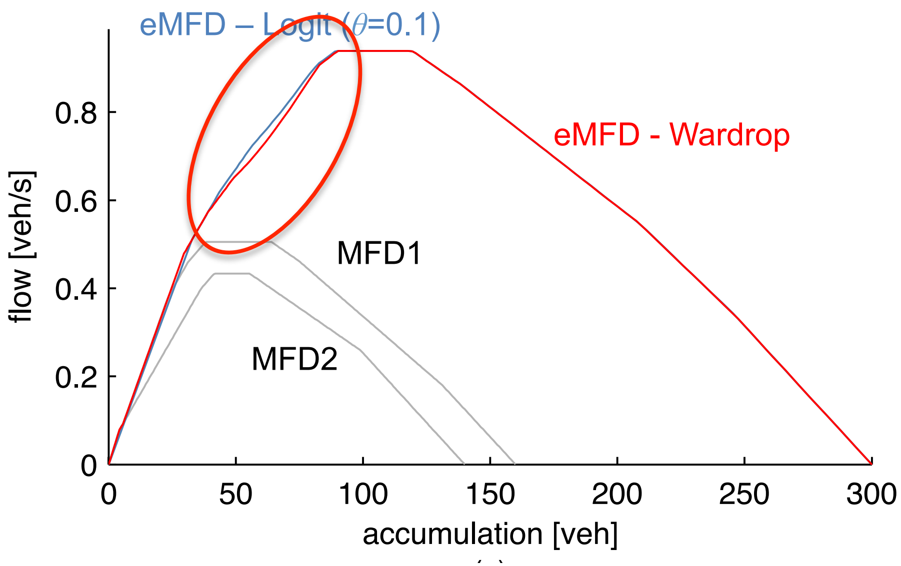
  - SO: equilibrium is defined by

$$\begin{cases} \min\left(\sum_i q_i \tau_i\right) = \min\left(\sum_i n_i\right) \\ \sum_i q_i = q \quad \text{with} \quad q_i \geq 0 \quad \text{and} \quad q_i = Q_i(n_i) \end{cases}$$

- Dynamic conditions
  - System dynamics is described by: $\frac{dn_i}{dt} = f_i - Q_i(n_i)$
  - We then apply the equilibrium rule

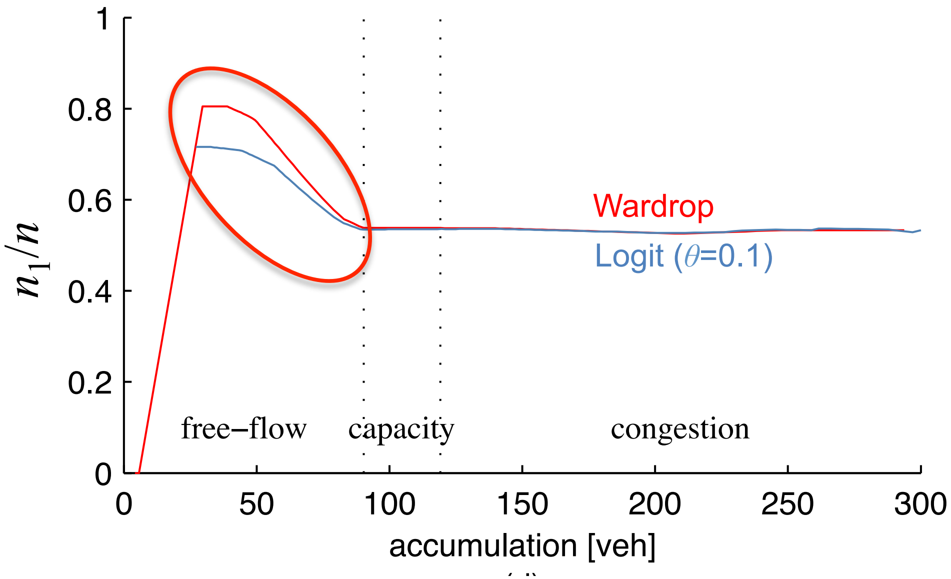
Details on aggregation methods and proofs are provided in the paper

Static User Equilibrium (M=2)

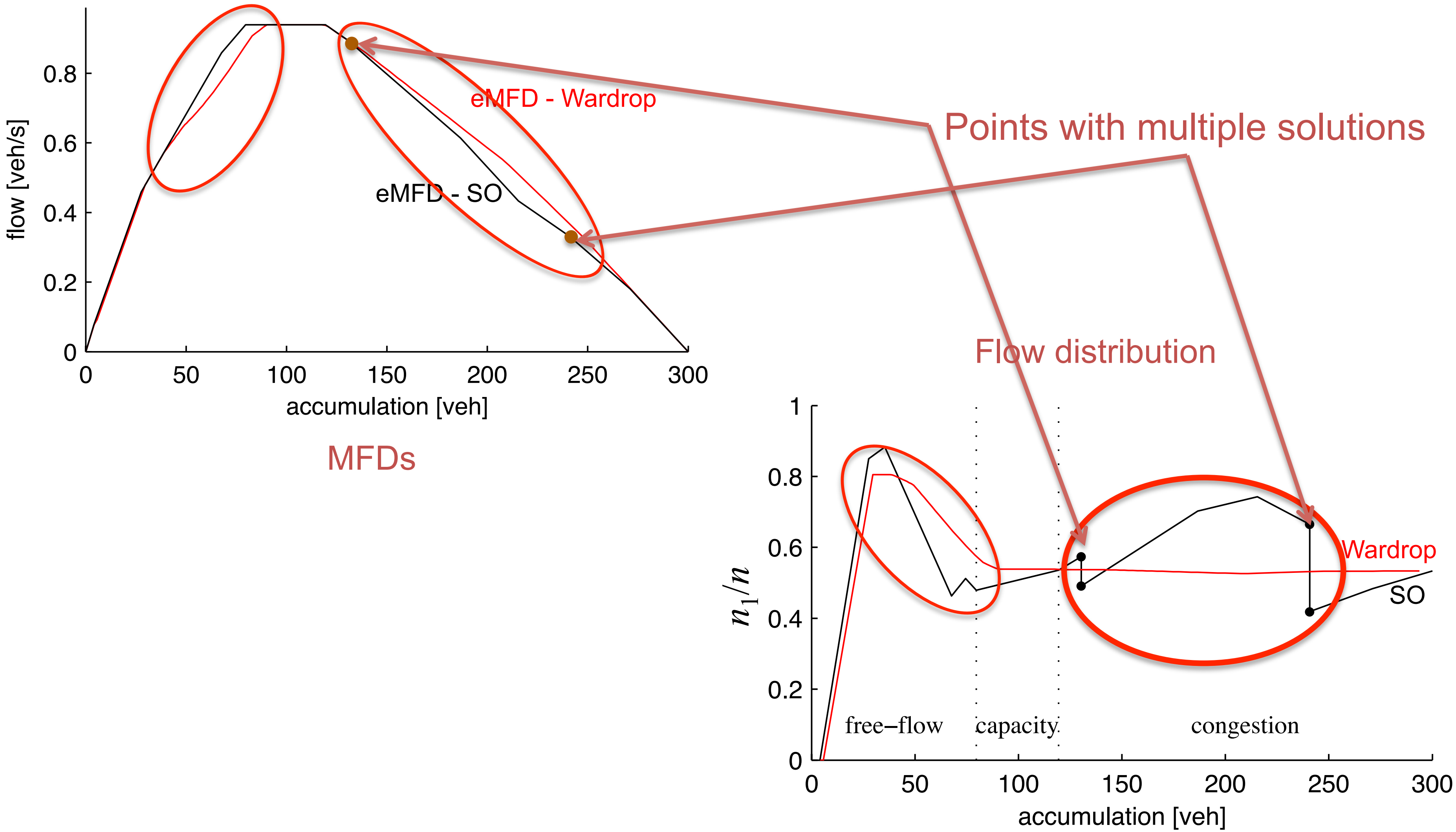


MFDs

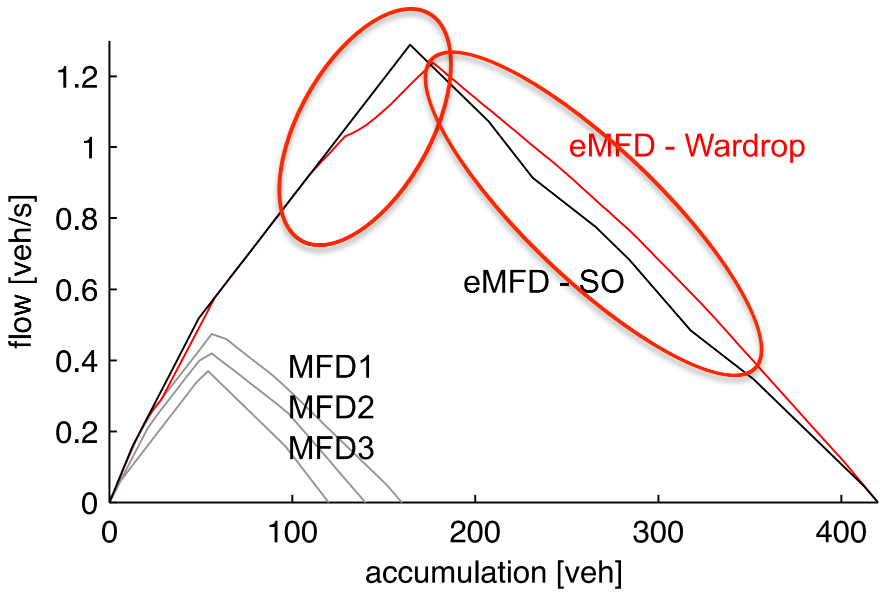
Flow distribution



Static System Optimum (M=2)

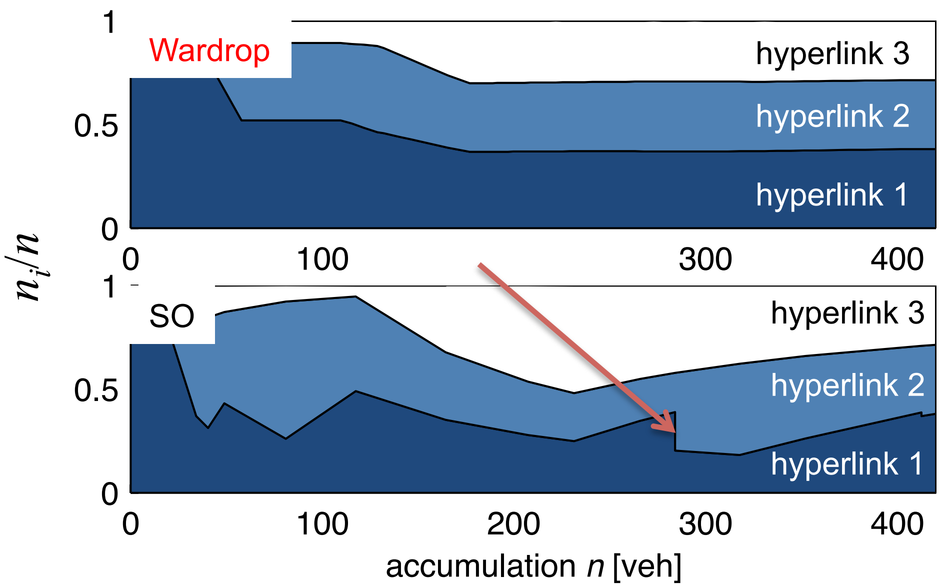


Static UE and SO (M=3)

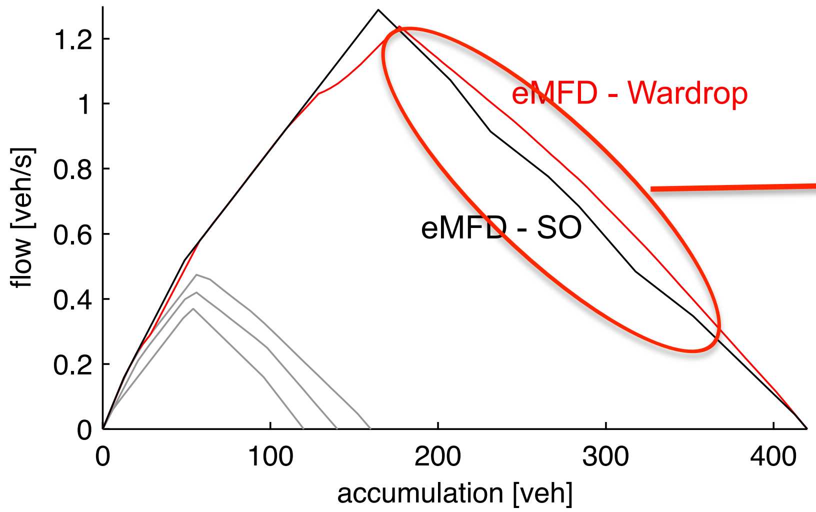


MFDs

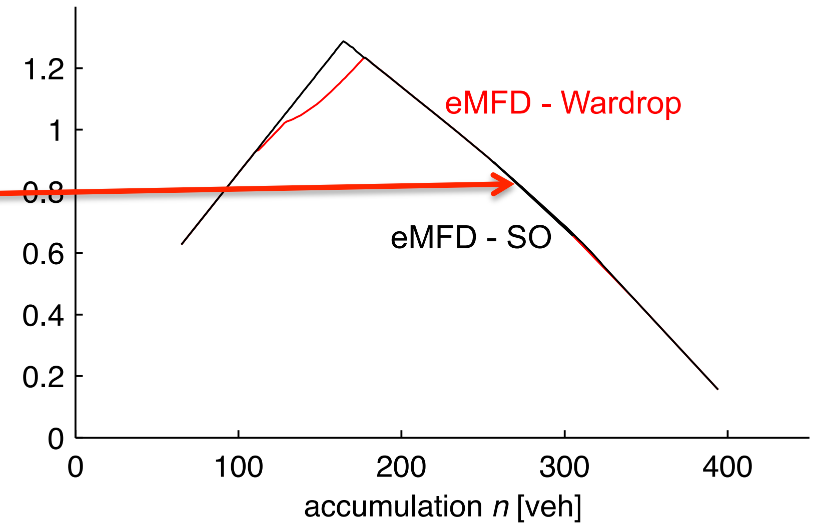
Flow distribution



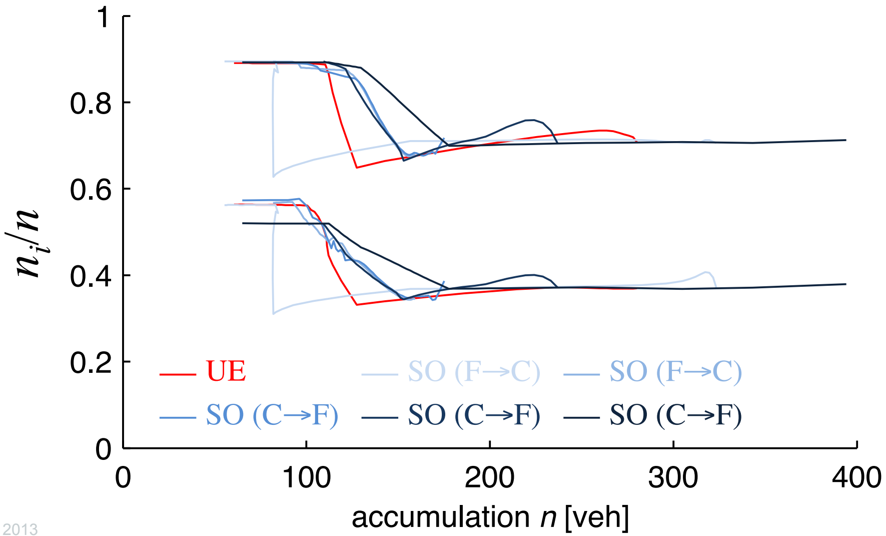
Dynamic loading (M=3)



Static case



Dynamic loading



Split ratios differently evolve during the onset and the offset of congestion

Final network state depend on the initial condition

Conclusion

- Contrary to previous statements, route choices influence the MFD especially close to the network capacity
- UE and SO are close in congestion
  => few potential savings with traffic management
- Static SO is not always reachable through dynamic loading

- Further researches are needed to extend results to more complex networks and to account for heterogeneous traffic conditions

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Thank you for your attention

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Some remarks

- The variational graphs include the practical cuts (observers that stop every k signals) defined in (Daganzo and Geroliminis, 2008)
- The variational graphs provide a tight bound for the MFD without any regularity conditions
- The method is still restricted to homogeneous case. Otherwise, it only provide an upper bound.