A multi-commodity LWR model of lane-changing traffic flow

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Outline

- Introduction
- Behavioral LC fundamental diagram
- A multi-commodity LWR model
- Application: lane-drop bottleneck
- Conclusion

The LWR model

- State variables: density $k(x,t)$, speed $v(x,t)$, flow-rate $q(x,t)$
- Five rules:

R1. Constitutive law: $q = kv$

R2. Fundamental diagram: $v = V(x,t,k)$ or $q = Q(x,t,k)$

R3. Traffic flow conservation: $\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0$

R4. Weak solution: discontinuous shock waves

R5. Entropy condition: unique solutions

- The LWR model: $\frac{\partial k}{\partial t} + \frac{\partial kV(x,t,k)}{\partial x} = 0$

Multi-commodity LWR model

- Commodity variables: vectors of densities $\boldsymbol{k}(x,t)$, speeds $\boldsymbol{v}(x,t)$, flow-rates $\boldsymbol{q}(x,t)$
- Total variables: $k = \sum_{\omega\in\Omega} k_\omega$, $q = \sum_{\omega\in\Omega} q_\omega$, $v = \frac{q}{k}$
- Five rules:

  R1'. Constitutive law: $\boldsymbol{q} = \boldsymbol{k} \circ \boldsymbol{v}$

  R2'. Fundamental diagram: $\boldsymbol{v} = \vec{V}(x,t,\boldsymbol{k})$ or $\boldsymbol{q} = \vec{Q}(x,t,\boldsymbol{k})$

  R3'. Traffic flow conservation: $\frac{\partial \boldsymbol{k}}{\partial t} + \frac{\partial \boldsymbol{q}}{\partial x} = 0$

  R4'. Weak solution: discontinuous shock waves

  R5'. Entropy condition: unique solutions
- The LWR model: $\frac{\partial \boldsymbol{k}}{\partial t} + \frac{\partial \boldsymbol{k}\circ\vec{V}(x,t,\boldsymbol{k})}{\partial x} = 0$

Multi-commodity LWR models (cont'd)

- **Partial flow** (Lebacque, 1996, Section 7): homogeneous FD; First-in-first-out (FIFO)
- **Special lane** (Daganzo, 1997): commodity-dependent FD; one-/two-pipe regimes
- **Multi-population** (Benzoni-Gavage and Colombo, 2003): heterogeneous drivers, commodity-dependent FD
- **Multi-class** (Wong and Wong, 2002): heterogeneous drivers, homogeneous FD
- **Mixed traffic** (Zhang and Jin, 2002): commodity-independent FD, but weighted by free-flow speeds; FIFO
- (Chanut and Buisson, 2003; Burger and Kozakevicius, 2007; Burger et al., 2008; van Lint et al., 2008; Ngoduy, 2010)

Lane-changing (LC) traffic

Lane-changing models

- Microscopic models (Gipps, 1986)
  - whether, why, when, where, and how
- Hybrid: moving bottleneck (Laval and Daganzo, 2006)
- Macroscopic weaving area capacity and characteristics (HCM): regression models
- Kinematic wave model (Jin, 2010)

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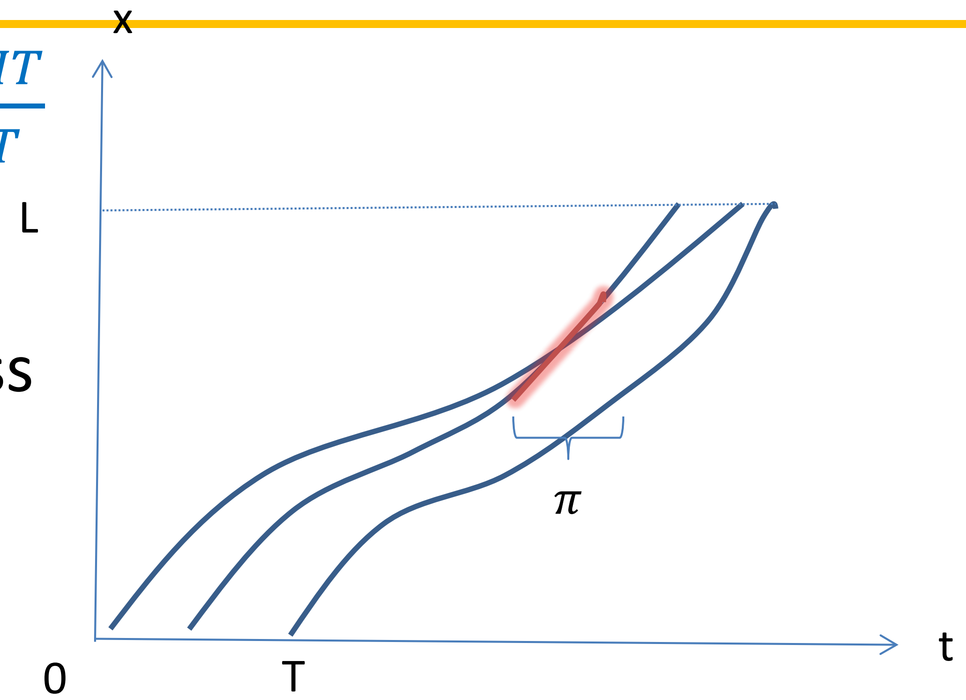
Lane-changing intensity: $\epsilon(x,t)$

- Edie's formula: $k = \frac{VHT}{LT}$
- **Observation #1**: one vehicle occupies two lanes during LC process
- Effective density: $\bar{k} = \frac{VLHT}{LT} = \frac{VHT+N_{LC}\cdot\pi}{LT}$
- LC intensity: $\epsilon = \frac{N_{LC}\cdot\pi}{kLT} = \frac{LC\ Time}{VHT}$
- Speed-density relation: $v = V(\bar{k}) = V\left(k(1+\epsilon)\right)$



The lane-changing LWR model

- The LWR model:

$$\frac{\partial k}{\partial t} + \frac{\partial k\, V\big(k(1+\epsilon)\big)}{\partial x} = 0$$

- **Intensity-density relation**:

$$\epsilon = E(x, k)$$

- The LC-LWR model (**single commodity**):

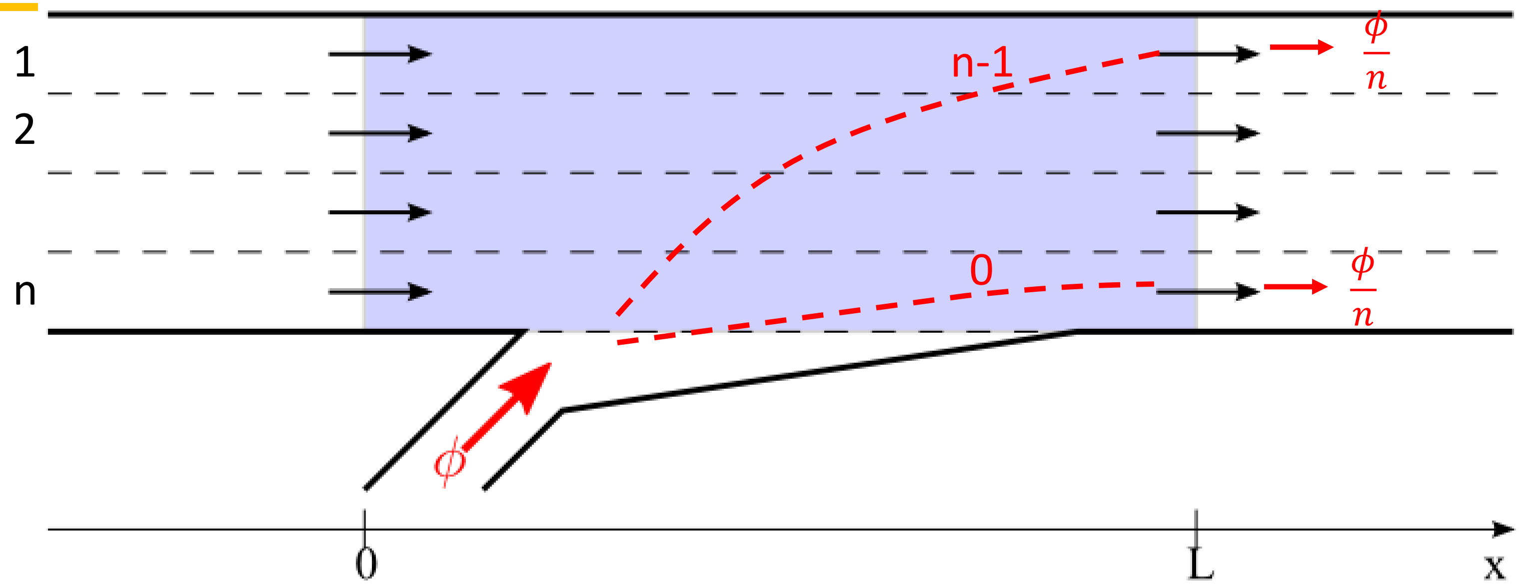
$$\frac{\partial k}{\partial t} + \frac{\partial k\, V\big(k(1+E(x,k))\big)}{\partial x} = 0$$

- An inhomogeneous LWR model:
  – Entropy condition by (Isaacson and Temple, 1992)

From single- to multi-commodity

- Single-commodity LC-LWR model simple but limited
  - Exogenous intensity-density relation
  - Phenomenological, not behavioral
- Two types of vehicles:
  - Weaving vehicles: who can NOT continue on their current lane
  - Non-weaving vehicles
- **Observation #2**:
  - Lane changes induced by weaving vehicles

Macroscopic LC model



- #/LC: $N_{LC} = \frac{\phi}{n} T \cdot 1 + \cdots + \frac{\phi}{n} T \cdot (n-1) = \frac{n-1}{2} \phi T$
- Constant lane-changing duration: π
- Intensity: $\epsilon = \alpha \frac{\phi}{k}$
  - $\alpha = \frac{n-1}{2L} \pi$

Behavioral LC fundamental diagram

- Car-following fundamental diagram: $v = F(z)$
  - z: per lane density
- LC fundamental diagram:

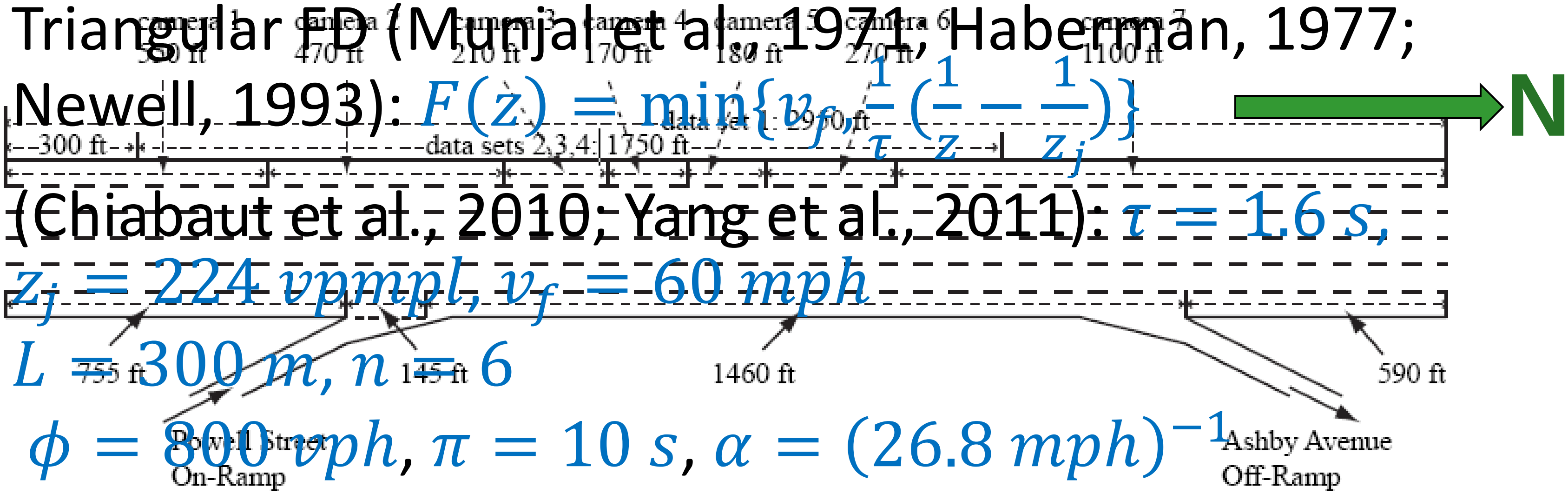
$$v = F\left(\frac{k(1+\epsilon)}{n}\right) = F\left(\frac{k+\alpha\phi}{n}\right) = F\left(\frac{(1+\alpha\xi v)k}{n}\right)$$

- $v = V(k, \phi)$ or $v = V(k, \xi)$

	Density	Flow-rate	Speed	Proportion
Total traffic	k	q	v	
Weaving traffic	ρ	ϕ	v	ξ

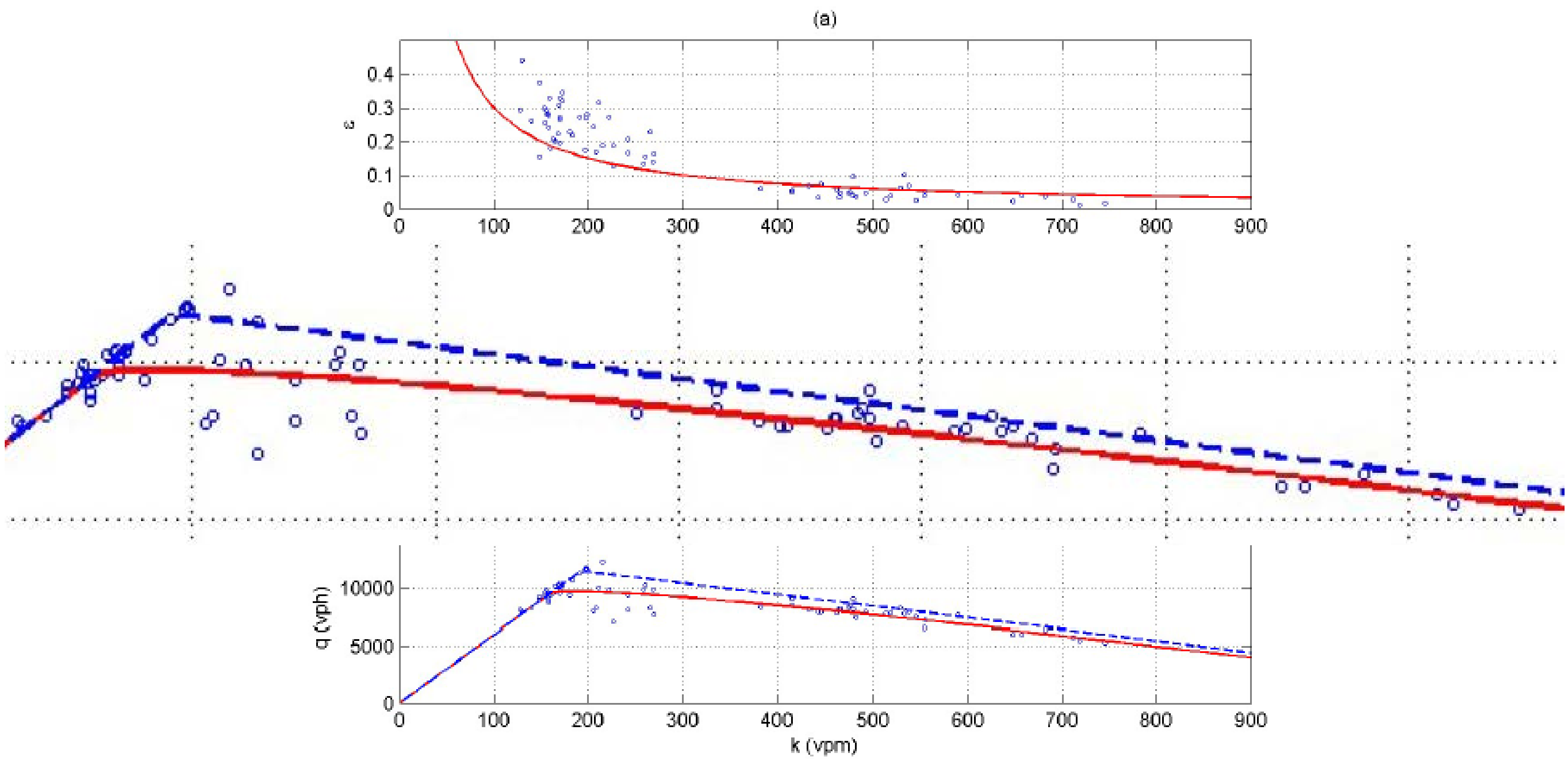
Calibration

- Triangular FD (Munjal et al., 1971; Haberman, 1977; Newell, 1993): $F(z) = \min\{v_f, \frac{1}{\tau}(\frac{1}{z} - \frac{1}{z_j})\}$
- (Chiabaut et al., 2010; Yang et al., 2011): $\tau = 1.6\ s$, $z_j = 224\ vpmpl$, $v_f = 60\ mph$
- $L = 300\ m, n = 6$
- $\phi = 800\ vph$, $\pi = 10\ s$, $\alpha = (26.8\ mph)^{-1}$



Data set	On-ramp flow	Time (seconds)	ϕ (vph)	π (seconds)	$\alpha\phi$ (vpm)
1	314	1742	649	11.0	26.4
2	190	882	775	8.5	24.4
3	206	864	858	9.8	31.1
4	215	907	853	12.4	39.2

Validation


(a)
ε
0.4
0.3
0.2
0.1
0
0 100 200 300 400 500 600 700 800 900
q (vph)
10000
5000
0
0 100 200 300 400 500 600 700 800 900
k (vpm)

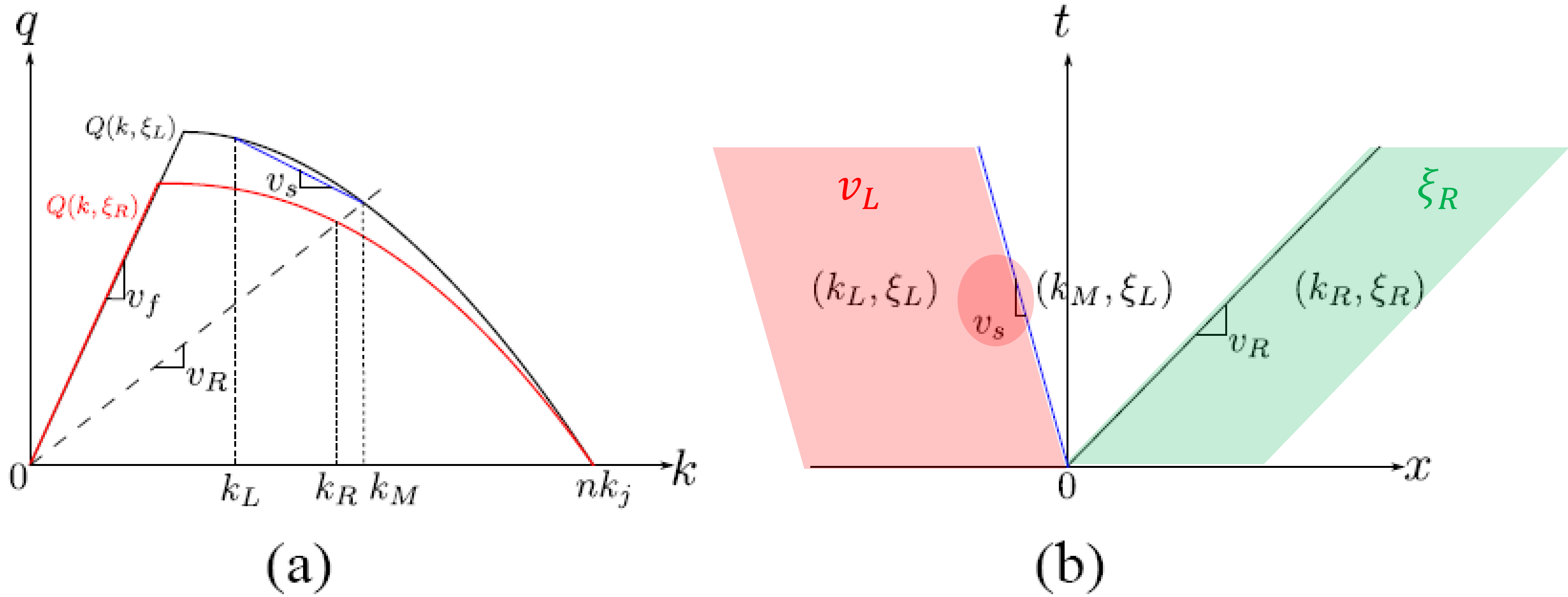
Multi-commodity LC-LWR model

- Multi-commodity model:

$$\frac{\partial k}{\partial t} + \frac{\partial kV(k,\rho)}{\partial x} = 0$$
$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho V(k,\rho)}{\partial x} = 0$$

- FIFO principle: weaving and non-weaving vehicles share the same speed
- Riemann problem: $(k,\xi) = \begin{cases} (k_L,\xi_L), & x < 0 \\ (k_R,\xi_R), & x > 0 \end{cases}$
- Entropy conditions:
  (i) Downstream speed not impacted by upstream traffic
  (ii) Deceleration=shock wave; acceleration=rarefaction wave (Ansorge, 1990)
  (iii) Upstream proportion propagates with vehicles

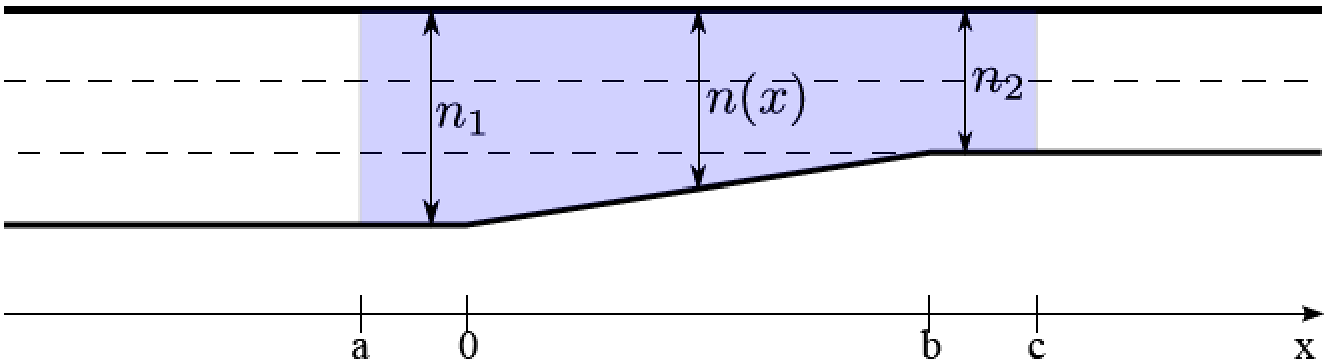
Riemann solution of shock waves



- Demand/supply:
  - Demand: $D(k_L,\xi_L) = Q(\min\{k_L, k_{c(\xi_L)}\}, \xi_L)$
  - Supply: $S(k_M,\xi_L) = Q(\max\{k_M, k_c(\xi_L)\}, \xi_L)$
- Boundary fluxes:

$$q(x=0,t) = \min\{D(k_L,\xi_L), S(k_M,\xi_L)\}$$
$$\phi(x=0,t) = \xi_L q(x=0,t)$$

Application: Lane-drop bottleneck



- Constant: $\xi = \frac{\rho}{k} = \frac{n_1 - n_2}{n_1}$
- The LWR model: $\frac{\partial k}{\partial t} + \frac{\partial Q(n(x), k, \xi)}{\partial x} = 0$
- Fundamental diagram:

$$k = K_2(n(x), v, \xi)$$
$$v = \min\{v_f, K_2^{-1}(n(x), k, \xi)\}$$
$$q = qv$$
$$\xi = \alpha(x)\xi v$$

Smoothing effects of HOV lanes

- Without HOV lanes: capacity=$Q^{max}(n_1)$
- One HOV lane: throughput=$Q_{HOV} + Q^{max}(n_1 - 1)$
- Increased throughput $\approx Q_{HOV} - 1400$=**500 vph**
- Observed: **300-600 vph** (Cassidy et al., 2010)

n_1	2	3	4	5	6	7	8	9	10
$(n_1-1)z_c v_f$	1926	3852	5778	7704	9630	11556	13482	15408	17334
$Q^{max}(n_1)$	1926	3352	4738	6126	7515	8903	10291	11679	13067
CR	0	13%	18%	20%	22%	23%	24%	24%	25%
$Q^{max}(n_1) - Q^{max}(n_1-1)$		1426	1386	1388	1389	1388	1388	1388	1388

Conclusion

- Behavioral LC fundamental diagram
  - Macroscopic lane-changing model
  - Calibrated and validated with NGSIM data
- Multi-commodity LWR model
  - Existence and uniqueness of Riemann solutions
- Applications to lane-drop bottleneck
  - Successfully predict smoothing effect of HOV lanes
- Model is behavioral, transferrable
  - LC area characteristics
  - Geometry
  - LC duration
  - Weaving flows

Future studies

- Validation with loop detector data
  - Compare with HCM models
- Merging bottlenecks
  - Interactions among merging and lane-changing traffic
  - Capacity drop
- Diverging, weaving and other bottlenecks
  - Capacity
  - Dynamics
- Microscopic lane-changing model
- Alleviate congestion effects of LC traffic
  - Lane-changing duration ⬇
  - Lane-changing area length ⬆
  - Lane-changing number ➡

Thank you!
Questions?