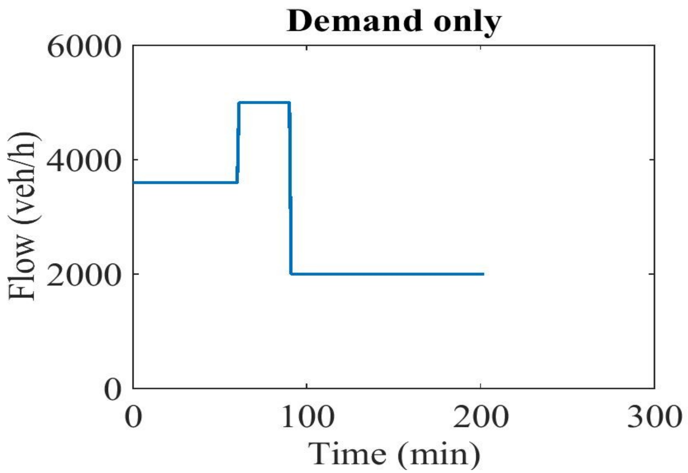
Cumulative curves

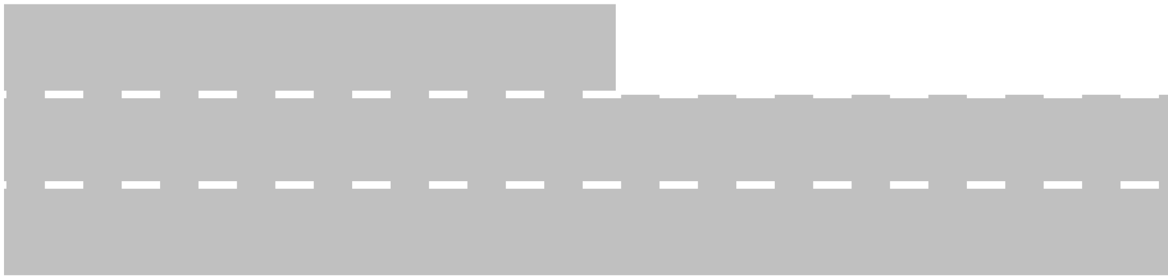
Calculation of delays and queues

01-02-17

Content for today

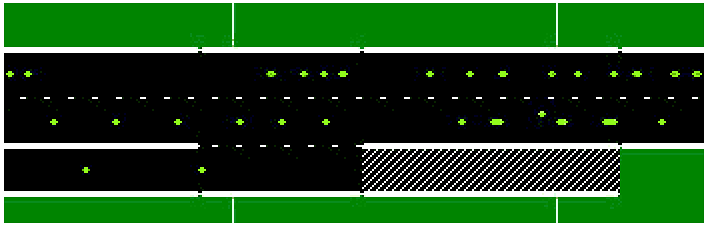
After this lecture, you are able to
Calculate delays and queue length with a vertical queuing model



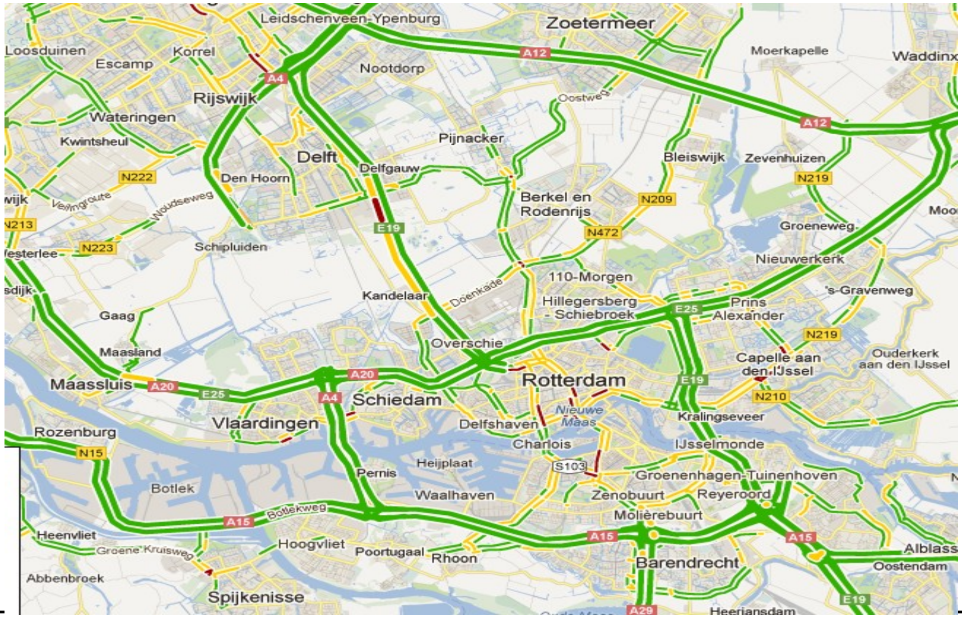


Levels of description

- Microscopic



- Macroscopic

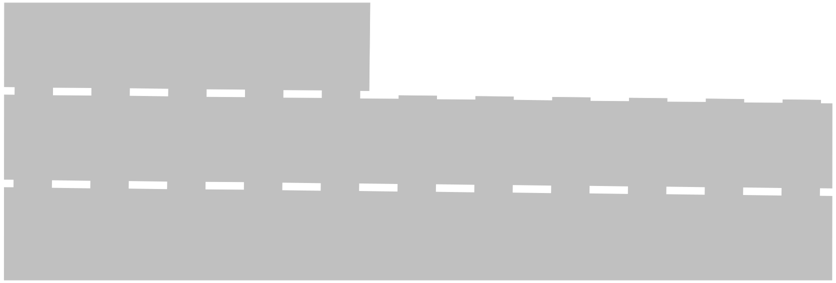


Capacity

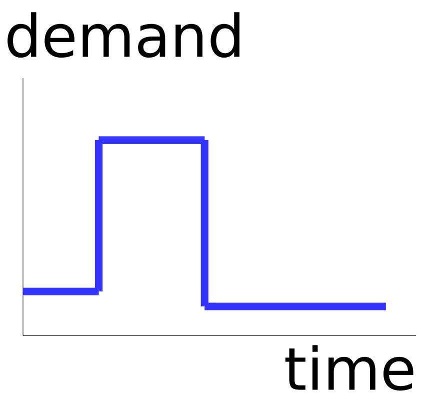
- Flow is the number of vehicles per unit of time (veh/h)
- **Capacity** is the maximum flow on a cross section
- Capacity is determined by
  - minimum headway (motorway: ~1.5-2 seconds)
  - number of lanes

Queuing model

- Given the following road profile, estimate the capacity



Content for today



1

2

Cumulative curves

Total delay
Av. travel time

3

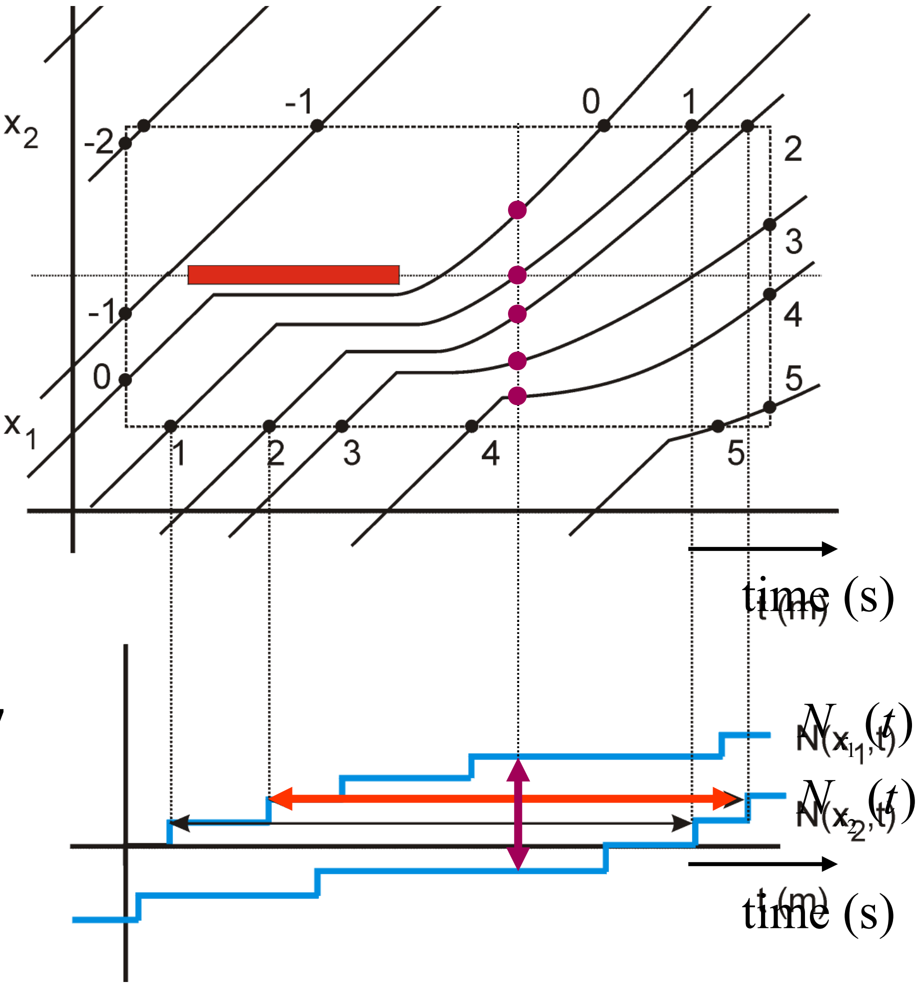
4

5

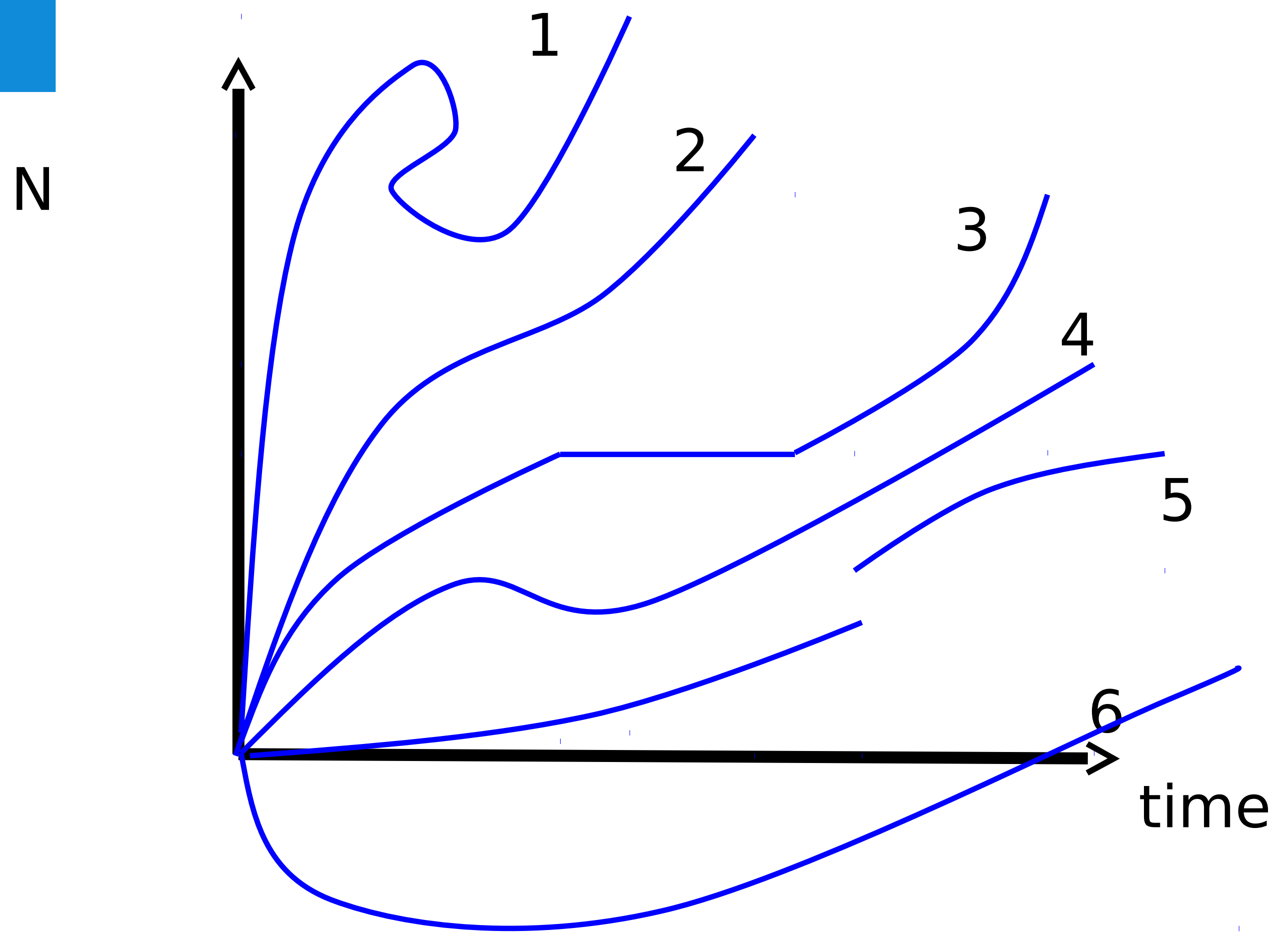
Cumulative curves

Cumulative vehicle plots

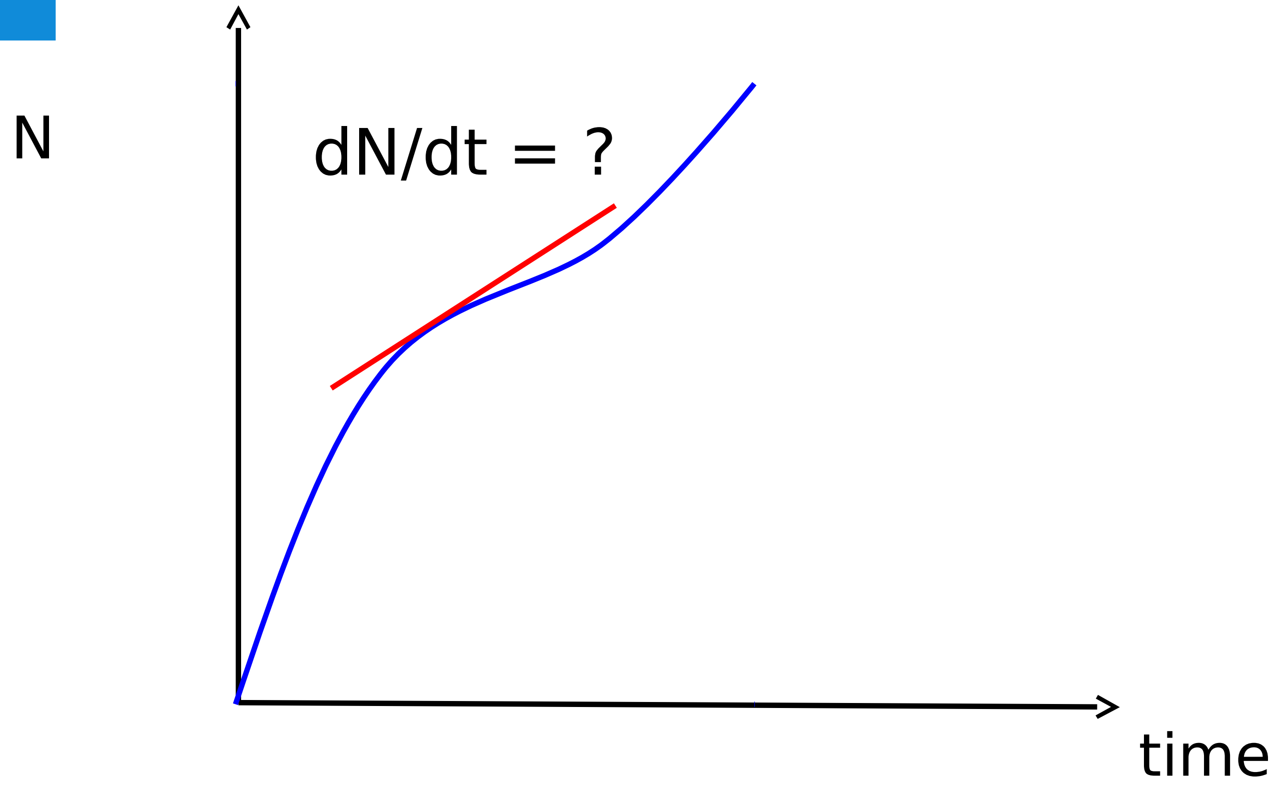
- Cumulative flow function $N_x(t)$: number of vehicles that have passed cross-section x at time instant t
- $N_x(t)$: step function that increases with 1 each time instant vehicle passes
- Often simplified to smooth curv



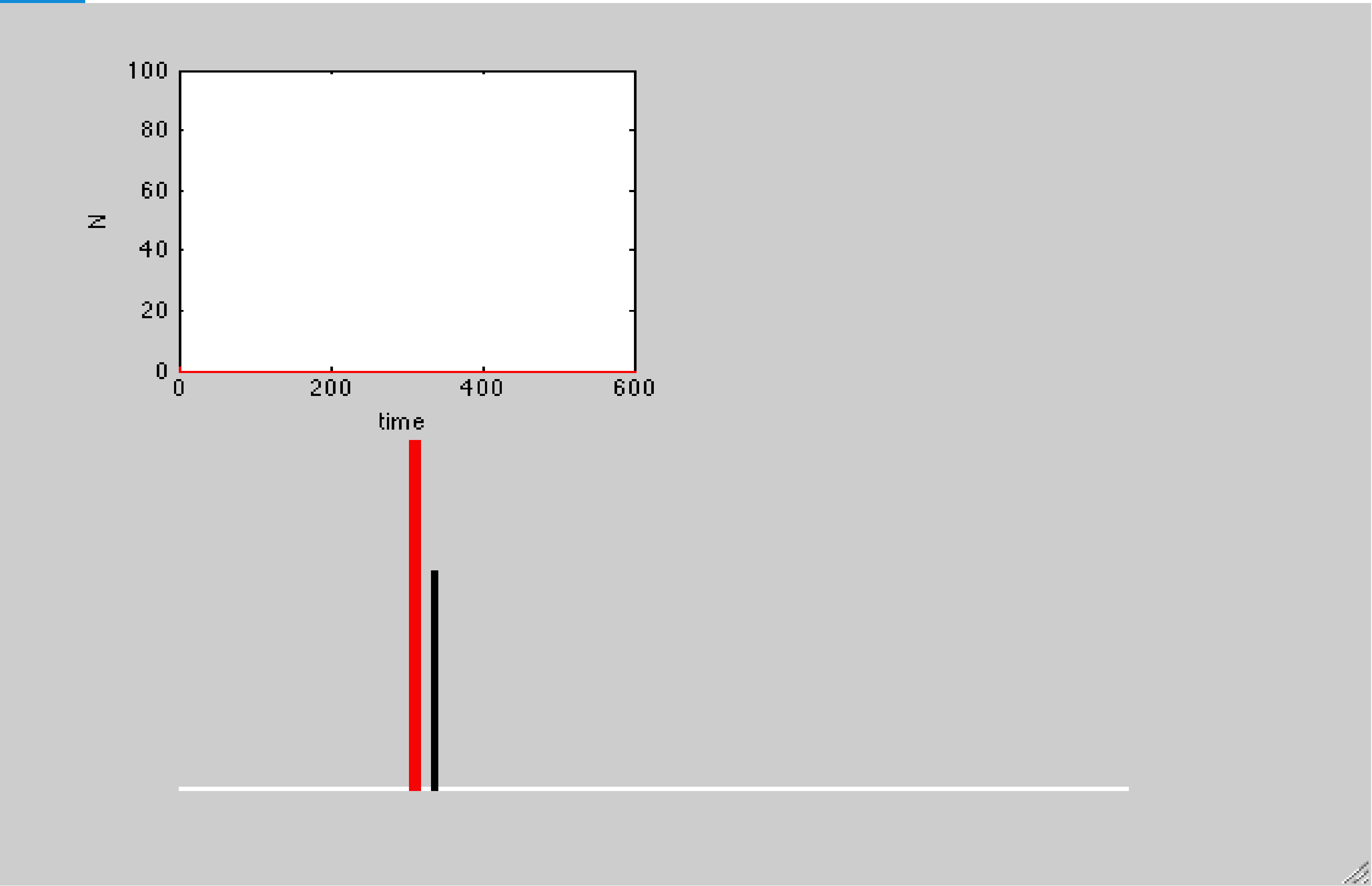
Examples of cumulative curves?



Information in cumulative curves

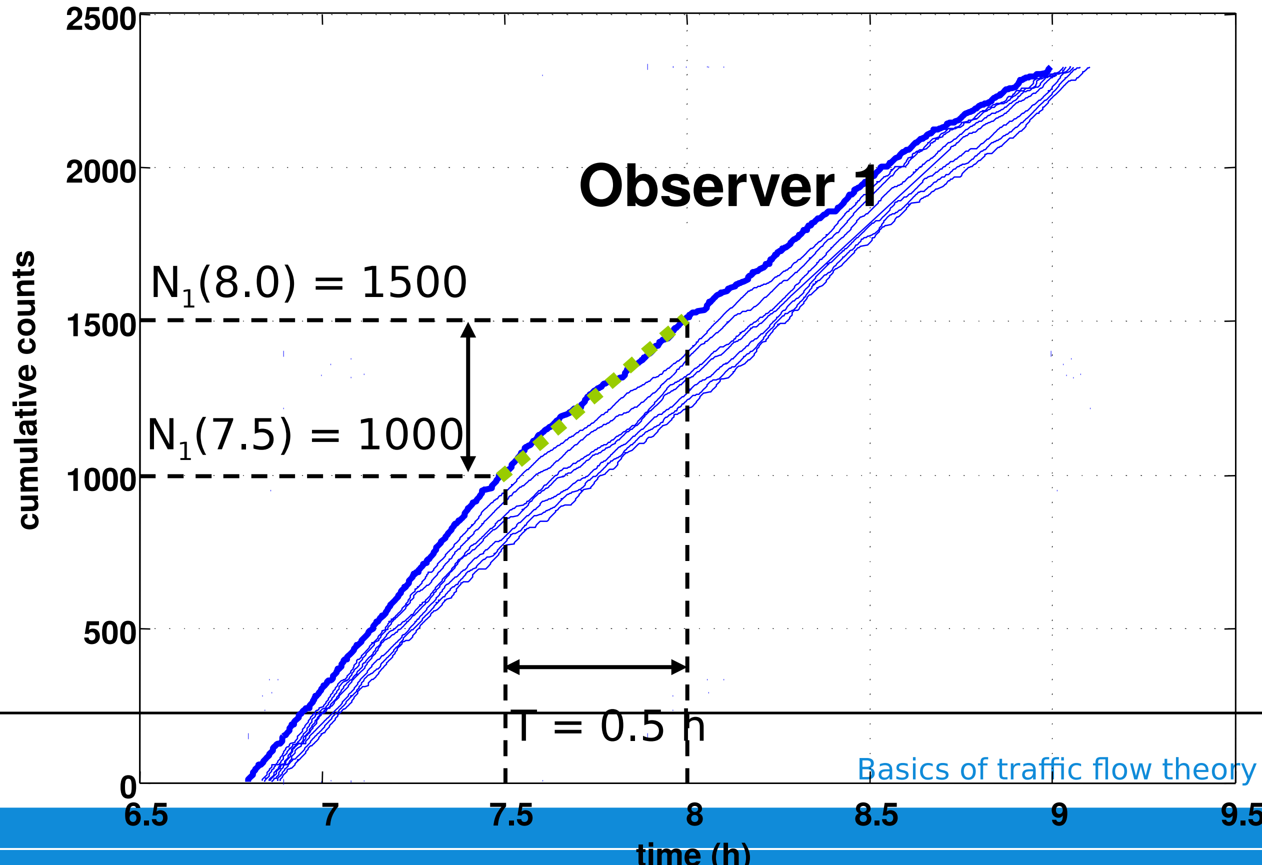


Construction of cumulative curves

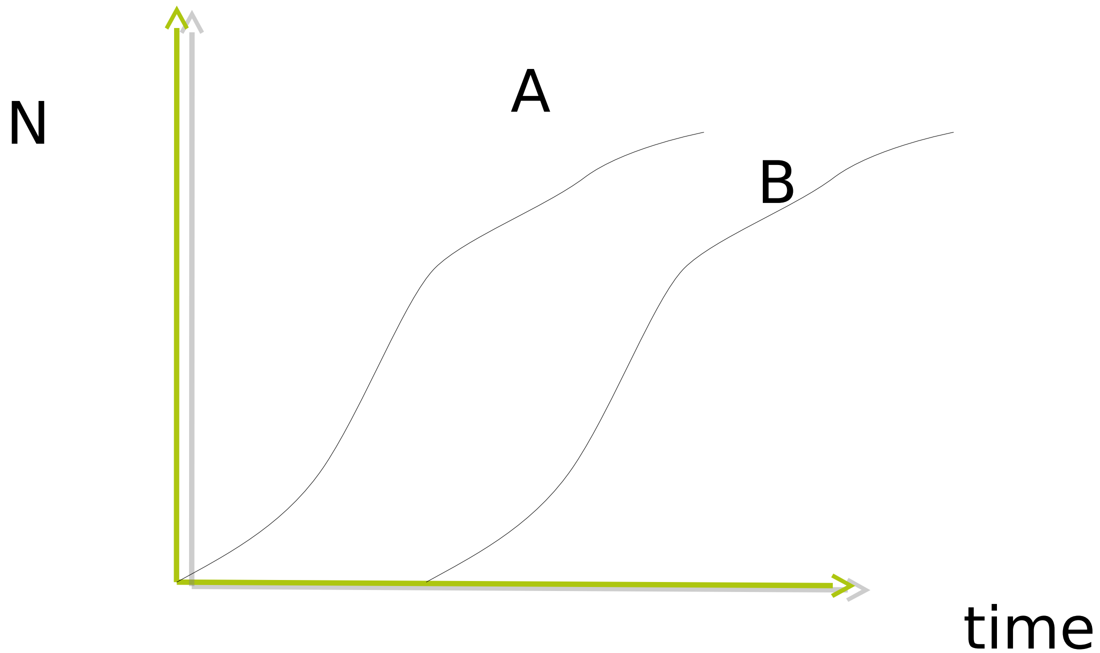


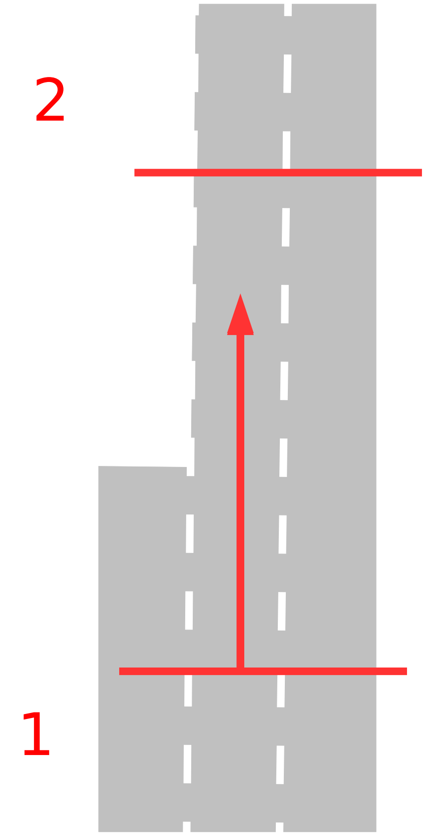
Cumulative vehicle plots[3]

- Flow = number of vehicles passing *x* (observer) during *T*
- *What is the flow in this case?*

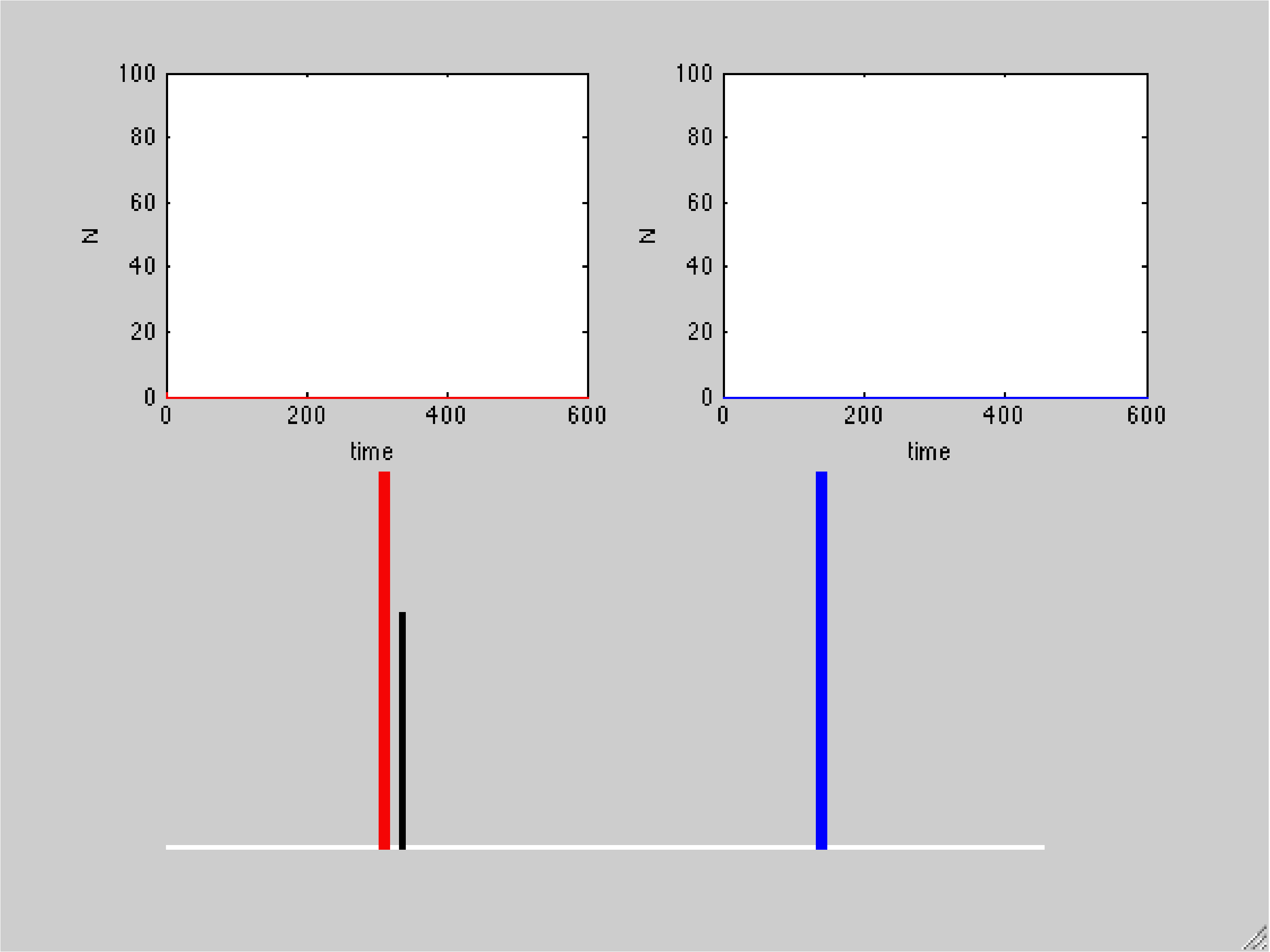


Information in cumulative curves

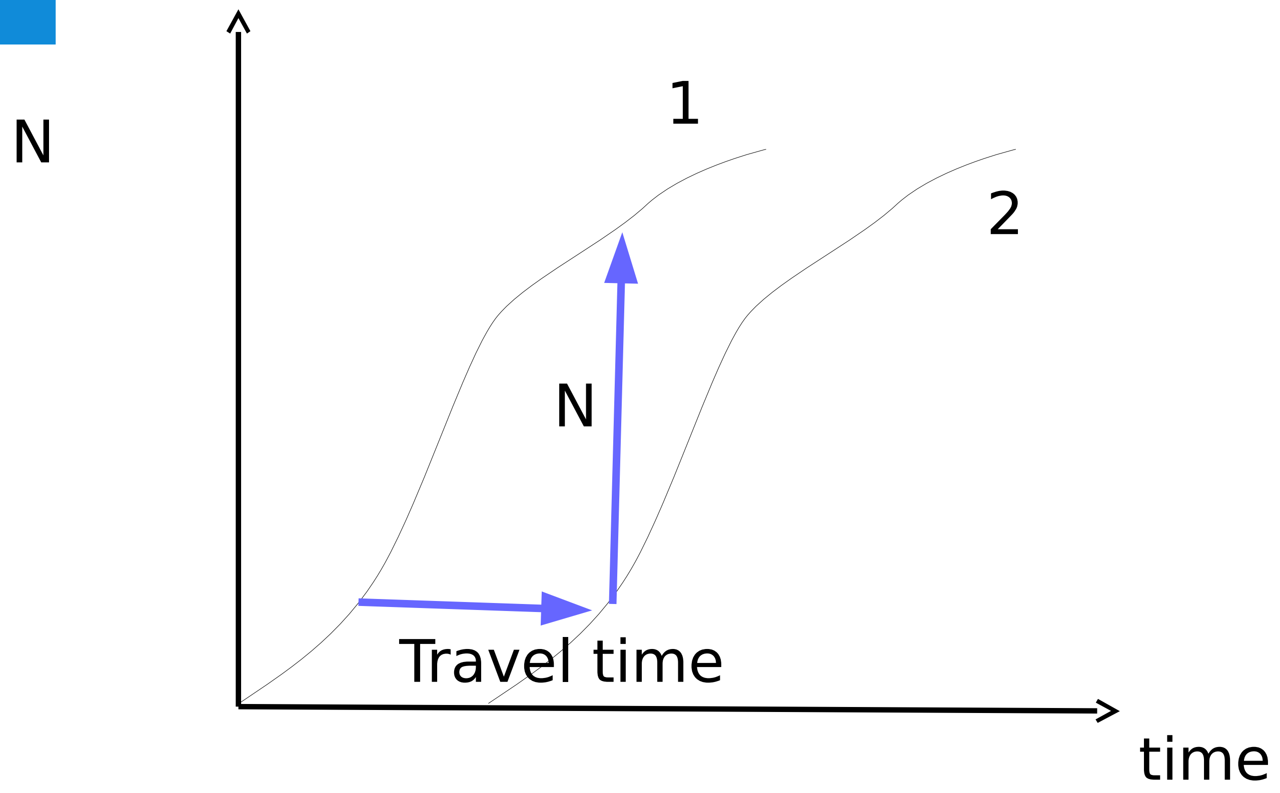


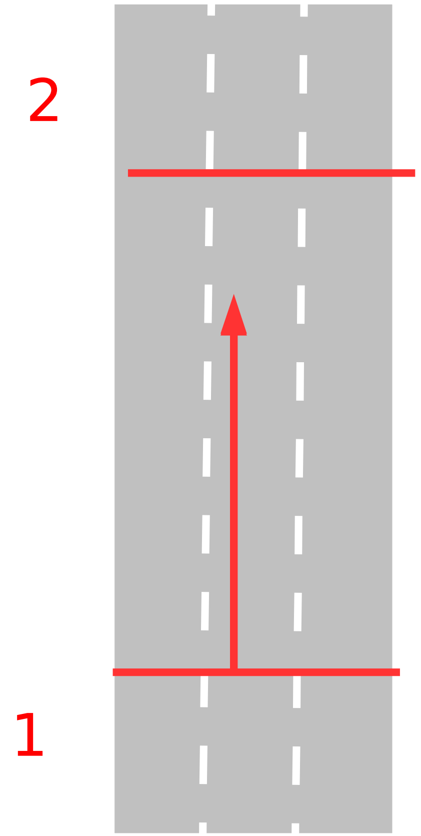


Which line corresponds with detector 1?

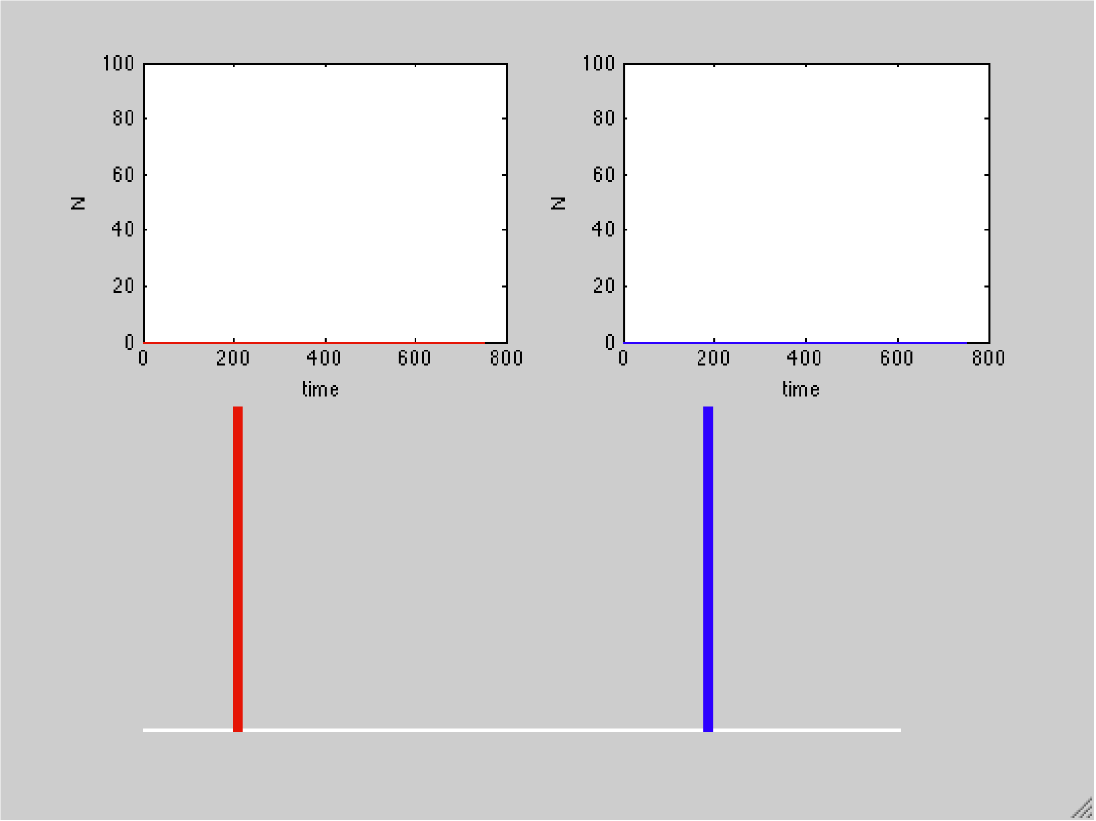

100
80
60
40
20
0
N
0
200
400
600
time
100
80
60
40
20
0
N
0
200
400
600
time

Information in cumulative curves

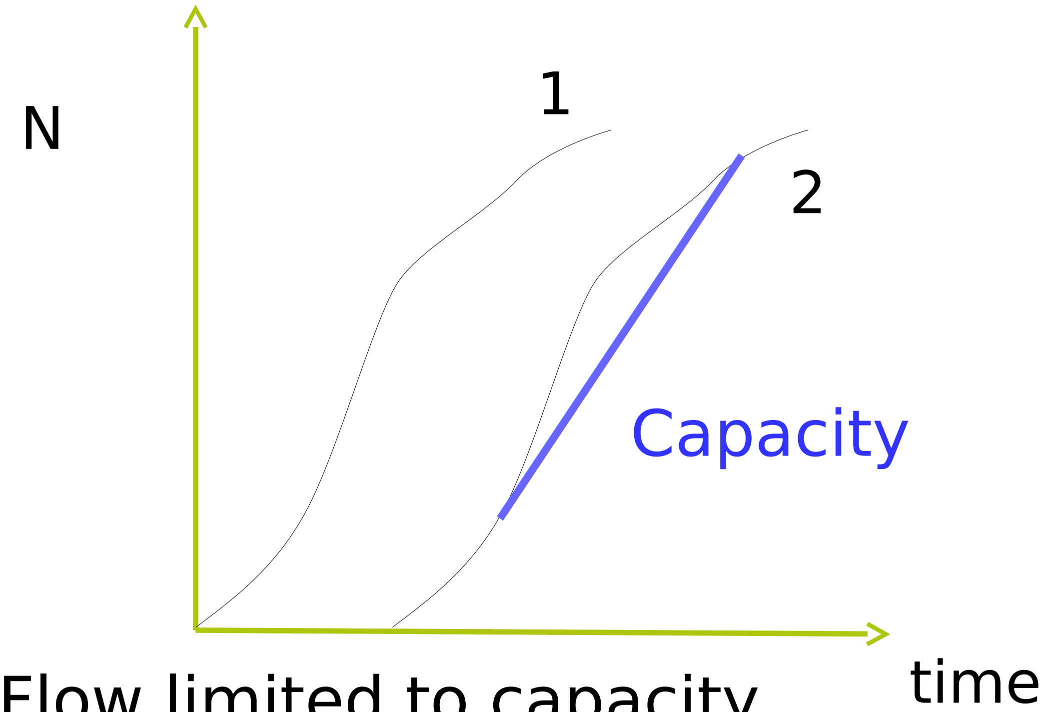




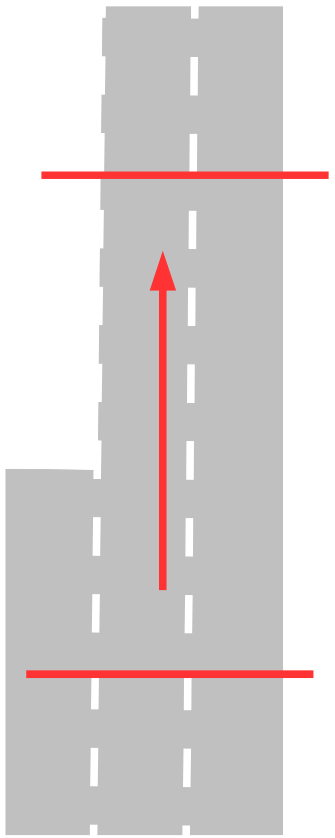
Bottleneck in section



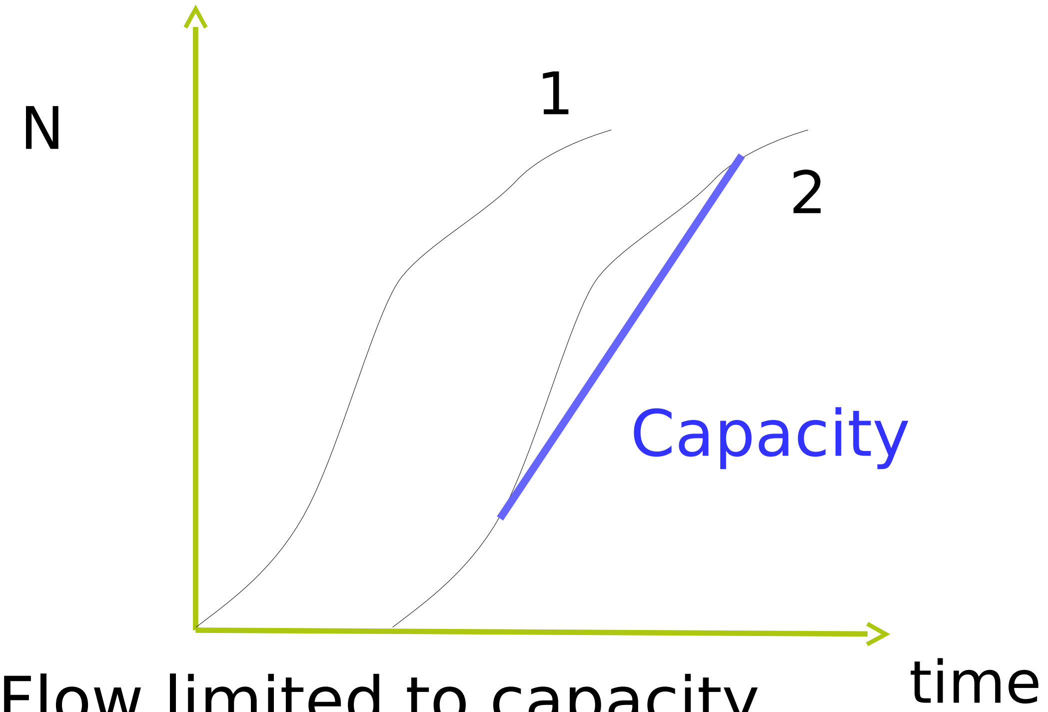
What if bottleneck is present



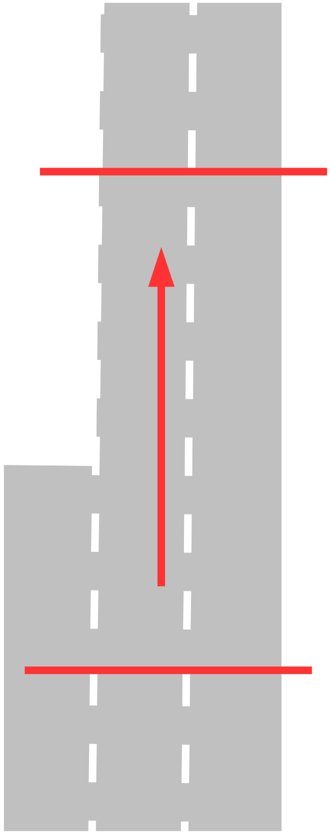
Flow limited to capacity



What if bottleneck is present

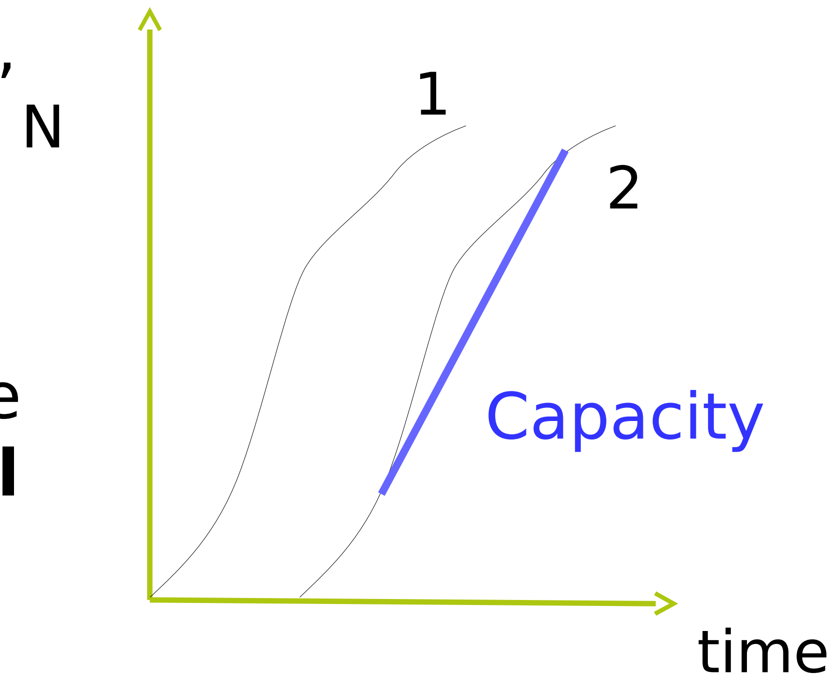


Flow limited to capacity
Travel time increases
More vehicles in the section



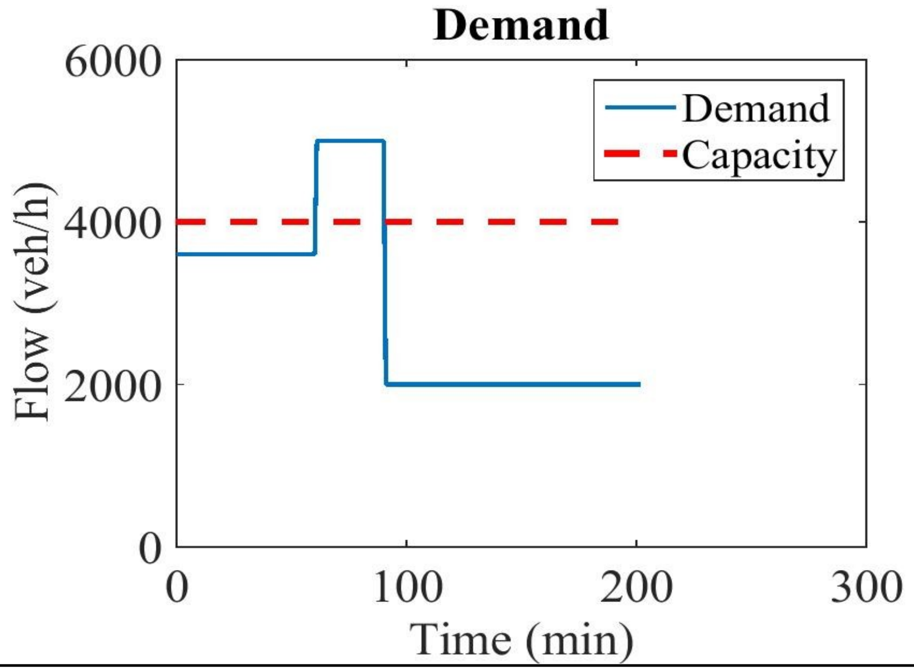
Queuing model

- "Vertical queuing model"
- Construct cumulative inflow curve
- Construct cumulative outflow curve
- **Ignore free flow travel time**

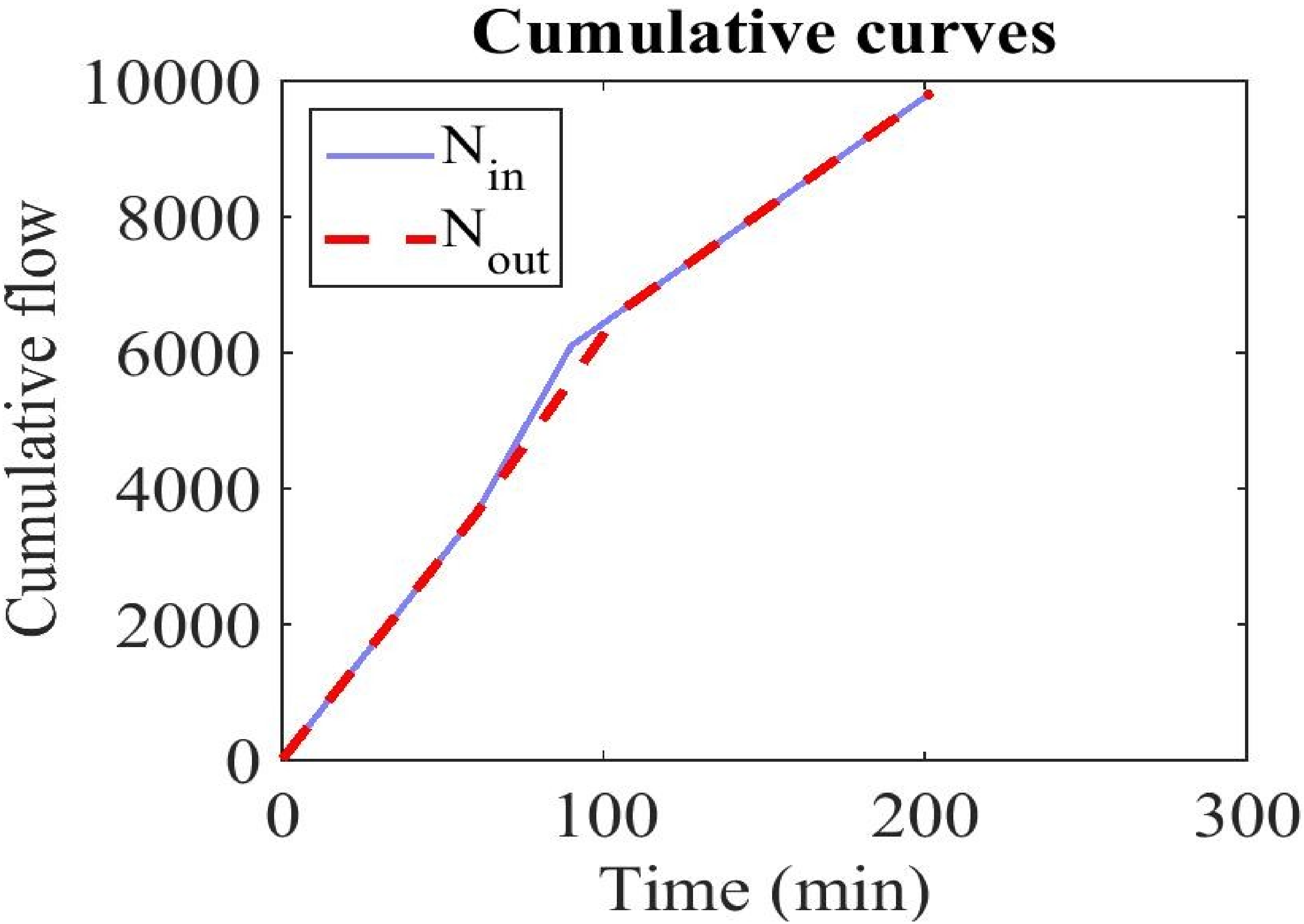


Queuing model

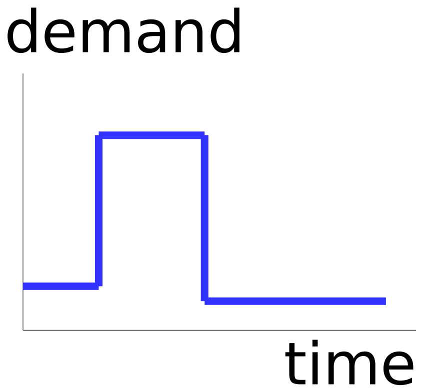
- Given demand and capacity, give the cumulative flow curves (ignore free flow travel time)



Answer: cumulative curves



Content for today



1

2

Cumulative curves

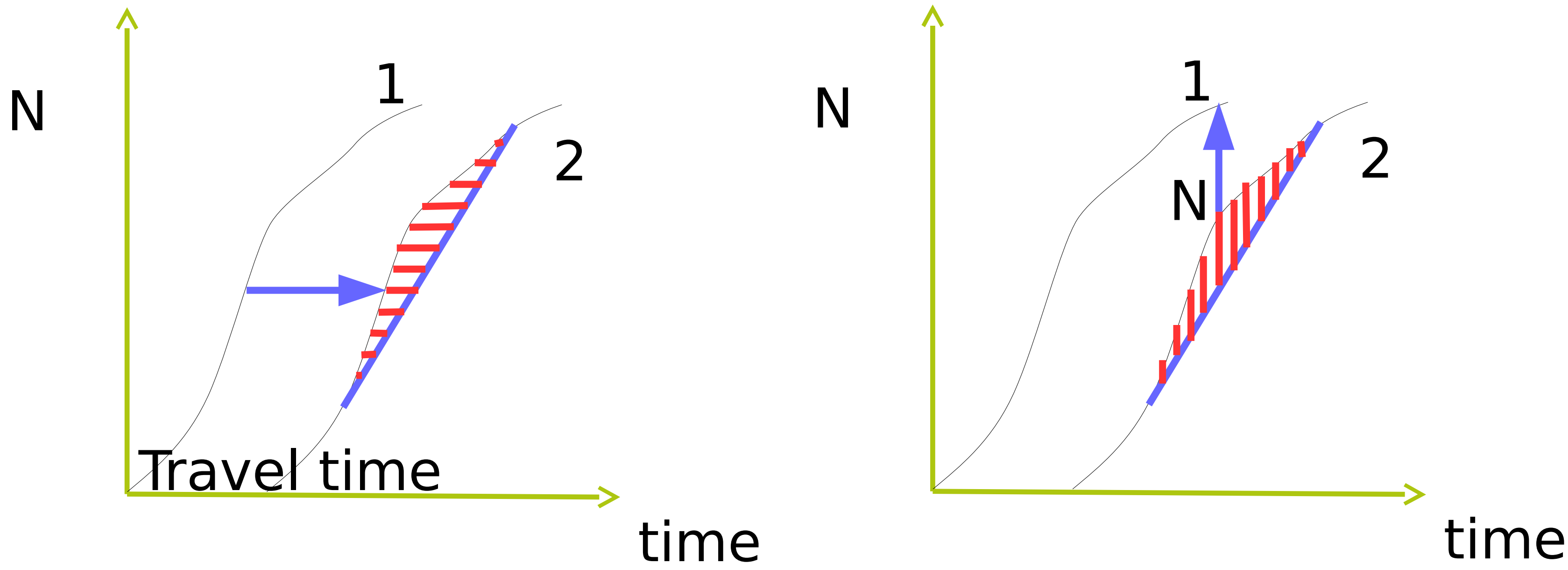
Total delay
Av. travel time

3

4

5

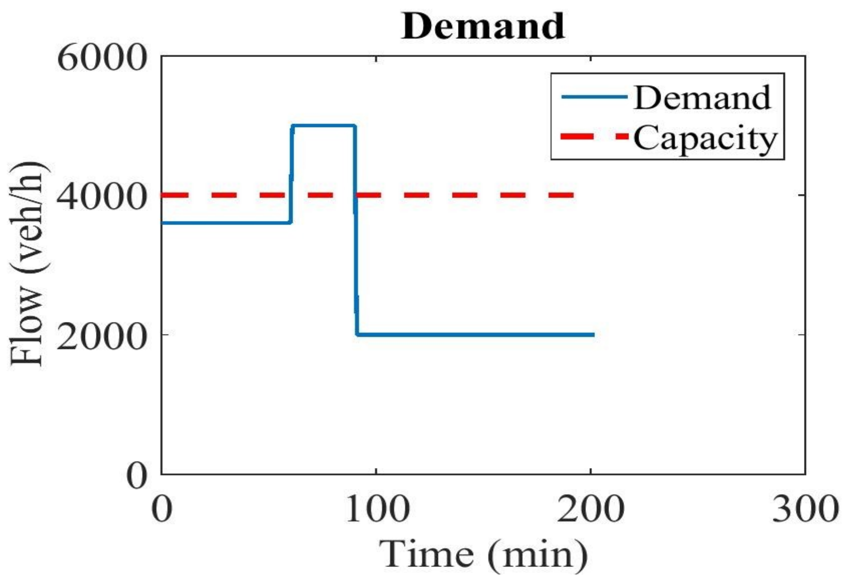
Delay = area

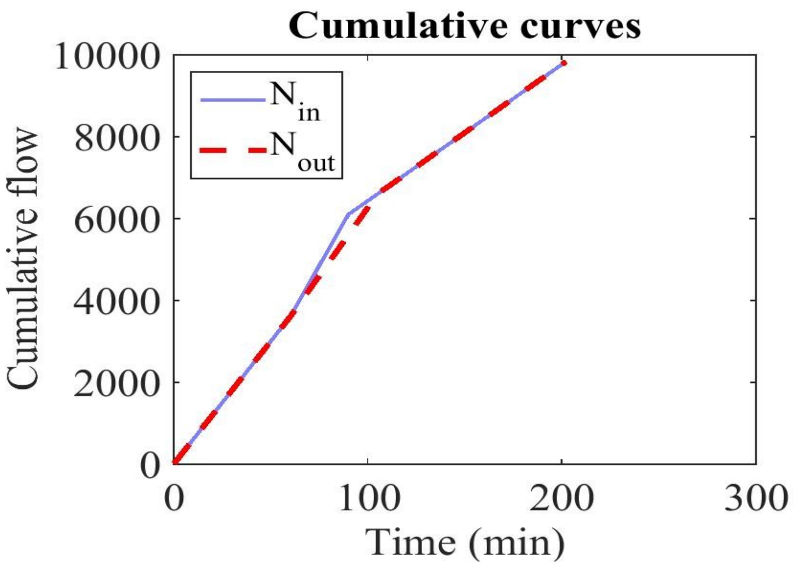


Total delay = sum delay over vehicles
Total delay = sum # extra vehicles in section

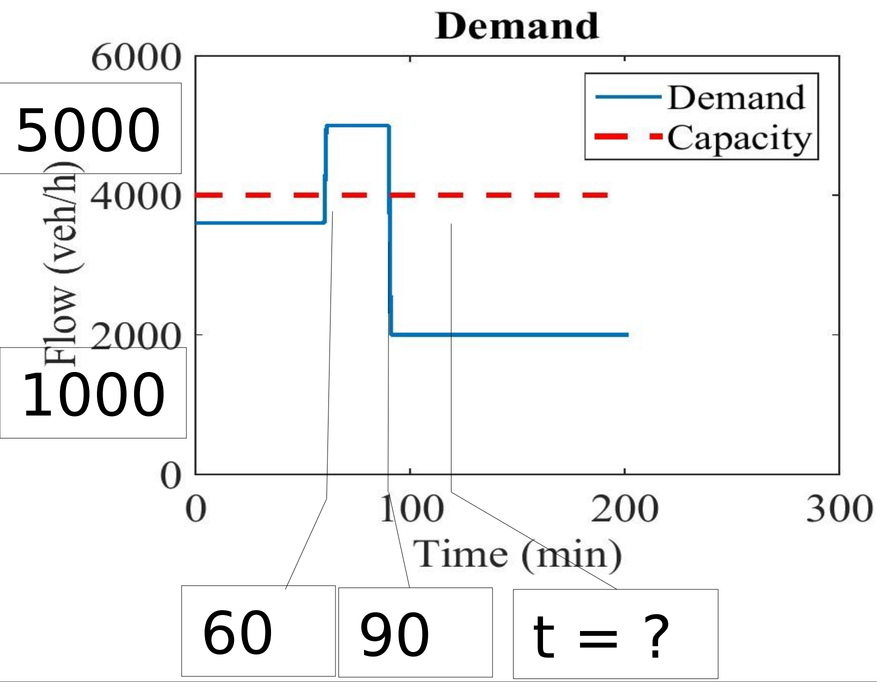
Delay for this case?

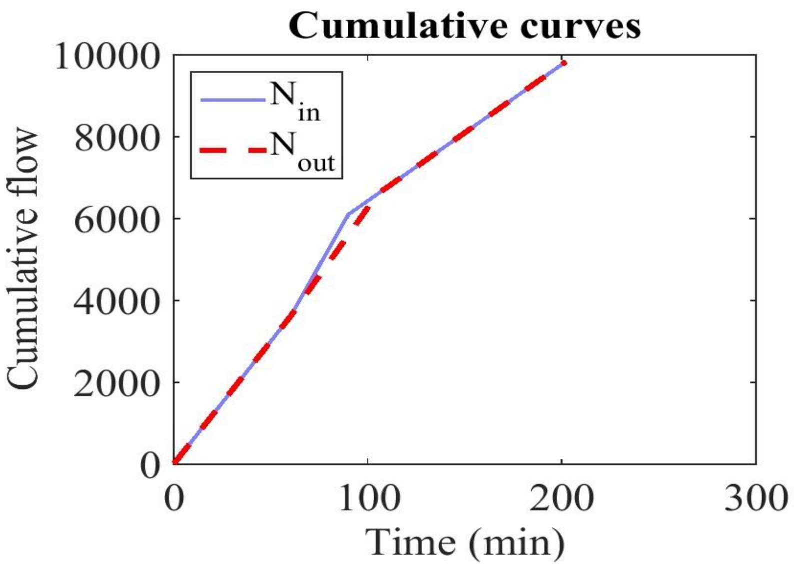
- Given demand and capacity, calculate the total delay





Computation tricks

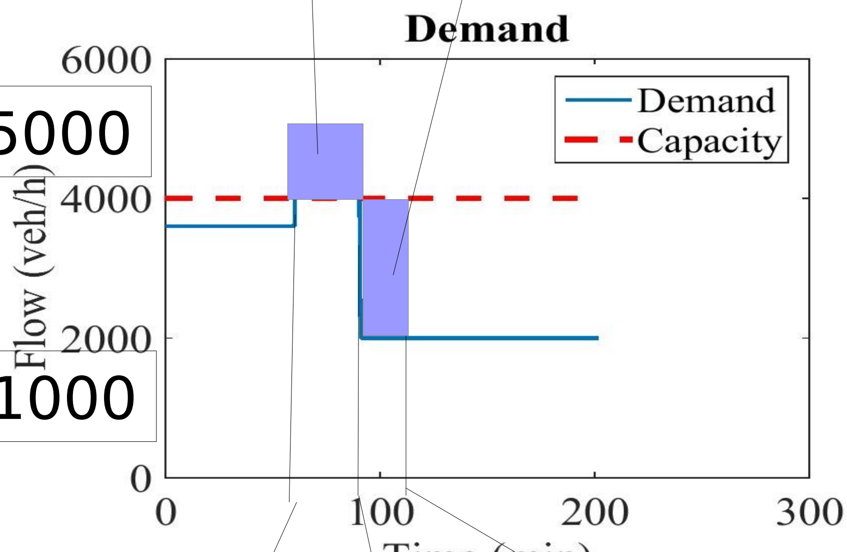




Delay for this case?

Nr of vehicles into queue = (90-60)/60*1000

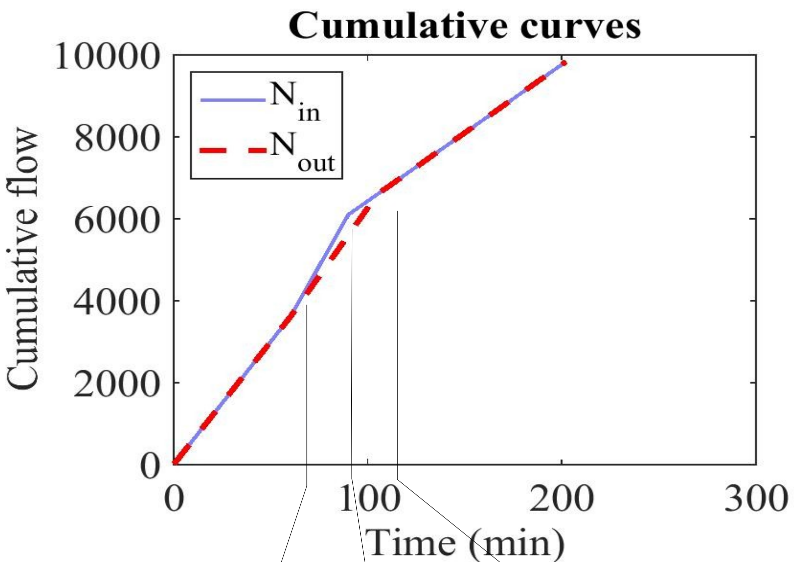
Nr of vehicles out of queue = (t-90)/60*2000



5000

1000

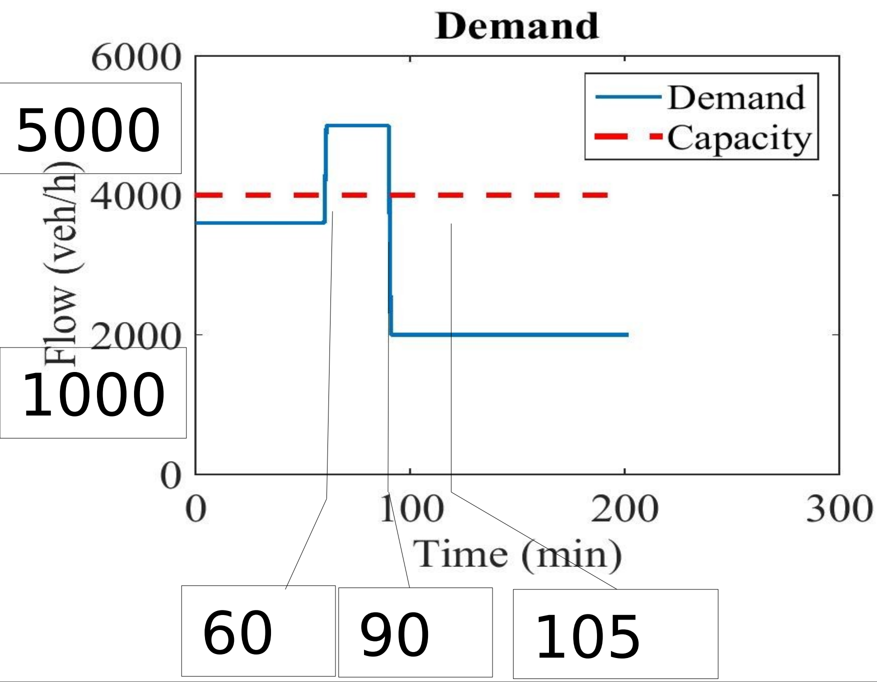
60 90 105

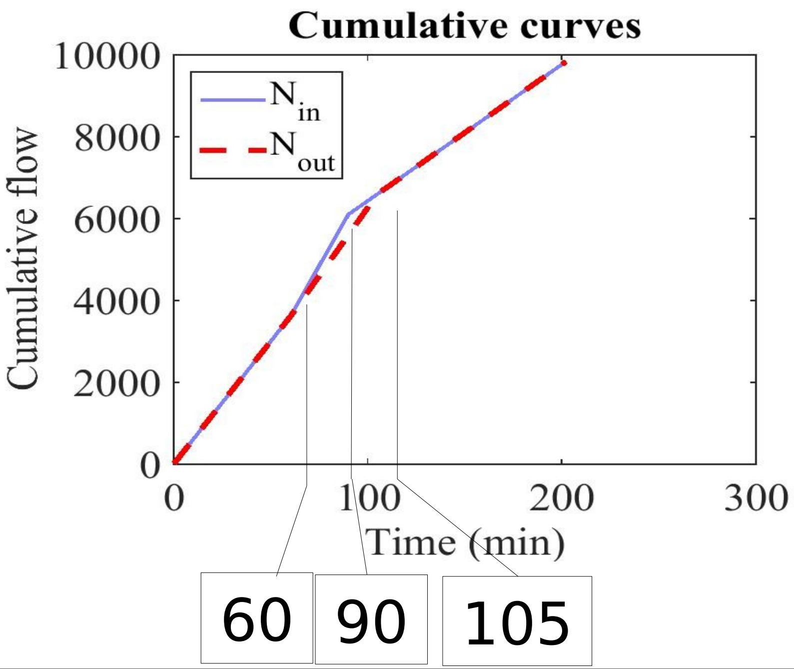


60 90 105

Delay for this case?

- Given demand and capacity, calculate the total delay



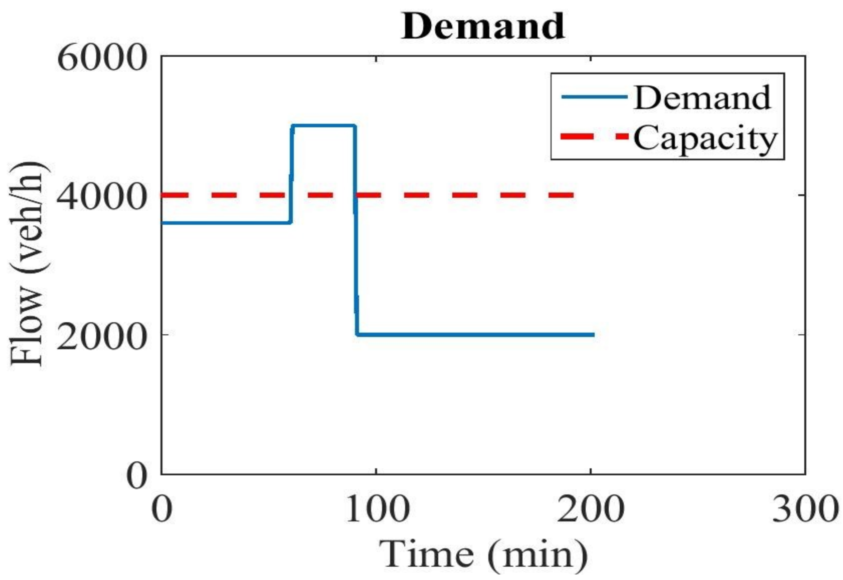


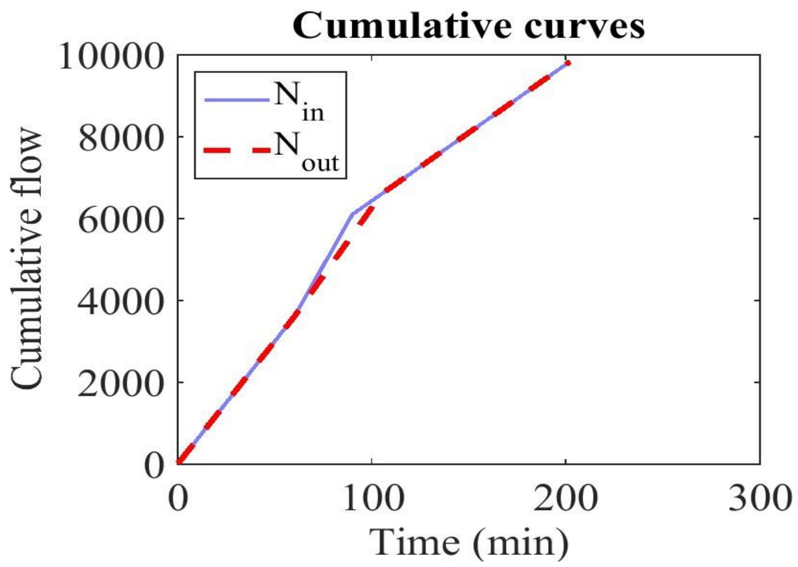
Answer

Total delay = area between cumulative curves
Triangle: area = ½ x base x height
Top at t=90 min = 30 min after start congestion:
height = Nin-Nout= 30 min/60 min/h x (5000-4000 veh/h)
1/2 x 1000 = 500 veh
Base = 105-60 = 45 min = 0.75h
Total delay = ½ 0,75*500 = 187.5 veh h

Delay for this case?

- Given demand and capacity, calculate the average delay





Answer

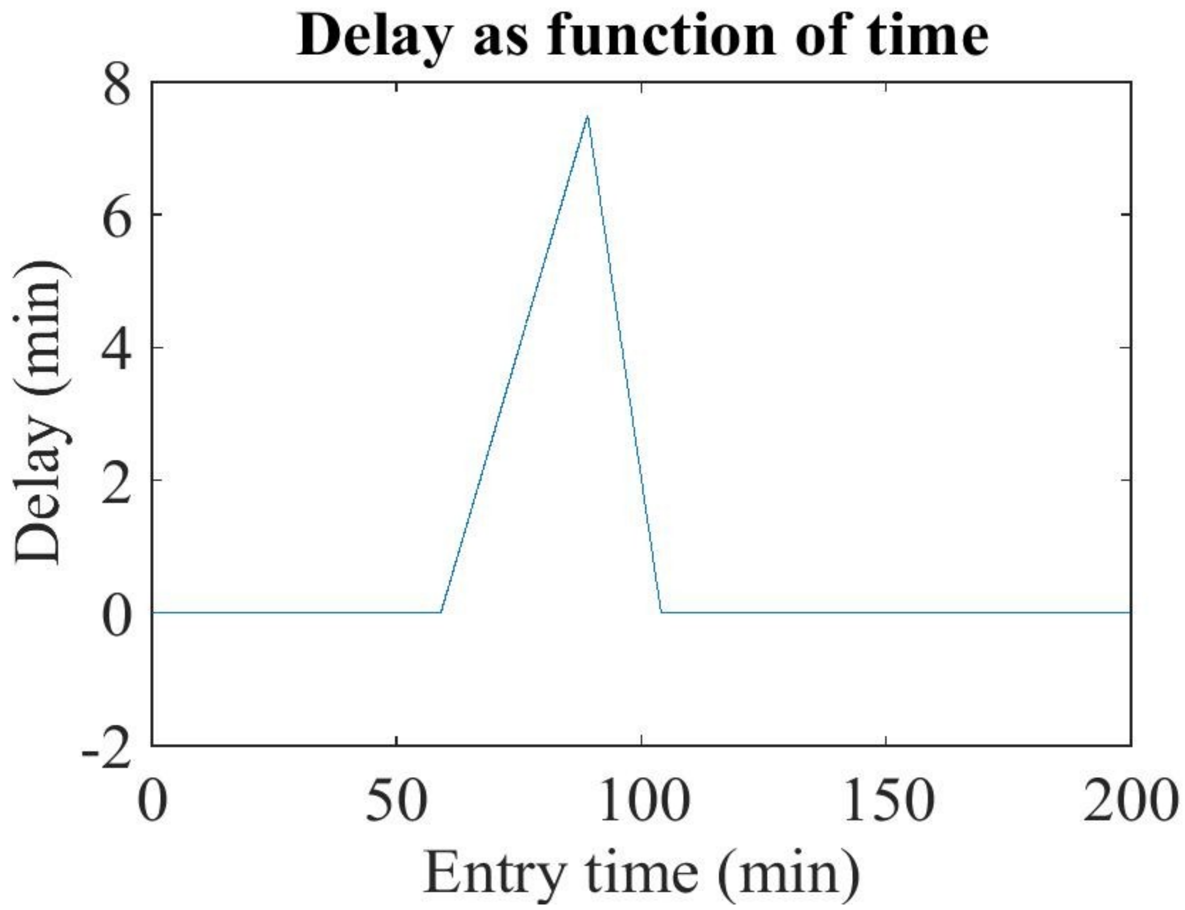
Total delay = area between cumulative curves
Triangle: area = ½ x base x height
Top at t=90:
height = Nin-Nout= 30 min/60 min/h x (5000-4000 veh/h)
1/2 x 1000 = 500 veh
Base = 105-60 = 45 min = 0,75h
Total delay = ½ 0,75*500 = 187,5 h

Total nr of veh: 9800 (integrate demand over time)
Average delay: 187,5/9800 = 0.0191 h = 1.1 min

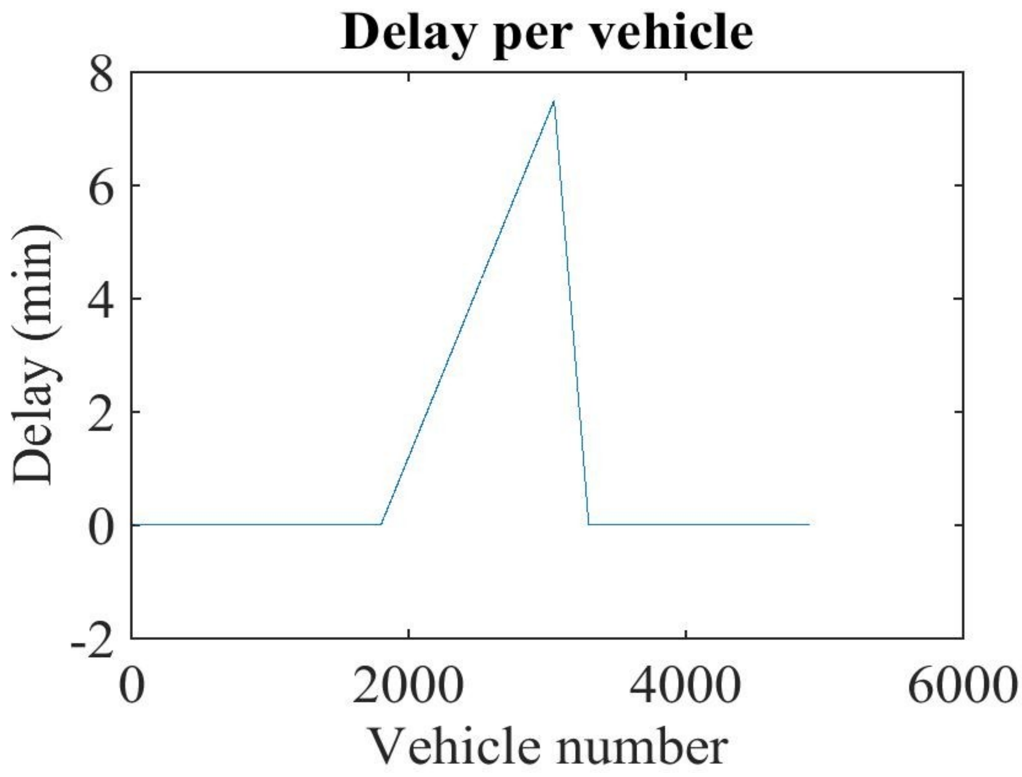
Summary

- Capacity derived from headways
- Construct cumulative curves
- Restrict outflow to capacity
- Travel times and queue length from curves
- Delay = area

After some manipulation (1)



After some manipulation (2)



Computations

Run the following Matlab code:

```
function quist_block()

q0=[3600;5000;2000];%the three demands
Tchange=[60;90];%times in minutes at which the demands change
c=4000;%capacity
T=0:200;%minutes
dt=1/60;%time steps (in hours: time step is 1 min)
dem=q0(end)*ones(size(T));%pre-allocate demand function to the last demand value
for(i=numel(Tchange):-1:1)
    dem(T<Tchange(i))=q0(i);%adapt the demand function
end

figure;
plot(dem,'linewidth',2)
hold on
plot(repmat(c,size(dem)),'r--','linewidth',3)
ylim([0 6000])
legend('Demand','Capacity','location','Northeast')
ylabel('Flow (veh/h)')
xlabel('Time (min)')
exportfig('Demand')
%%

Nin=dt*cumsum(dem);
qout=zeros(size(Nin));
qout(1)=dem(1);%in veh/h
queued=zeros(size(dem));
for(t=2:numel(T))
    qout(t)=1/dt*min(dt*c, dt*dem(t)+queued(t-1));
    queued(t)=queued(t-1)+dt*dem(t)-dt*qout(t);
end
Nout=dt*cumsum(qout);
figure;plot(Nin,'linewidth',2,'color',[0.5 0.5 1]);hold on;plot(Nout,'r--','linewidth',3)
legend('N_{in}','N_{out}','location','Northwest')
ylabel('Cumulative flow')
xlabel('Time (min)')
exportfig('Cumulative curves')
%%

%compute total delay:
TotalD=dt*sum(queued)%then the total delay in hours
NrVeh=Nin(end);
AvgDelay=TotalD/NrVeh%then the total delay in hours
AvgDelayMin=60*AvgDelay;%then the total delay in hours

%%
Tin=interp1(Nin,T,1:NrVeh);%time to enter for each vehicle -- interpolation
Tout=interp1(Nout,T,1:NrVeh);%time to exit for each vehicle -- interpolation
DT=Tout-Tin;%additional travel time
figure;
plot(1:NrVeh,DT,'linewidth',2)
xlabel('Vehicle number')
ylabel('Delay (min)')
exportfig('Delay per vehicle')

figure;
plot(Tin,DT,'linewidth',2)
xlabel('Entry time (min)')
ylabel('Delay (min)')
exportfig('Delay as function of time')
```