A stochastic modeling of traffic breakdown for freeway merge bottlenecks
VSL
Variable Speed Limit

TO PREVENT BREAKDOWN
- Prevent bottleneck activation
  - e.g., Carlson et al. (2010)

TO INCREASE DISCHARGE RATE
- From moving jams or fixed bottlenecks
  - e.g., Hegyi et al. (2008); Chen et al. (2014)

Main Principle
- Speed Limitation
- Inflow Regulation
  - Below bottleneck capacity
- Breakdown prevention & Capacity Drop

Stochastic BN capacity
STOCHASTIC CAPACITY

Capacity can vary

EMPIRICAL OBSERVATIONS

1 minute flow (pcphpl)

MEDIAN LANE FLOWS

Probability of Breakdown

1 minute flow (pcphpl)

1. INTRODUCTION

PREVIOUS FINDING

• Breakdown probability increases with flow
• It typically has S-shape

LIMITATION

• Mainly focus on flow effect
• Other factors?
• Insight for traffic control?

Persuad et al. (1998)
Elefteriadou (1994), Evans et al. (2001), Son et al. (2004), Brilon et al. (2005), Chen et al. (2014)
## 1. INTRODUCTION

### PROBLEM STATEMENTS

<table>
<thead>
<tr>
<th>1. Stochastic capacity, but…</th>
</tr>
</thead>
<tbody>
<tr>
<td>- We have a limited understanding of breakdown mechanisms and the effects of various factors</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. VSL can regulate inflow to a bottleneck, but…</th>
</tr>
</thead>
<tbody>
<tr>
<td>- VSL causes controlled-congestion that propagates upstream</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. VSL can prevent traffic breakdown or minimize capacity drop, but…</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Existing strategies do not consider stochastic BN capacity for determining control parameters</td>
</tr>
<tr>
<td>- Conservative control → system underutilization</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

## RESEARCH OBJECTIVES
Model, Strategies, Optimization

<table>
<thead>
<tr>
<th>BREAKDOWN MODEL</th>
<th>CONTROL STRATEGIES</th>
<th>SYSTEM OPTIMIZATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>- To analyze <em>breakdown mechanisms</em> and <em>effects</em> of various factors</td>
<td>- <strong>CV</strong> (+VMS): effective for low penetration rate</td>
<td>- To achieve <em>maximum</em> delay saving</td>
</tr>
<tr>
<td>- → Offer <em>insight</em> for traffic control</td>
<td>- <strong>Proactive</strong>: to minimize breakdown probability</td>
<td>- Considering <em>stochastic capacity</em> and control failure probability</td>
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<td></td>
<td>- <strong>Reactive</strong>: to resolve existing queue / prevent additional breakdown</td>
<td></td>
</tr>
</tbody>
</table>
1. INTRODUCTION

RESEARCH OBJECTIVES
Model, Strategies, Optimization

**BREAKDOWN MODEL**
- To analyze *breakdown mechanisms* and *effects* of various factors
- → Offer *insight* for traffic control

**CONTROL STRATEGIES**
- **CV (+VMS):** effective for low penetration rate
- **Proactive:** to minimize breakdown probability
- **Reactive:** to resolve existing queue / prevent additional breakdown

**SYSTEM OPTIMIZATION**
- To achieve *maximum* delay saving
- Considering *stochastic capacity* and control failure probability
Breakdown Model

1. INTRODUCTION
2. BREAKDOWN MODEL
3. MODEL PROPERTIES
4. PROACTIVE CONTROL
5. CONCLUSION
DRIVER BEHAVIOR
Newell’s Car-Following Model (Newell, 2002)

VEHICLE TRAJECTORIES
Same except translation

SPACING AND SPEED
Linear relation

\[ S_F(v') \]
\[ S_F(v'') \]
\[ V_F \]
(Desired speed)
(=Free flow speed, \( u \))
DRIVER BEHAVIOR
Vehicle behavior with merge

CASE 1: MERGING SPACING \((s_F) \geq \text{DESIRED SPACING } (S^*_F(u))\)
ID mode → CF mode

\[
s_F(\geq S^*_F(u))
\]

(Merging) \(v_{in}\)

Vehicle \(M\)

\[
s_F(\geq S^*_F(u))
\]

(ID mode)

\[
v_F^*(u)
\]

\[
v_{in}
\]

\[
S^*_F(v_{in})
\]

\[
d_F
\]

\[
\frac{d_F}{2}
\]

\[
\tau_F
\]

\[
u
\]

\[
v
\]

(Free flow speed)

ID mode

CF mode
Driver Behavior

Vehicle behavior with merge

Case 1: Merging Spacing \((s_F) \geq\) Desired Spacing \((S_F^*(u))\)

ID mode → CF mode

2. Breakdown Model
DRIVER BEHAVIOR
Vehicle behavior with merge

CASE 1: MERGING SPACING \( (s_F) \geq \) DESIRED SPACING \( (S_F^*(u)) \)
ID mode → CF mode

2. BREAKDOWN MODEL
CASE 2: MERGING SPACING ($s_F < S_F^*(u)$)

Prompt CF mode

Vehicle behavior with merge
**DRIVER BEHAVIOR**

Vehicle behavior with merge

**CASE 2 : MERGING SPACING ($s_F$) < DESIRED SPACING ($S^*_F(u)$)**

Prompt CF mode

---

**Rule 1**

Merging spacing decides the speed

$$v_r = \frac{s_F - d_F}{\tau_F}$$

---

Diagram showing:
- Merging spacing
- ID mode
- CF mode
- Speed ($v$)
- Spacing ($s$)
- Distance ($d_F$)
- Time ($\tau_F$)
- Vehicle $F$ and $M$
**DRIVER BEHAVIOR**

Vehicle behavior of merging vehicle

---

**Duration of \( v_{in} (t_{in}) \)**

- Acceleration time
  \[
  t_a = \frac{u - v_{in}}{2a}
  \]
DRIVER BEHAVIOR
Vehicle behavior of merging vehicle

Duration of $v_{in}$ ($t_{in}$)

- Acceleration time
  \[ t_a = \frac{u - v_{in}}{2a} \]
DRIVER BEHAVIOR
Vehicle behavior of merging vehicle

Duration of $v_{in} (t_{in})$

- Acceleration time
  \[ t_a = \frac{u - v_{in}}{2a} \]

- Time to reach desired spacing
  \[ t_{ds} = \frac{s_M^*(u) - s_M}{u - v_{in}} \]

\[ \therefore t_{in} = \max [t_a, t_{ds}] \]

Rule 2
DURATION OF MERGING SPEED
BREAKDOWN PROCESS
Two elements of traffic breakdown

TRIGGER
Speed reduction by merging vehicle

PROPAGATION
Wide spread congestion

Both Trigger and propagation are necessary to cause traffic breakdown
TRIGGER
Degree of disturbance by merge vehicle

**h**: headway between vehicle L and F before merging

\[ h > h_{cr} \]

- \[ h_{cr} = \frac{s_M(u) + S_F(u)}{u} \]
- No speed reduction (vehicle F)

2. BREAKDOWN MODEL

NO TRIGGER
TRIGGER
Degree of disturbance by merge vehicle

TYPE I: MILD

- $h_1 < h \leq h_{cr}$
  - $h_1 = \frac{s_M + S_F(u)}{u}$
- $v_{tr} = v_{in}$
- $t_{tr} = \frac{h-h_1}{h_{cr}-h_1} t_{in}$

**$h$: headway between vehicle $L$ and $F$ before merging**
TRIGGER
Degree of disturbance by merge vehicle

**TRIGGER**

- **Type II: Moderate**
  - $h_{II} < h \leq h_{I}$
    - $h_{II} = \frac{s_M + d_F}{u}$
  - $t_{tr1} = \tau_F, t_{tr2} = t_{in}$
    - $t_{tr} = t_{tr1} + t_{tr2}$
  - $v_{tr1} = \frac{(h-h_{II})u}{\tau_F}, v_{tr2} = v_{in}$
  - $v_{tr} = \frac{t_{tr1}v_{tr1} + t_{tr2}v_{tr2}}{t_{tr1} + t_{tr2}}$

**Notes:**
- $h$: headway between vehicle $L$ and $F$ before merging
- $L \rightarrow M \rightarrow F$
TRIGGER
Degree of disturbance by merge vehicle

**h**: headway between vehicle $L$ and $F$ before merging

### TYPE III: SEVERE

- $h_{III} < h \leq h_{II}$
  - $h_{III} = \frac{s_M}{u}$
- $t_{tr1} = \frac{d_F-S_F}{v_{in}} + \tau_F$
- $t_{tr2} = t_{in} + \tau_F - t_{tr1}$
  - $t_{tr} = t_{tr1} + t_{tr2}$
- $v_{tr1} = 0$, $v_{tr2} = v_{in}$
  - $v_{tr} = \frac{t_{tr1}v_{tr1} + t_{tr2}v_{tr2}}{t_{tr1} + t_{tr2}}$
PROPAGATION
Persistence of disturbance until next merging vehicle

"Disturbance will cumulate over time / lead to widespread congestion"
PROPAGATION
To resolve triggered disturbance

Minimum headway
\( h_{\text{min}} = \tau_{F_2} + \frac{d_{F_2}}{u} \)

Critical buffer time
\( (B_{cr}) \)

\( t_{tr} \)

\( v_{tr} \)

\( M_1 \quad F_1 \quad F_2 \)

\( v_{in} \)

\( M_1 \quad F_1 \quad F_2 \quad F_3 \quad F_4 \)

\( > \quad \text{Sum of Buffer headway (during } t_M \text{)} \)

*Buffer headway of vehicle \( i \) \( (b_i)\)

\[ b_i = h_i - \frac{s_i^*(u)}{u} = h_i - (\tau_i + \frac{d_i}{u}) \]
## PROPAGATION

### Probability calculation

### Definition of buffer headway

\[
b_i = h_i - \left( \tau_i + \frac{d_i}{u} \right)
\]

### Divide by \( n \)

\[
B(h) = \frac{B_{cr}}{n} + \left( E(\tau) + \frac{E(d)}{u} \right)
\]

### Central limit theorem

\[
p \left( \sum_{i=1}^{n} b_i < B_{cr} \right) = p \left( \sum_{i=1}^{n} \left[ h_i - \left( \tau_i + \frac{d_i}{u} \right) \right] < B_{cr} \right)
\]

\[
= p \left( \sum_{i=1}^{n} h_i < B_{cr} + \sum_{i=1}^{n} \left( \tau_i + \frac{d_i}{u} \right) \right)
\]

\[
= p \left( E(h) < \frac{B_{cr}}{n} + \left( E(\tau) + \frac{E(d)}{u} \right) \right)
\]

\[
= p \left( E(h) < B(h) \right)
\]

\[
p \left( Z < \frac{B(h) - \mu_h}{\sigma_h / \sqrt{n}} \right)
\]

Z-score (standard normal distribution)
**BREAKDOWN PROBABILITY**

Integrating of Trigger and Propagation

**BREAKDOWN = “TRIGGER” AND “PROPAGATION”**

\[
p(BD) = \sum_{i=1}^{III} p(\text{Type } i \text{ trigger}) \times p(\text{Propagation} | \text{Type } i \text{ trigger})
\]

(Probability of breakdown)

\[
= \sum_{i=1}^{III} \int_{a_i}^{b_i} p(h) \, dh \times \frac{\int_{a_i}^{b_i} p(h) \, p(PR|h) \, dh}{\int_{a_i}^{b_i} p(h) \, dh}
\]

\[
= \sum_{i=1}^{III} \int_{a_i}^{b_i} p(h) \, p(PR|h) \, dh = \int_{h_{III}}^{h_{cr}} p(h) \, p(PR|h) \, dh
\]
1. INTRODUCTION
2. BREAKDOWN MODEL
3. MODEL PROPERTIES
4. PROACTIVE CONTROL
5. CONCLUSION

Model Properties
And breakdown mechanism
MODEL PROPERTIES

Understand Breakdown Mechanisms → Insight for Control

Breakdown probability decreases with

1) Low flow rate (mainline, merging)
2) High merging speed
3) Low deviation of headway
4) Small $\tau$ (aggressive driver)
MODEL PROPERTIES
Effects of Headway Distribution (Deviation)

3 scenarios of headway distribution

- Same mean, different deviations
  (i) Exponential distribution: $\sigma_h = \mu_h$
  (ii) Gamma distribution: $\sigma_h = \mu_h/2$
  (iii) Gamma distribution: $\sigma_h = \mu_h/4$

MECHANISM
✓ Small headway deviation

→ Low propagation  → Low breakdown
   (Before critical flow)
Proactive VSL Control

1. INTRODUCTION
2. BREAKDOWN MODEL
3. MODEL PROPERTIES
4. PROACTIVE CONTROL
5. CONCLUSION
4. PROACTIVE CONTROL

PROACTIVE VSL CONTROL
Headway Adjustment Area

OBJECTIVES
- To achieve uniform headways in free flow state
- Not to generate controlled congestion that propagates upstream

MEANS
- Temporary deceleration of CV
- Combination with VMS control
4. PROACTIVE CONTROL

Control Procedure

- Flow rate ($q_a$) vs. density ($u$)
- Merging area
- Headway Adjustment Area

(Before) Random headway

(distance)
4. PROACTIVE CONTROL

Control Procedure

1. CV reduce speed as $v_{a'}$

Headway Adjustment Area

(flow rate vs. density)

Distance

Merging area

(Before) Random headway

$u$ $u$ $u$ $u$ $u$ $u$
**PROACTIVE VSL CONTROL**

Control Procedure

1. CV reduce speed as $v_{a'}$

2. VMS VSL = $v_{a'}$

3. CV $= u$

4. PROACTIVE CONTROL

**Diagram:**

- **Flow Rate:** $q_a$
- **Density:** $u$
- **Speed:** $v_{a'}$
- **CV:** $u$
- **Merging area**
- **Headway Adjustment Area**
- **(Before) Random headway**

**Equations:**

- $o = \begin{cases} 1. CV reduce speed as v_{a'} & \text{if} \ a \leq a' \\ 2. VMS VSL = v_{a'} & \text{if} \ a > a' \end{cases}$
4. PROACTIVE CONTROL

Control Procedure

1. CV reduce speed as $v_{a'}$

2. VMS VSL = $v_{a'}$

3. PROACTIVE CONTROL

(Merging area)

(Before) Random headway

(After) Uniform headway

Headway Adjustment Area

(flow rate)

(density)

(distance)

(time)
Conclusions
And Future works
CONCLUSIONS / CONTRIBUTIONS

**BREAKDOWN MODEL**
- Better describes the *mechanisms* of traffic breakdown
- Obtain *insight* for traffic control: small headway deviation

**PROACTIVE CONTROL**
- *Harmonize vehicle headways* using CV (CAV) and VMS
- Decrease breakdown probability *without* controlled-congestion propagating upstream
LIMITATIONS / FUTURE WORKS
Multilane and Autonomous Vehicle

MULTILANE ISSUE

- **Breakdown model**: Following vehicles can change lanes
- **Traffic control**: Vehicle might change lanes to avoid speed reduction

- **Model**: Consider probability of LC
- **Low CV**: Additional control by VMS
- **High CV**: Simultaneous control of multiple CVs in different lanes

7. CONCLUSIONS
Thank You!

Source: U.S. DOT
**CAPACITY DROP**
Discharge rate from bottleneck diminishes after onset of congestion

<table>
<thead>
<tr>
<th>Authors (Year)</th>
<th>Type</th>
<th>Capacity Drop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edie (1961)</td>
<td>Tunnel</td>
<td>-12.5 %</td>
</tr>
<tr>
<td>Banks (1991)</td>
<td>On-ramp</td>
<td>-3.0~10.0 %</td>
</tr>
<tr>
<td>Hall and Agyemang-Duah (1991)</td>
<td>On-ramp</td>
<td>-5.0~6.0 %</td>
</tr>
<tr>
<td>Cassidy and Bertini (1999)</td>
<td>On-ramp</td>
<td>-8.0~9.0 %</td>
</tr>
<tr>
<td></td>
<td>On-ramp</td>
<td>-4.0~10.0 %</td>
</tr>
<tr>
<td>Bertini and Leal (2005)</td>
<td>Lane drop</td>
<td>-10.0 %</td>
</tr>
</tbody>
</table>
STOCHASTIC CAPACITY
Capacity can vary

MECHANISM

- Elefteriadou (1994): Effect of merging flow rate (by Breakdown process)
- Evans et al. (2001): Effect of mainline flow (by Markov Chains)
- Son et al. (2004): Effect of merging vehicle (by Wave propagation model)
- X. Chen et al. (2014): Effect of merging vehicle (by Queueing theory)
2. BREAKDOWN MODEL

**Duration of $v_{in} (t_{in})$**

- **Acceleration time**
  
  $$t_a = \frac{u - v_{in}}{2a}$$
## TRIGGER

Degree of disturbance by merge vehicle

<table>
<thead>
<tr>
<th>Trigger Type</th>
<th>Range of $h$ ($a_i &lt; h &lt; b_i$)</th>
<th>$v_{tr}$</th>
<th>$t_{tr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$v_{tr1}$</td>
<td>$v_{tr2}$</td>
</tr>
<tr>
<td>No trigger</td>
<td>$h &gt; h_{cr}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>I</td>
<td>$h_1 &lt; h \leq h_{cr}$</td>
<td>$v_{in}$</td>
<td>$\frac{h-h_1}{h_{cr}-h_1}t_{in}$</td>
</tr>
<tr>
<td>II</td>
<td>$h_{II} &lt; h \leq h_{I}$</td>
<td>$\frac{(h-h_{II})u}{\tau_F}$</td>
<td>$v_{in}$</td>
</tr>
<tr>
<td>III</td>
<td>$h_{III} &lt; h \leq h_{II}$</td>
<td>0</td>
<td>$v_{in}$</td>
</tr>
</tbody>
</table>

*The magnitude of trigger depends on headway, $h$, between leading and following vehicle.
PROPAGATION
Probability calculation

Probability of propagation

\[ p(PR) = p(\sum_{i=1}^{n} b_i < B_{cr}) \]

- \( n \): vehicle number during merging interval, \( t_M \)
  \[ n = q t_M \]

- \( b_i \): buffer headway of vehicle \( i \)
  \[ b_i = h_i - \frac{s^*_i(u)}{u} = h_i - (\tau_i + \frac{d_i}{u}) \]

- \( B_{cr} \): Critical buffer time
  \[ B_{cr} = t_{tr} \left( 1 - \frac{v_{tr}}{u} \right) \]
BREAKDOWN PROBABILITY

Process Summary

Step 0 [Parameters]
- Flow rate \( (q) \)
- Headway distribution \( (p(h|q)) \)

Step 1 [Trigger]
- Decide trigger type by \( h \)
- Estimate \( v_{tr} \) and \( t_{tr} \) as function of \( h \)
- Calculate \( B_{cr} \) by \( v_{tr} \) and \( t_{tr} \)
- Estimate \( p(PR|h) \) by \( B_{cr} \)

Step 2 [Propagation]
- Estimate \( p(BD|q) \)

Parameters
- Driver: \( \tau \) (and \( d \))
- Vehicle: \( acc \)
- Traffic: \( u, v_{in}, s_M, t_M \)

2. BREAKDOWN MODEL
3. MODEL PROPERTIES

Effects of Flow rate

- **Higher mainline flow** → Severe trigger, Higher prob. propagation → Higher prob. breakdown
- **Higher merge flow rate** → Higher prob. propagation → Higher prob. breakdown

(Shorter merging interval → Fewer vehicles to resolve disturbance )
MODEL PROPERTIES

Effects of Merging behavior

**By merging speed**

- \( v_{in} = 40 \text{ mph} \)
- \( v_{in} = 42 \text{ mph} \)
- \( v_{in} = 44 \text{ mph} \)
- \( v_{in} = 46 \text{ mph} \)
- \( v_{in} \geq 48 \text{ mph} \)

**By merging spacing**

(Leading and merging vehicle)

- \( s_M = 150 \text{ ft} \)
- \( s_M = 100 \text{ ft} \)
- \( s_M = 50 \text{ ft} \)

**MECHANISM**

- ✔️ High merging speed → Mild trigger → Low breakdown
- ✔️ Small merging spacing (close to leading vehicle) → Mild trigger → Low breakdown
MODEL PROPERTIES
Effects of Driver Characteristics

Effect of Driver characteristic \( \tau \)
- Trigger (magnitude of \( v_{tr} \) and \( t_{tr} \)) is affected by \( \tau_F \)
- Propagation is affected by \( E(\tau) \)

MECHANISM
- Small \( E(\tau) \) (aggressive driver)
  - Mild trigger, Low prob. propagation
  - Low breakdown

Effects of \( E(\tau) \)

- \( \tau = 1.6 \text{ sec} \)
- \( \tau = 1.5 \text{ sec} \)
- \( \tau = 1.4 \text{ sec} \)
Breakdown probability decrease with:

1) Low flow rate (mainline, merging) ➔ Demand & traffic management
2) High merging speed ➔ Geometric design
3) Low deviation of headway ➔ Controllable (proactive control)
4) Low $\tau$ (aggressive driver) ➔ Connected automated vehicle
PROACTIVE VSL CONTROL

Contribution

Decrease Breakdown Probability without Propagated Congestion

Discuss for VMS

- Required at low CV penetration to control regular vehicles
- Phase-out with CV penetration increasing
## ONGOING WORKS

### Model Validation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>From</th>
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<tbody>
<tr>
<td>Free flow speed</td>
<td>$u$</td>
</tr>
<tr>
<td></td>
<td>Trajectory or Historical data (free flow state)</td>
</tr>
<tr>
<td>Wave speed</td>
<td>$w$</td>
</tr>
<tr>
<td></td>
<td>Trajectory or Literature</td>
</tr>
<tr>
<td>Headway distribution</td>
<td>$p(h)$</td>
</tr>
<tr>
<td></td>
<td>Trajectory or Detector data</td>
</tr>
<tr>
<td>Average tau</td>
<td>$E(\tau)$</td>
</tr>
<tr>
<td></td>
<td>Trajectory or Literature</td>
</tr>
<tr>
<td>Merge flow rate</td>
<td>$q_M$</td>
</tr>
<tr>
<td></td>
<td>Historical data</td>
</tr>
<tr>
<td>Merge headway distribution</td>
<td>$p(t_M)$</td>
</tr>
<tr>
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<td>Trajectory data</td>
</tr>
<tr>
<td>Merge speed</td>
<td>$v_{in}$</td>
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<td></td>
<td>Trajectory data</td>
</tr>
<tr>
<td>Acceleration rate</td>
<td>$a$</td>
</tr>
<tr>
<td></td>
<td>Trajectory data</td>
</tr>
<tr>
<td>Merge spacing</td>
<td>$s_M$</td>
</tr>
<tr>
<td></td>
<td>Trajectory data</td>
</tr>
</tbody>
</table>
STOCHASTIC BREAKDOWN MODEL

Results / Features

- Develop model to link microscopic behavior to breakdown probability
- Increases with flow rate (S-shape)
  → Consistent with previous result
- General model to incorporate other factors as well as flow rate
  (→ Next Chapter)