Introduction to Traffic Flow Theory

13-09-16

Victor L. Knoop
Learning goals

• After this lecture, the students are able to:
  - describe the traffic on a microscopic and macroscopic level
  - apply the relationship $q = ku$
  - draw the fundamental diagram, i.e. $q = q(k)$
  - argue the differences and similarities between relationships on the network level and road level
  - explain the steps in numerical traffic flow models
Part 1: Traffic description
Zone description

- Speed in the zone dependent on nr of vehicles
Traffic variables

- Macroscopic equivalents:
  1) Speed \( (v) \sim \text{Average speed} \ (u) \)
  2) Distance headway \( (s) \sim \text{density} \ (k) \)
  3) Time headway \( (h) \sim \text{flow} \ (q) \)
From micro to macro

- Average speed \( u = \langle v \rangle \)
- Density \( k = 1/\langle s \rangle \)
- Flow \( q = 1/\langle h \rangle \)
- Pay attention to units!
From micro to macro

- Average speed $u = \langle v \rangle$
- density $k = 1/\langle s \rangle$
- Flow $q = 1/\langle h \rangle$

- A road has a density of 20 veh/km, what is the average distance headway?
- The average time headway is 4s what is the flow?
- The speed is 1 miles/second and the gross headway is 5s, what is the flow?
Part 2: Traffic relationships
Traffic relationships

- Macroscopic
Traffic relationships

- Macroscopic

\[ q = C \times u \]
Traffic relationships

- Macroscopic

\[ q = C \times k \times u \]
Traffic relationships

• Macroscopic

Example:

\[
k = 10 \text{ veh/km} \\
v = 20 \text{ km/h}
\]

Take a 2x20 km road

In the first 20 km, \(20 \times 10 = 200 \text{ veh}\)

All these vehicles pass the detector in one hour \(\Rightarrow q = 200 \text{ veh/h}\)
Traffic relationships

- Microscopic
- Differentiation between gross and net space/time headways

What is the space headway expressed from the time headway and the speed?
## Relationships

<table>
<thead>
<tr>
<th>Microscopic (vehicle-based)</th>
<th>Macroscopic (flow-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space headway (s [m])</td>
<td>Density (k [veh/km])</td>
</tr>
<tr>
<td>Time headway (h [s])</td>
<td>Flow (q [veh/h])</td>
</tr>
<tr>
<td>Speed (v [m/s])</td>
<td>Average speed (u [km/h])</td>
</tr>
</tbody>
</table>

\[
s = h \times v \quad \quad q = k \times u
\]
Apply \( q = ku \)

A road has a density of 20 veh/km, and the average speed is 100 km/h: what is the flow?

- The flow is 1500 veh/h and the density is 30 veh/km, what is the speed?
- The speed is 1 miles/minute and the headway is 5s, what is the density?
Part 2b: Behavioral relationships
## Relationships

<table>
<thead>
<tr>
<th>Microscopic (vehicle-based)</th>
<th>Macroscopic (flow-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space headway (s [m])</td>
<td>Density (k [veh/km])</td>
</tr>
<tr>
<td>Time headway (h [s])</td>
<td>Flow (q [veh/h])</td>
</tr>
<tr>
<td>Speed (v [m/s])</td>
<td>Average speed (u [km/h])</td>
</tr>
</tbody>
</table>

\[
s = h \times v \\
q = k \times u
\]

- q is found from k and u (theoretically)
- Additionally is there a (behavioral) relation between k and u
Zone description

Traditional

<table>
<thead>
<tr>
<th>Microscopic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Macroscopic</th>
</tr>
</thead>
</table>

New

Zones

- Speed in the zone dependent on nr of vehicles
Relationships variables

- Given $q = ku$
- Given one more relationship, e.g. $u = u(k)$
  -- does that make sense? --
  traffic state is determined by one variable
Exercise

- Determine a realistic relationship between two variables (First qualitatively, maybe add typical points later)
- Derive the fundamental diagrams:
Exercise

- Determine a realistic relationship between two variables (First qualitatively, maybe add typical points later)

- Derive the fundamental density-flow diagram.
Points characterising the FD

- Often modelled: triangular in flow-density
- Critical speed: 100 km/h
- Jam density: 125 veh/km (8 m/veh)
- Capacity: \( \frac{1}{1.5} \times 3600 = 2400 \) veh/h/lane (=min time headway 1.5 s)
Triangular
Greenshields
Challenge the future
Differences

- Mainly in free flow branch
  - Speed reduction
  - Functional form
- Most have a straight line in q-k for the congested branch (except Greenshields)
- Capacity drop
How about fundamental diagrams?
How about fundamental diagrams?
How about fundamental diagrams?
In excessive demand, where is the queue?
Simple road with increasing demand

What happens if the demand increases
What observations can be made?

A) Whole FD  
B) Increasing part of FD (free flow)  
C) Decreasing part of FD (congestion)
Simple road with varying demand

Flow q (veh/h)

Density k (veh/km)
Part 3: Network description
Zone description

- Speed in the zone dependent on nr of vehicles
Stochasticity in local data

- Macroscopic fundamental diagram
- “Average” fundamental diagram for an area

\[
\text{Flow} = \text{Density} \times \text{Average density}
\]

Fig: (Geroliminis and Daganzo)
Not so simple road

- Origins and destinations everywhere
- By increasing input $\Rightarrow$ congestion
- Major difference with road!
Averaging traffic states leads to lower FD

Flow q (veh/h) vs Density k (veh/km)
Averaging traffic states leads to lower FD

But there is much more...
Network with periodic boundary
Build up of congestion
Fitting a functional form

\[ P(A) = A*(c_1 + c_2A + c_3A^2) - c_4\sigma \]

Homogeneous traffic situation

Inhomogeneous traffic situation
Fitting a functional form

\[ P(A) = A^2(c_1 + c_2 A + c_3 A^2) - c_4 \sigma \]
Causes of decrease with inhomogeneity

Different traffic conditions

![Graph showing the relationship between inhomogeneity and accumulation](image-url)
Challenge the future

Improvement: Generalised NFD

Generalised Macroscopic Fundamental Diagram

Std of density =>

Accumulation =>

TU Delft

Challenge the future
Use for your desktop

GMFD top view fit

Challenge the future
Part 4: Traffic dynamics
Why numerical solutions are needed

• Only applicable in relatively simple situations, e.g. with respect to upstream traffic demand, off-ramps and on-ramps, etc.

• What to do when demand on main-road and on-ramps is changing dynamically? Use numerical approximations!

• Practical applications, e.g. use for network simulation
Basic principles

• Various approaches exist to trace traffic dynamics

• Simplest:
  - Divide roadway into cells $i$, length $dx$
  - Divide time into steps with length $dt$

\[
\begin{align*}
  x_{i-1/2} & \quad k_i \quad x_{i+1/2} \\
  q_{i-1/2} & \quad q_{i+1/2}
\end{align*}
\]

interface at $x_{i+1/2}$
Assumptions

- Cells are homogeneous, length $dx$
- Within a time step $(dt)$, traffic flow is stationary
- Express $k_{i,t+1} = f(k_{i,t}, q_{i-1/2,t}, q_{i+1/2,t}, dt, dx)$
Basic principles (2)

• For the slides: this is the answer...

\[ k_{i,t+1} = k_{i,t} + (q_{i-1/2,t} - q_{i+1/2,t}) \frac{dt}{dx} \]

\[ q_{i+1/2,j} = \bar{q}(k_{i,j}, k_{i+1,j}, k_{i,j+1}, k_{i+1,j+1}) \]

• But how to get to:

\[ q_{i+1/2,j} = k_{i,j} u_{i,j} = Q(k_{i,j}) \]
What if uncongested?

- What determines the flow from A to B?
  - A – state in cell A
  - B – state in cell B
What if congested?

- What determines the flow from A to B?
  - A – state in cell A
  - B – state in cell B
The trick:

- Demand of region A and supply of region B

\[ D_L = \begin{cases} 
q_A & k < k_c \\
c & k \geq k_c 
\end{cases} \quad S_B = \begin{cases} 
c & k < k_c \\
q_B & k \geq k_c 
\end{cases} \]

- \( D_L \) = maximum flow out of region L (bounded by the capacity of the road)
- \( S_R \) = maximum flow into region R (bounded by road capacity and the space becoming available during one time-step)
- Actual flow at \( x=0 \) : \( \min(D_L, S_R) \)
Graphically

- Flow based on Demand & Supply
- => fundamental diagram
Dynamic MFD model

Network Transmission model
Network-wide traffic management

• Microscopic and macroscopic models are OK to simulate small sections
• In cities, congestion spreads and the whole city need to be described
• Another way of describing is needed
Network Transmission Model

Base qualities

• Computational speed: few steps

• **Dynamic** traffic patterns
  - Road closure, rerouting

• Scalability
  - bigger or smaller zones
  - use other scale within one zone
Process

• Implementation in OpenTraffic Sim, TU Delft simulator, enabling link to other models

• Link to static model: ao zone-boundaries, road length per zone
Oorspronkelijke zones
Zones
Calibration

- Data shows incomplete and biased view (location of measurements, roads?)
  => no proper NFDs

- Estimate model parameters based on theoretical considerations
Solution:

- 1st estimate for NFD: based on properties of road (from static model)
- Then adapt based on speeds
- Adapt as well: boundary capacity
Result:

- Dynamic model voor:
  1) Normal day
  2) Beach traffic (adapt OD matrix)
  3) Normal day with incident (capacity adapted)
Learning goals

• After this lecture, the students are able to:
  - describe the traffic on a microscopic and macroscopic level
  - apply the relationship $q = ku$
  - draw the fundamental diagram, i.e. $q = q(k)$
  - argue the differences and similarities between relationships on the network level and road level
  - explain the steps in numerical traffic flow models
Model: possibilities

- Fast: 15s for 3 h simulation on (old) laptop. Expected: factor 10 improvement by recoding? Fast enough for on-line computation (including optimalisation!)
- Change OD matrix for events
- Boundary capacity and zone properties change, for instance in incident
- Schaling:
  - zone size (bigger / smaller) (?)
  - combine with other modelling scales (eg, one zone on microscopic level)
Stochasticity in local data

- Macroscopic fundamental diagram
- “Average” fundamental diagram for an area

Fig: (Geroliminis and Daganzo)
Name giving

• Macroscopic Fundamental Diagram = Network Fundamental Diagram
• Name giving
  Average density = Accumulation
  Average (internal) flow = production
  Outflow = performance
Model: volgende stappen

- Kwaliteitsmaten & -eisen definieren
- Inclusie zoekverkeer
- “Stroomwegen” apart houden
- Beperkingen in routekeus (nu alleen wel/niet tussen zones, maar geen opeenvolging: wel A=>B en B=>C, maar niet A=>B=>C)
- Hoe kunnen regelscenario’s in het model meegenomen worden: welke parameters moeten hoe aangepast worden?
- Eventueel schakelbaarheid tussen niveaus
- Data-feed en toestandsschatting (incl HB)
Name giving

- Macroscopic Fundamental Diagram = Network Fundamental Diagram
- Name giving
  Average density = Accumulation
  Average (internal) flow = production
  Outflow = performance
Shape

• Qualitatively like FD (but differences...)

[Graph showing Macroscopic Fundamental Diagram with axes labeled "Av. density (veh/km/lane)" and "Av. flow (veh/km/h)".]
Relation performance - production

Correlation - Geroliminis and Daganzo (2008)

Outbound flow (veh/s) vs. Average flow (veh/s)