Mathematical view on traffic flow theory

The use of variational theory to compute capacity

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Goals for today

• Show techniques:
  • Count in vehicle number (or pedestrian nr, cycle number)
  • Use dimensional analysis to exclude parameters of your problem (simplification)
  • Moving observers can help for analysis
  • Transformation of capacity-problem to shortest-path problem

• Applied to a problem of road capacity under pedestrian crossings:
  • Capacity can be calculated
  • Capacity depends on frequency x (crossing time)²
Traffic relationships
Traffic relationships

- Often modelled: triangular in flow-density
Moving observers
Moving observers

• How does the relation \( q = k u \) change for a moving observer for the relative flow compared to the moving observer?
• Video

• \( q = k(u-v) \)
Variational theory
Method

• Construct **N-plane** $N(x,t)$, showing how many vehicles have passed location $x$ at time $t$
• Limitation to $N$ can come from
  • demand (no-one wants to go)
  • Supply (traffic jams)
• Check all possible limitations, and the most strict limitation is the final N-number
Construction of $N(x,t)$

Space

Time

$n=1$  $n=2$  $n=8$

blocking
Construction of $N(x,t)$

- Check all possible limitations, and the most strict limitation is the final N-number.
Construction of $N(x,t)$

- Check all possible limitations, and the most strict limitation is the final N-number.
- Can be converted to a shortest-path problem with moving observers, and overtaking rates as costs.
Construction of $N(x,t)$

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- Can be converted to a shortest-path problem with moving observers, and overtaking rates as costs.

Theory: for triangular fundamental diagram, all shortest path go forward with free flow speed, follow a bottleneck or go back with wave speed.
Capacity with crossing pedestrians
Problem definition

• Find the capacity of the road under crossing pedestrians

• Assumptions:
  • Homogeneous flow of pedestrians
  • Equal crossing time
  • Triangular fundamental diagram

• Pedestrian flow in peds/meter/second
Problem units: reduce dimensionality

- Units: time, space, and vehicle number
- Without loss of generality, choose units: $q_o = k_j = \tau = 1$
- Pedestrian flow now in units: $\text{peds}/\tau/(v_f \tau)$, with $v_f$ free flow speed
- **Increase of crossing duration**
  same effect as square of increase of pedestrian flow
Trajectories
Blockings

- Find the shortest path between two points at the same line
- Costs are:
  - 0 at a pedestrian
  - 0 moving at free flow
  - R moving at backwards at wave speed
Shortest path

- Find the shortest path between two points at the same point in space
- Costs are:
  - 0 at a pedestrian
  - 0 moving at free flow
  - $r$ moving at backwards at wave speed
After clever shortest path choice

- Analytical boundaries and expression for capacity
Concluding...
Lessons and conclusions

• Count in vehicle number (or pedestrian nr, cycle number)
• Use dimensional analysis to exclude parameters of your problem (simplification)
• Moving observers can help for analysis
• Transformation of capacity-problem to shortest-path problem

• Road capacity under pedestrian crossing can be calculated
• Capacity depends on frequency $x$ (crossing time)$^2$