Mathematical view on traffic flow theory

The use of variational theory to compute capacity

02-12-16

Goals for today

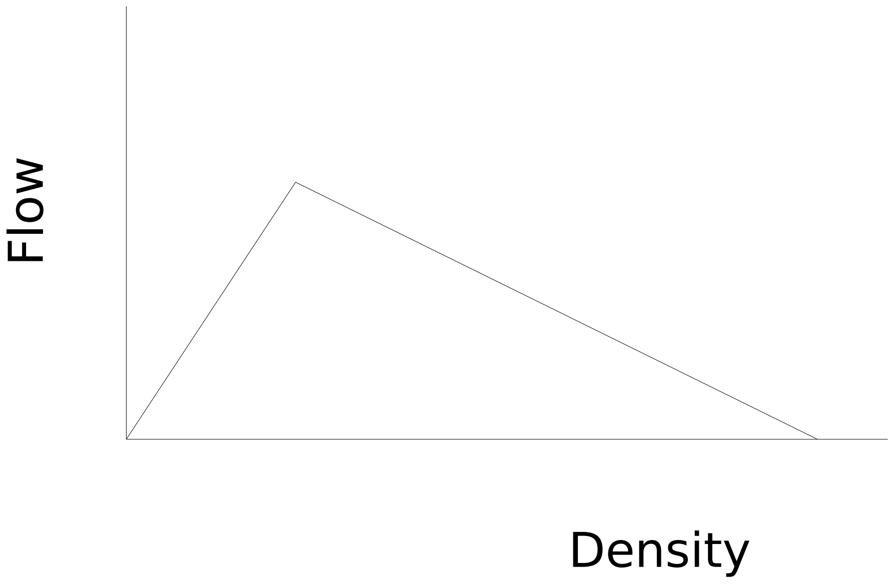
- Show techniques:
  - Count in vehicle number (or pedestrian nr, cycle number)
  - Use dimensional analysis to exclude parameters of your problem (simplification)
  - Moving observers can help for analysis
  - Transformation of capacity-problem to shortest-path problem

- Applied to a problem of road capacity under pedestrian crossings:
  - Capacity can be calculated
  - Capacity depends on frequency x (crossing time)^2

Traffic relationships

Traffic relationships

- Often modelled: triangular in flow-density



Moving observers

Moving observers

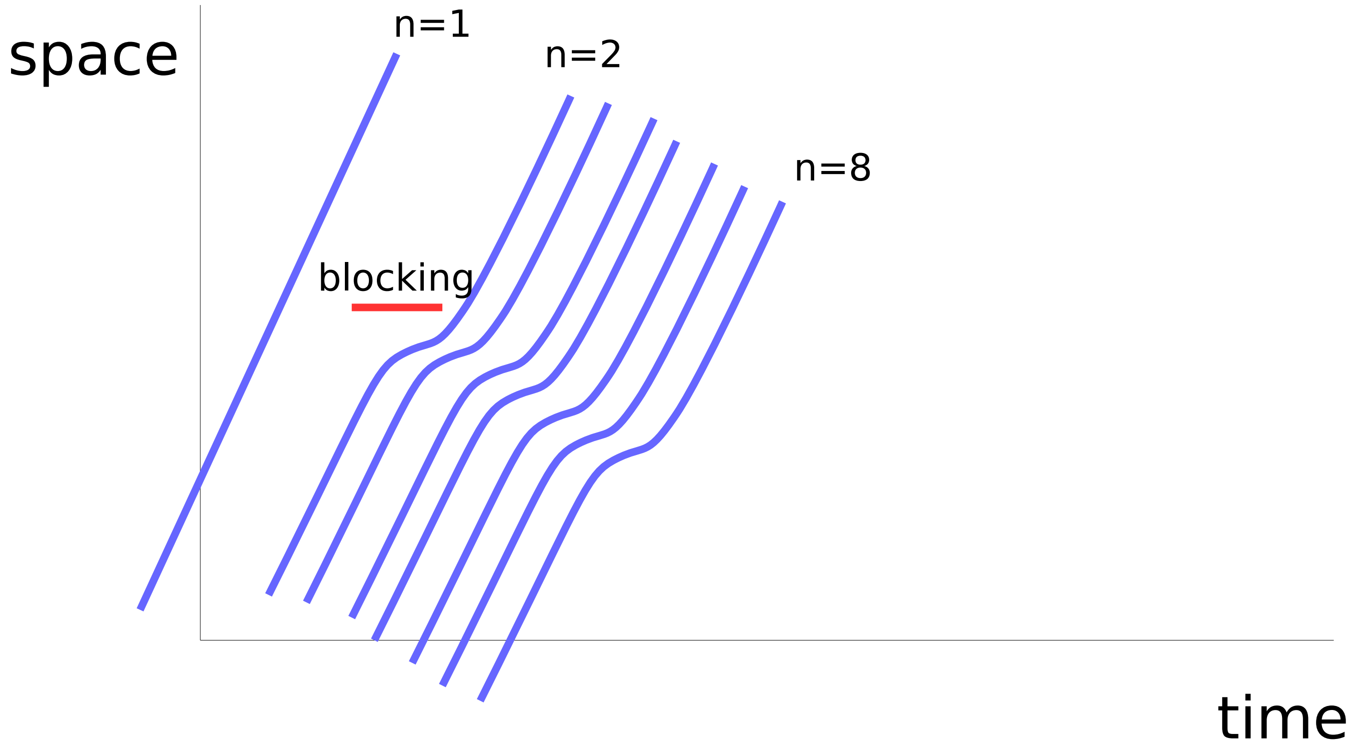
- How does the relation q=ku change for a moving observer for the relative flow compared to the moving observer?
- Video
- q=k(u-v)

Variational theory

Method

- Construct **N-plane** N(x,t), showing
  how many vehicles have pased location x at time t
- Limitation to *N* can come from
  - demand (no-one wants to go)
  - Supply (traffic jams)
- Check all possible limitations,
  and the most strict limitation is the final N-number

Construction of N(x,t)



Construction of N(x,t)

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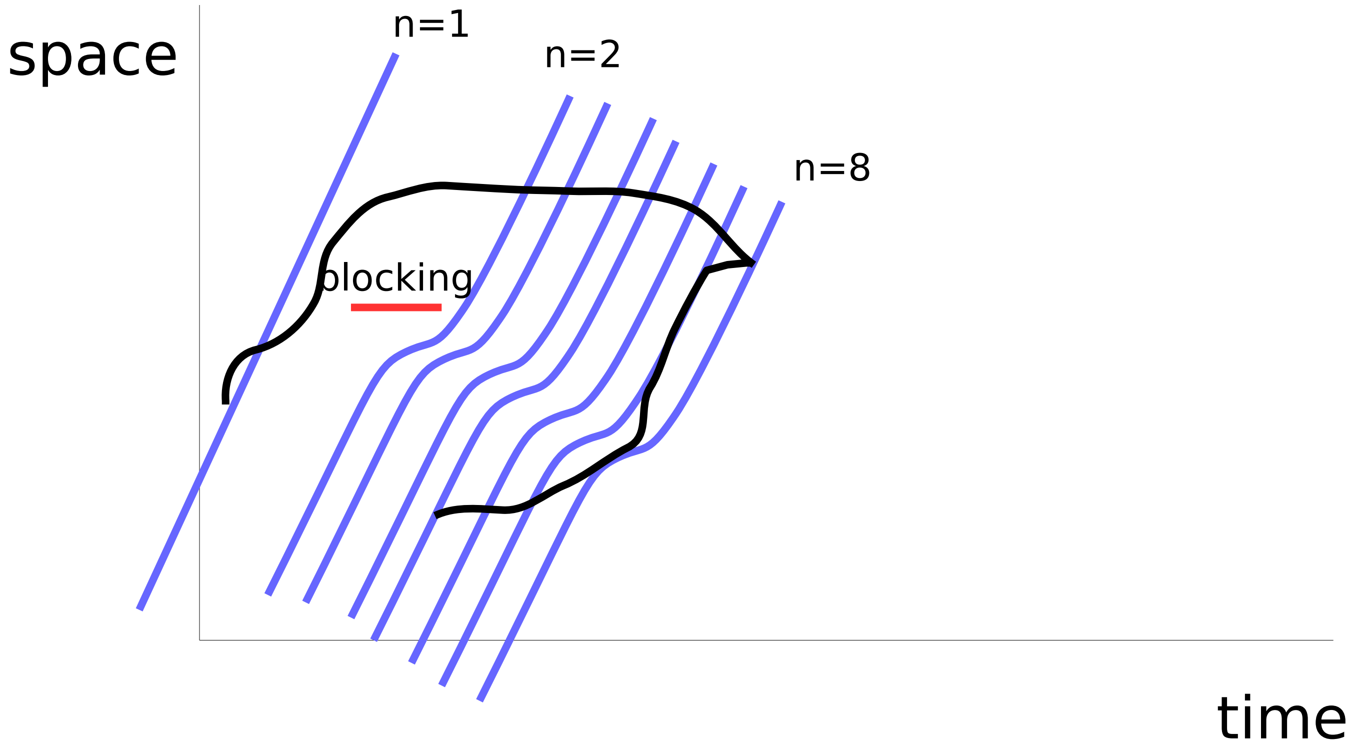
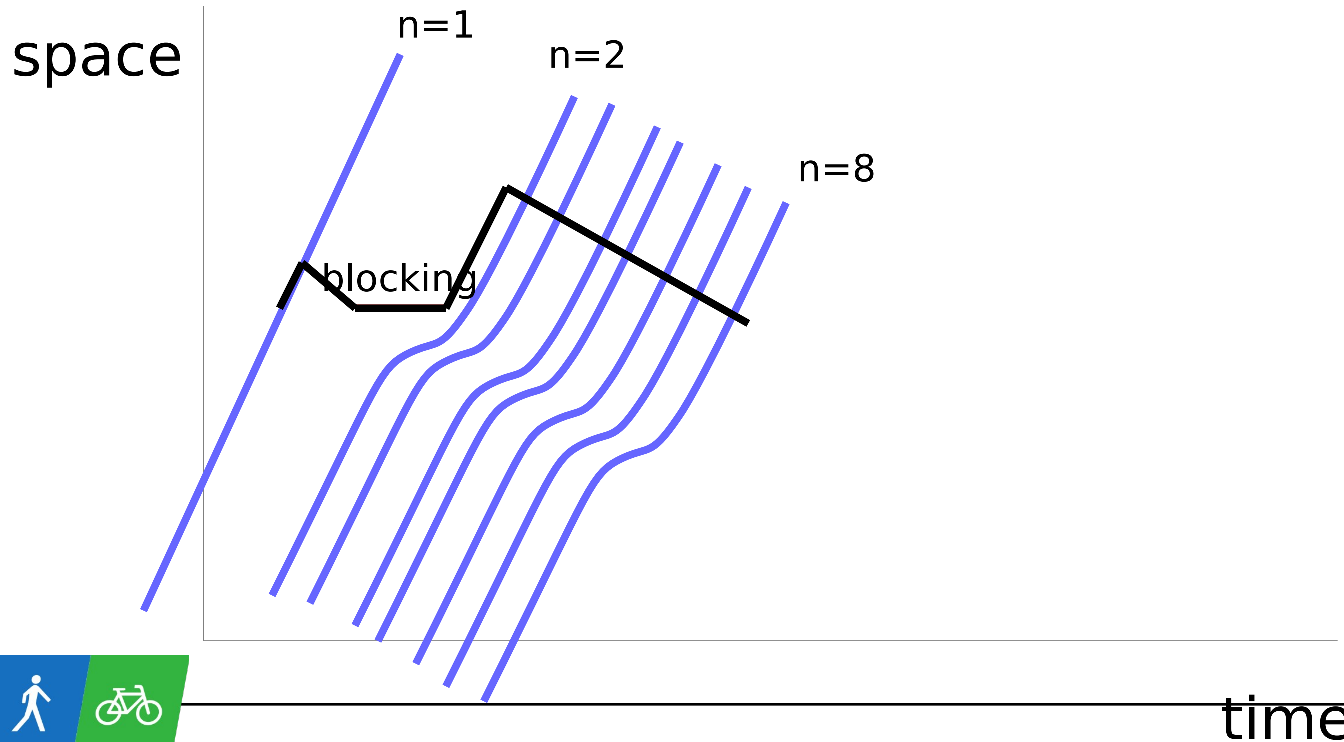


Construction of N(x,t)

- Check all possible limitations,
  and the most strict limitation is the final N-number
- Can be converted to a shortest-path problem with moving observers, and overtaking rates as costs



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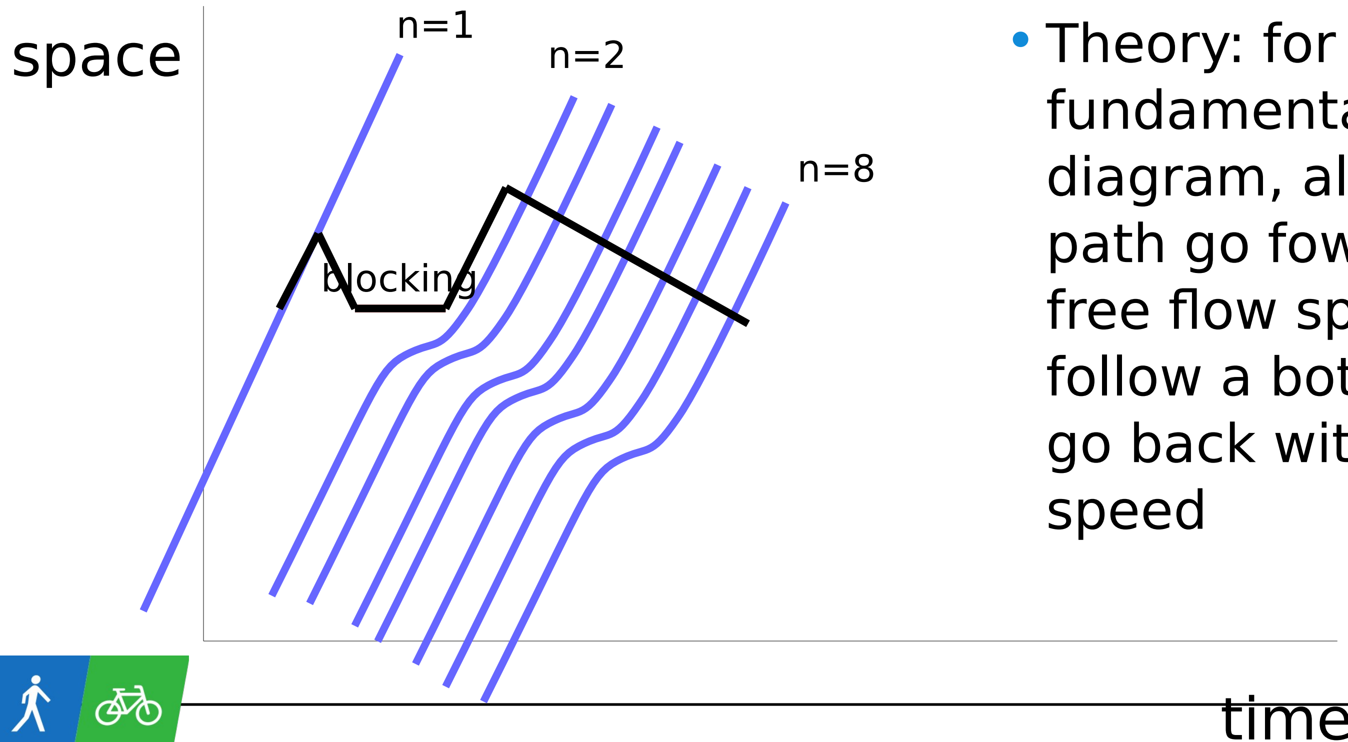


- Theory: for triangular fundamental diagram, all shortest path go foward with free flow speed, follow a bottleneck or go back with wave speed

Capacity with crossing pedestrians

Problem definition

- Find the capacity of the road under crossing pedestrians

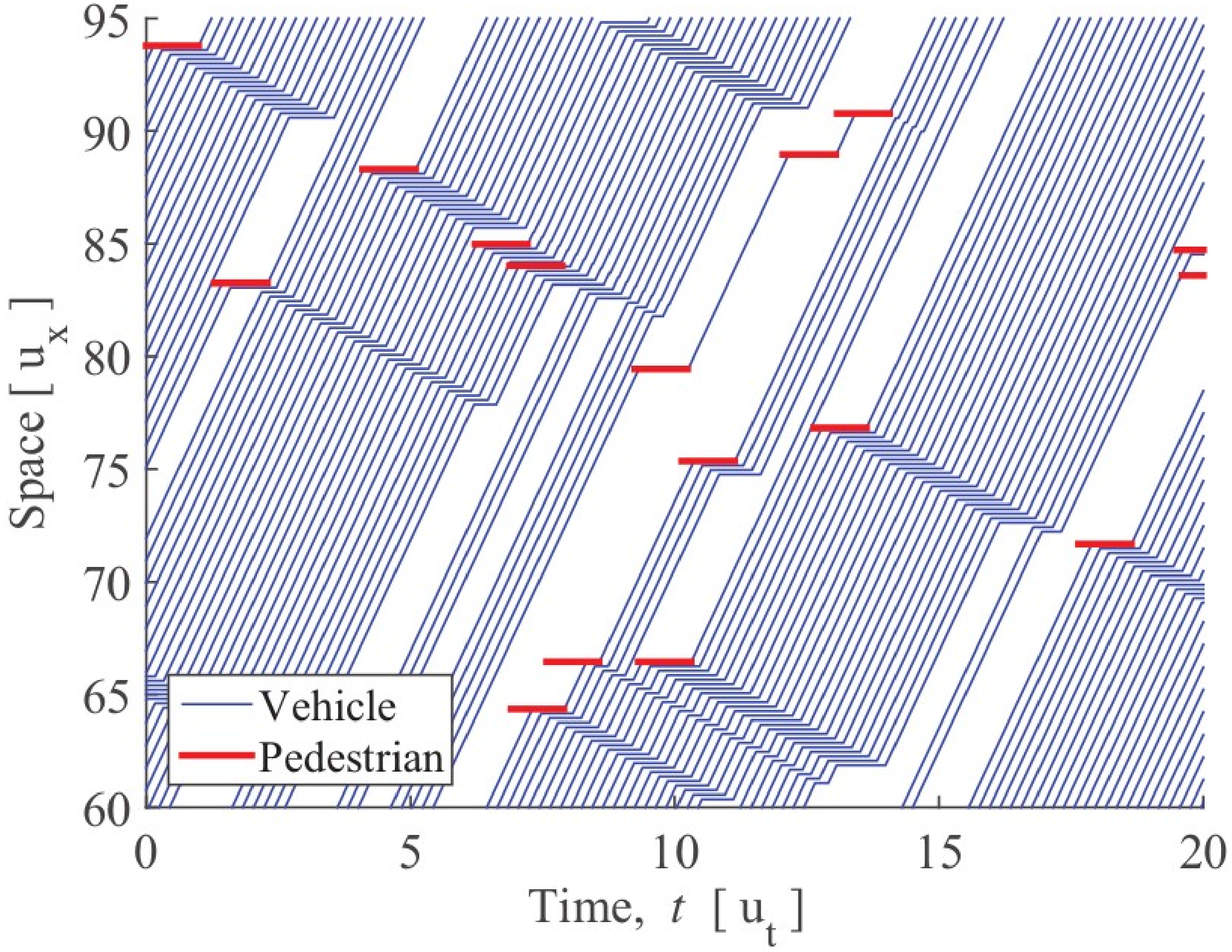
- Assumptions:
  - Homogeneous flow of pedestrians
  - Equal crossing time
  - Triangular fundamental diagram

- Pedestrian flow in peds/meter/second

Problem units: reduce dimensionality

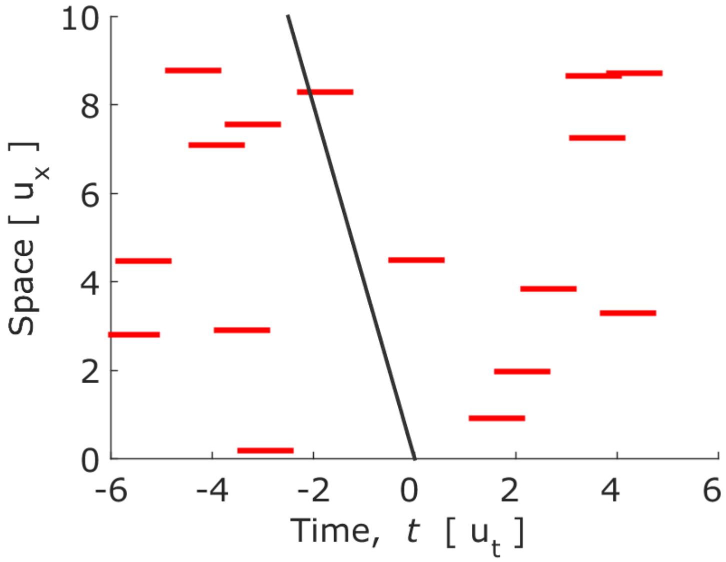
- Units: time, space, and vehicle number
- Without loss of generality, choose units: $q_o = k_j = \tau = 1$
- Pedestrian flow now in units: peds/τ/($v_f\tau$), with v_f free flow speed
- **Increase of crossing duration same effect as square of increase of pedestrian flow**

Trajectories



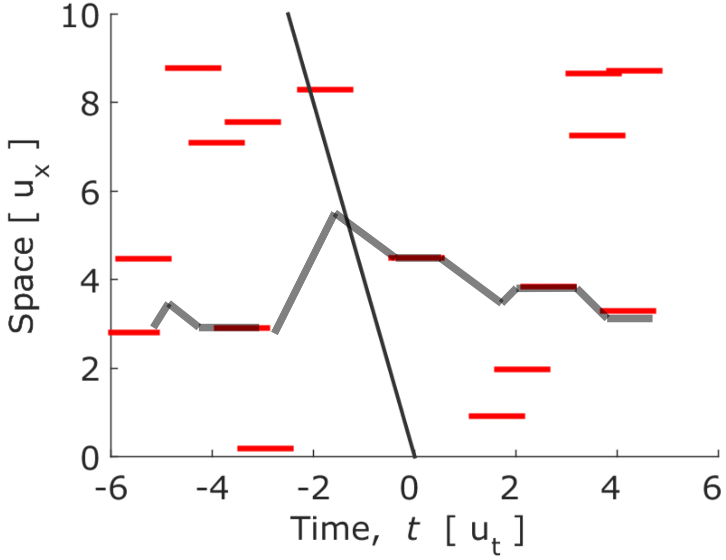
Blockings

- Find the shortest path between two points at the same line
- Costs are:
  - 0 at a pedestrian
  - 0 moving at free flow
  - R moving at backwards at wave speed



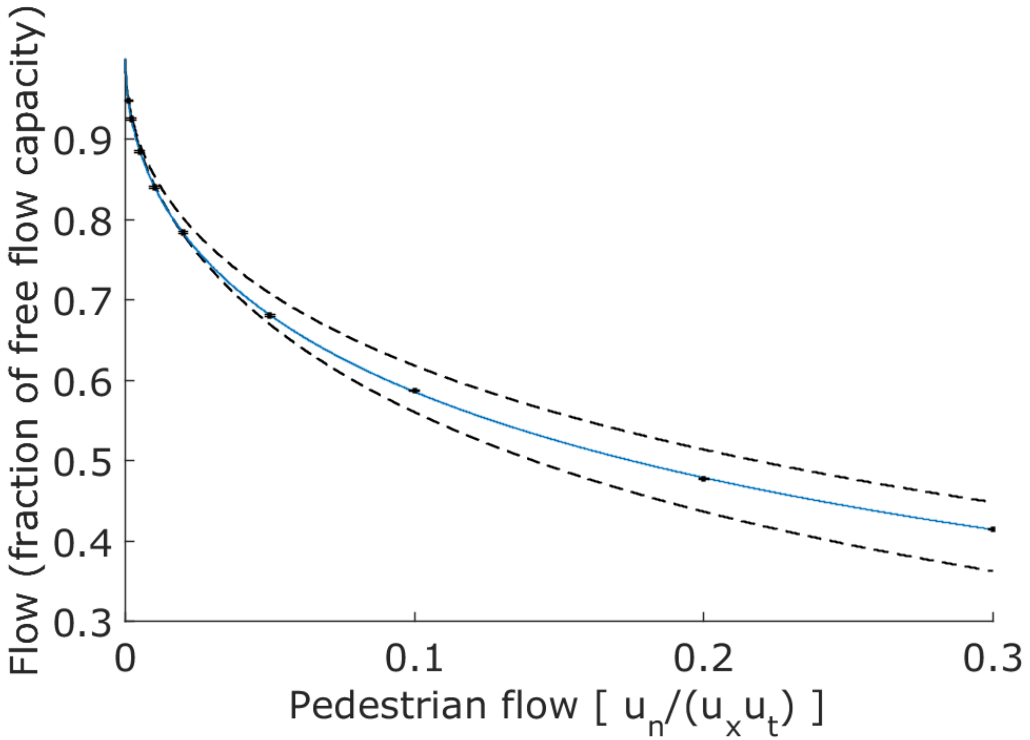
Shortest path

- Find the shortest path between two points at the same point in space
- Costs are:
  - 0 at a pedestrian
  - 0 moving at free flow
  - r moving at backwards
    at wave speed



After clever shortest path choice

- Analytical boundaries and expression for capacity



Concluding...

Lessons and conclusions

- Count in vehicle number (or pedestrian nr, cycle number)
- Use dimensional analysis to exclude parameters of your problem (simplification)
- Moving observers can help for analysis
- Transformation of capacity-problem to shortest-path problem

- Road capacity under pedestrian crossing can be calculated
- Capacity depends on frequency x $(\text{crossing time})^2$