

# Generalized Network Fundamental Diagram

Network Dynamics, Level of Service and Resilience Applications

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## ABSTRACT

This paper presents different applications of the so-called **generalized Network Fundamental Diagram (g-NFD)**, which extends recent work on the Network Fundamental Diagram by considering the impact of spatial heterogeneity of the network density.

In doing so, it turns out that the network production (weighted average flows) or exit rates from the network can be described accurately as a function of both the average network density and its spatial variation.

This paper discusses some of the new insights stemming from this representation in particular focusing on network dynamics. Furthermore, we propose two applications of the g-NFD, namely as a network level-of-service indicator and as a measure for network resilience.

## INTRODUCTION INTO THE G-NFD

The generalized Network Fundamental Diagram represents the relation between the average network density  $\rho(t)$  and a measure of the spatial heterogeneity  $\sigma(t)$  evaluated at instant  $t$ . The average network density describes the average number of vehicles per kilometer-lane of roadway. For the spatial heterogeneity, we use the standard deviation of the density  $\rho_i(t)$  for all measurement sections  $i$ :

$$\sigma(t) = \sqrt{\frac{1}{n} \sum_i (\rho_i(t) - \rho(t))^2}$$

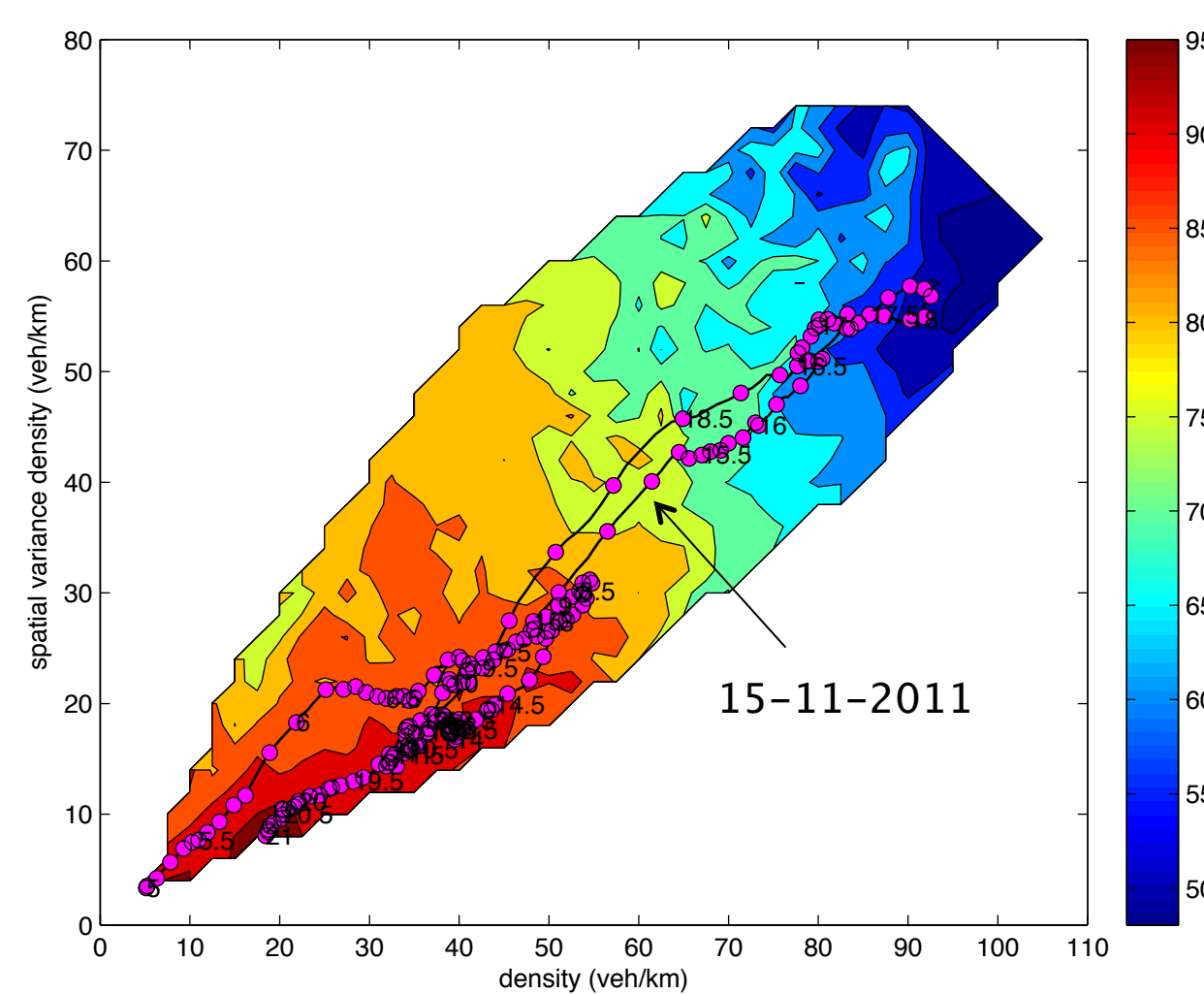
Let  $Q(t)$  denote the (weighted) average network flow and let  $V(t)$  denote the average network speeds at  $t$ . The g-NFD expresses  $Q$  (or  $V$ ) to  $\rho$  and  $\sigma$ , i.e.:

$$Q = Q(\rho, \sigma) \quad \text{and} \quad V = V(\rho, \sigma)$$

## EXAMPLE RINGROAD G-NFD

Let us now take a look at the representation of actual traffic data. To this end, we took data from the entire A10 ringroad around the Dutch city of Amsterdam.

The figure below shows the g-NFD representation of half a year of data collected on A10 ring-road.



The relation between average network speed  $V$ , density  $\rho$  and spatial heterogeneity  $\sigma$  is clear: the higher the average density and/or spatial heterogeneity, the lower the speed.

Using these data, we established the following relation:

$$V(\rho, \sigma) = v_0 \cdot \left( 1 - \left( \frac{\rho}{\rho_{jam}} \right)^n \right) \cdot \sqrt{1 - \frac{\sigma}{\sigma_0}}$$

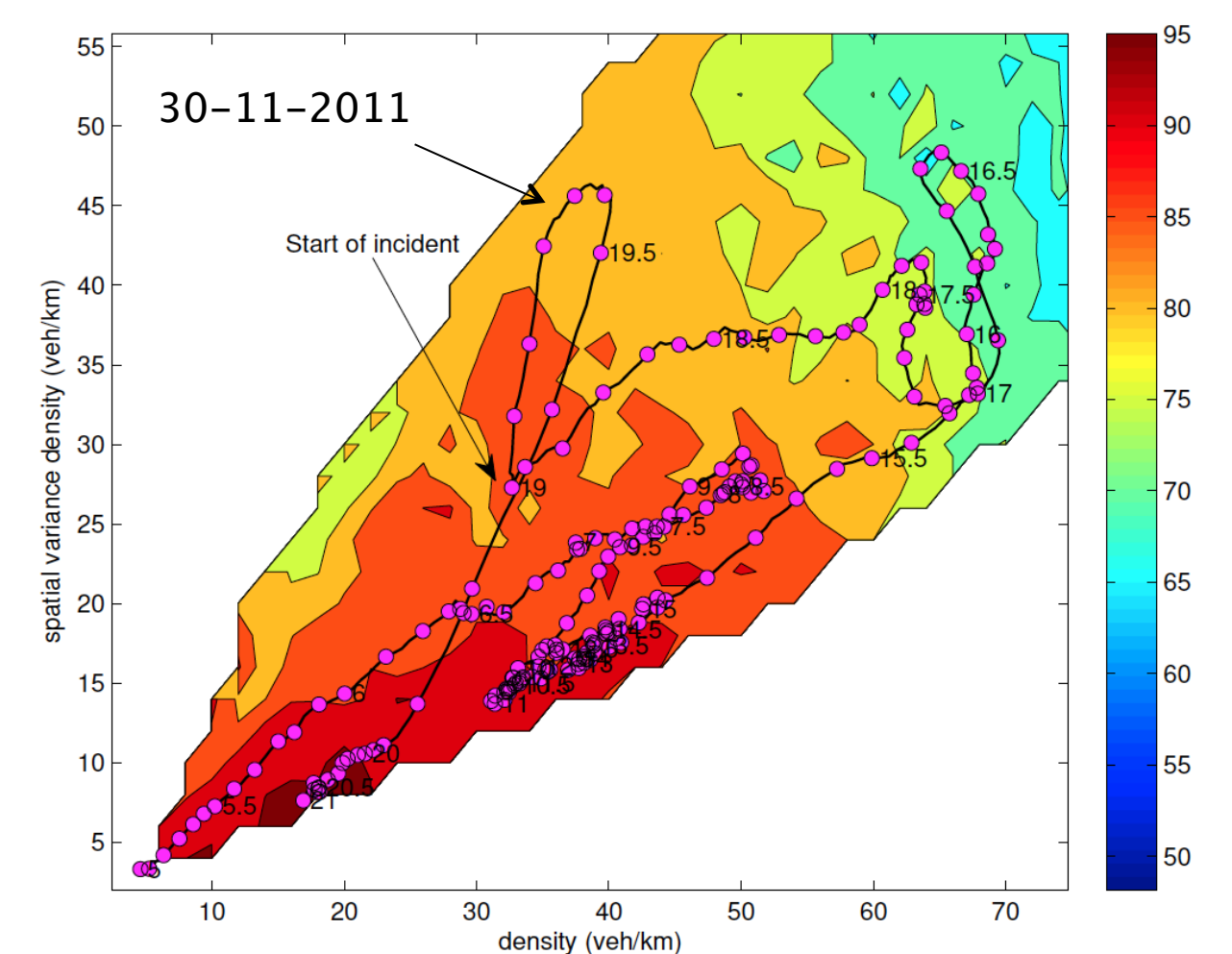
The fitting method yielded an adjusted rho-square of 0.93 and all parameters were statistically significant. Most parameters have a nice interpretation.

$V_0$ (km/h)	$\rho_{jam}$ (veh/h/lane)	$n$	$\sigma_0$ (veh/h/lane)
102.1	130	3.3	1244

Next to the half-year average contour, the picture shows the  $(\rho(t), \sigma(t))$  path for a particular day, showing the dynamics of density and heterogeneity during a regular day.

## INCIDENT CONDITIONS

The picture below shows the path during incident conditions (incident occurred at 19:00), clearly illustrating how the path breaks away from the recuperation line. We clearly see that while the average network density stays relatively constant, the heterogeneity increases sharply. At the same time, the average network speed diminishes substantially.



## APPLICATIONS

Both examples shown here reveal important applications of the g-NFD. First of all, the overall level of service can be expressed as a function of  $\rho$  and  $\sigma$ , giving a rough indication what the quality of the network conditions is.

More importantly, however, is the application in describing network resilience. As an incident causes an increase in  $\sigma$ , which in turn is accompanied a reduction in the average network speed. In the paper, we propose using the partial derivative:

$$\lambda(\rho, \sigma) = \frac{\partial}{\partial \sigma} V(\rho, \sigma)$$

as a state-dependent indicator for the reduction of network performance in case of large disturbances. This indicator shows the deterioration of average network performance as the spatial heterogeneity  $\sigma$  increases.