Risk-averse Traffic Assignment in a Dynamic Traffic Simulator

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1 Introduction and literature review

Often, travellers needs to be certain to arrive before a certain time, regardless of the road conditions. They therefore tend to avoid routes that can have long delays, even if such delays are seldom. This paper discusses a traffic assignment which takes risk avoidance into account. However, the probability distribution of travel times on links and the probability of incidents on links is not known to the travellers beforehand. They therefore have to make assumptions based on their risk-attitude.

The approach to risk-averse route choice behaviour presented in this paper is introduced by Bell [1]. He assumes that risk-averse drivers anticipate worst cases and minimise their exposure to these. In fact the traffic loads influence which cases are worst. Bell and Cassir [2] extend the concept using an demand-capacity relationship to determine the travel cost rather than two possible fixed costs. In this paper, a blockade means a lower capacity, resulting in a new travel time on the blocked link.

Nagae and Akamatsu [3] continue on this line of research. They point out that it might be too extreme to expect the worst case situation to happen. They relax the assumption of people being completely risk-averse. They add two extra terms to spread the breakdown chances over the different scenarios. This changes the perspective on route choice behaviour slightly, but moreover, it makes the mathematical framework much easier to solve. Bell et al. [4] use the same but, compared to the strictly risk-averse simulation, they just relax the perception of link failure probabilities (and not the route choice). The mathematical advantage then still holds.

Until now, this game theoretical approach to risk-averse assignment has only been combined with static models and not with a dynamic traffic simulation. This paper fills this gap. The risk-averse route choice model proposed here can be used in traffic assignment models. In the future, this risk-averseness might be implemented in journey planners. One can conceive of an on-line journey planner with a slide bar for risk-averseness. In general, risk-minimizing behaviour is relevant for transport of special goods or persons, such as hazardous materials or VIPs.

2 Methodology

The model proposed here extends the work of risk-averse traffic modelling [1, 2, 3, 4]. For the sake of simplicity, we formulate the model for one destination. A multi-destination network is easily fit into this model. All route-related variables should then be considered for each destination separately.

This method aims at computing the route choice vector ($\mathbf{h}(t)$), giving the route fractions in percentages over the different paths. Therefore, at each time instant t the elements of $\mathbf{h}(t)$ add up to 1. The route choice depends on the travel time in each scenario ($\mathrm{tt}_{ijt}^{\mathrm{link}}$) and the anticipated probability of each scenario, vector $\mathbf{f}$, which size is the number scenarios. In this paper, a scenario is a possible incident blocking link j somewhere in the network, and therefore the number of scenarios equals the number of links. In the context of this paper it is important to note that $\mathrm{tt}_{ijt}^{\mathrm{link}}$ is time dependent and can be calculated using any traffic simulator. Also, influences from downstream links might influence $\mathrm{tt}_{ijt}^{\mathrm{link}}$. The total cost for travelling under scenario j, T_j, can be obtained from the traffic simulator.

The key of a risk-averse approach, as introduced by Bell [1], is that a risk averse user would count on the worst scenario to happen most likely. Therefore, the following equations have to be solved:

$$\begin{aligned} \mathbf{f}^* &= \operatorname*{argmax}_{\mathbf{f}} \left(\langle T(\mathbf{f}, \mathbf{h}^*(t))\rangle\right) \qquad (1) \\ \mathbf{h}^*(t) &= \operatorname*{argmin}_{\mathbf{h}} \left(\langle T(\mathbf{f}^*, \mathbf{h}(t))\rangle\right) \qquad (2) \end{aligned}$$

With Nagae and Akamatsu [3] we argue that maximisation is perhaps too extreme and even risk-averse users have a more balanced expectation of the scenarios. Due to the limited length of this extended abstract, we will here only show the solution of the system; however, the full paper will also show the derivation of this solution. The key is that the scenario which gives the worst performance (anticipated cost of travelling), gets the largest anticipation in the risk-averse users' perception. In this paper, this performance is chosen to be the travel time T. Nagae and Akamatsu [3] show that with this slightly modified equation 1 there is a logit-like solution for the "incident anticipation" ($\mathbf{f}$), which is, for the case at hand, given by the following equation:

$$f_j = \exp\left(\vartheta T_j\right) / \sum_j \exp\left(\vartheta T_j\right). \tag{3}$$

Note the way the parameter ϑ works out in the solution: it indicates how smoothly the incident probability is distributed over the scenarios. $\vartheta = 0$ gives an equal probability to each scenario and $\vartheta = \infty$ gives only weight to the scenario with the highest disruption.

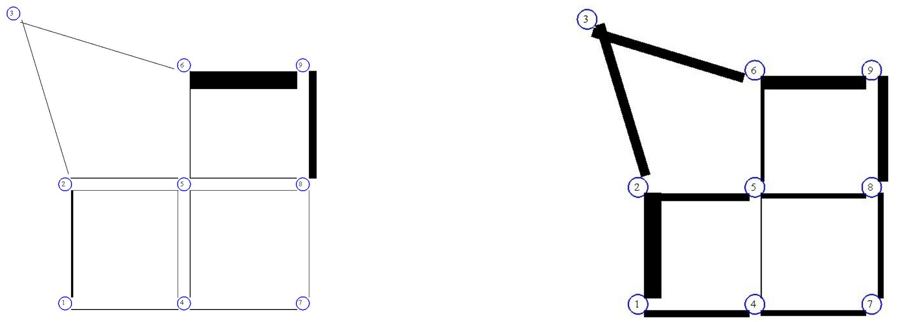
For the traffic assignment it is needed to have the anticipated link travel times, $\left\langle \mathrm{tt}_{it}^{\mathrm{link}} \right\rangle$. These can be obtained by taking a weighted average of the link travel times in each scenario:

$$\left\langle \mathrm{tt}_{it}^{\mathrm{link}} \right\rangle = \sum_j f_j \mathrm{tt}_{ijt}^{\mathrm{link}} \tag{4}$$

Using the above elements, traffic can be assigned risk-aversely in the following loop. The initial step is to start with an equal probability for each scenario, meaning all elements of $\mathbf{f}$ are equal. Then, we fix the assignment of traffic over the fastest route according to the free-flow travel time. With these initial values we start the optimisation loop. Using the traffic assignment, we calculate the link travel costs for each scenario using a dynamic traffic simulation program which gives the link travel times and the total travel times under each scenario. These, in turn, cause a new anticipation on each scenario (equation 3), thereby causing new anticipated link travel times (equation 4). With these travel times, a new one-shot traffic assignment (fastest route) is calculated which is averaged with the previous one using a Method of Successive Averages [5]. Now, a new iteration in the optimisation loop can be started. This loop converges to a solution for both the route choice and the anticipated scenario.

3 Results

The method is implemented and tested on a test-network, shown in figure 1, in which traffic has to go from the bottom left to the upper right corner. The network consists of a motorway (links at the left and top) and a secondary road (roads at the bottom and right) around a city centre (links in the middle). The capacity and the speeds have been adapted to the type of link. Most travellers would take the motorway in non risk-averse equilibrium conditions. Because of this high load, the impact of a blockade on the motorway would be high and therefore risk-averse people anticipate most on a blockade there. The distribution of the weights of the blockade is shown in figure 1a. As expected, the highest anticipation is put on the motorway link at the end. Note that a blockade at the end is more disruptive because that also blocks travellers which already passed the first link, but that have not yet passed the last link at the moment of happening. Figure 1b shows the resulting traffic assignment. This shows that not all travellers will take the motorway, which is the faster route if risk-averseness was not taken into account, indicating that adding risk-averseness changes the actual traffic assignment.



(a) The anticipated blocking probability **f** – the width of the line indicates the anticipated blocking probability

(b) Routes chosen by the travellers, **h** – the width of the link indicates the number of drivers taking that route

Figure 1: The results of the test network

4 Conclusions

This paper introduces dynamic queuing into risk-averse traffic simulation. It is shown that with an innovative approach, it is possible to integrate risk-averse traffic assignment with a traffic simulation with realistic queuing dynamics. In this simulation the influence of the incident will expend spatially and temporally, meaning travellers will avoid also other links than the incident site also in other times than the incident itself.

References

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