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The need of spillback simulation in assessing robustness: concepts and a case study with dynamic route choice

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ABSTRACT
Robustness of a network is, as main aim for road network managers these days, becoming an important study area for transportation scientists now. This paper discusses one specific aspect of robustness: the consequences of the blocking of a link in a road network using a traffic simulator. In a regional size road network simulation study, sequentially links are blocked and the simulation program determines the network performance, both with a route choice adapted to daily congestion and a route choice adapted to the actual situation. The paper focuses on spillback effects. A special feature in the new simulator proposed here is that the representation of spillback can be switched on and off; thus, effects of spillback can be examined explicitly. Road network robustness and characteristics of vulnerable links are evaluated for both spillback and non-spillback cases. It is found that spillback simulation is necessary for the estimation of the robustness of the network as a whole. Simulation of spillback is also needed to assess the impact of the blocking of a specific links; furthermore, without simulating spillback, it is not possible to identify correctly the most vulnerable links for the network performance.

INTRODUCTION
Reliable road networks are valued by both travelers and network authorities [1, 2]. Bates found, for example, that one minute reduction of standard deviation of travel time and two minutes of actual travel time are equally valued [3]. Bogers and Van Zuylen [4] showed that drivers avoid routes that are on the average the quickest but have a probability of exceptional high travel times. Robustness of networks is the ability of a network to cope with variations in demand or network capacity without much influence on travel times and is as such a corner stone for travel time reliability. The mentioned variations can be caused by normal daily fluctuations in demand and supply as empirically shown by Tu et al. [5]. Another cause for this variation is the blocking of a link by an incident or road maintenance. This is not part of the normal daily fluctuations and reduces the capacity. This paper analyzes this reduction of performance of a network caused by an incident. Different research projects assessing road network robustness use different traffic simulation models. The models differ in the dynamics of traffic flow and congestion. Due to the complexity of the network modeling, sometimes spillback, i.e. congestion propagation to a more upstream link, is not modeled. This contribution compares link network robustness for scenarios with and without spillback in order to assess the need of proper spillback modeling. It also focuses on methods for identifying critical links and compare methods with and without spillback. This study makes a distinction for cases in which road users adapt their route choice to the new situation with a blocked link and congestion and a situation in which they take their usual routes.

The method we use is as follows. We sequentially block all links, one at a time, and compare the results in network performance. The robustness is inverse proportional to the number of links that reduce the network performance in a considerable way. The robustness of the network depends on the internal structure and the flows in the network.

The main question in this paper is if conclusions about robustness and properties of important links differ for simulations with spillback and without spillback and if robustness studies using network models without spillback have any value in real life. It is found that the robustness assessment differs for the two approaches. Therefore spillback should be well modeled in robustness studies.
STATE OF THE ART

A considerable part of delays, about 25% at least, is caused by incidents [6]. In this article, we discuss the consequences of an incident in detail. We will find the most vulnerable links in a network (both with and without spillback modeled); furthermore, we will calculate the robustness of a network. The robustness against link failure is usually approached in one of the following two ways:

The first one, used in [7, 8], is an analytical, game theoretical approach. An “evil entity” wants to destroy a network and is given the possibility to destroy a limited number of links. The road users choose their routes in order to get a short travel time. Usually, it is assumed that drivers take the route such that they can not unilaterally change their route and get a higher utility, usually based on travel time [9]. Those links that reduce the network performance most if they are blocked are called vulnerable. The lower the effect of the dropout of links, the more robust a network is considered to be. In this approach, results are based on an analytical approach and a mathematical framework is set; in both articles [7, 8], a simple network is used as test case. The same approach is also used in [10], but here, the travel times also depend on the flow on the links.

The other approach found in literature, is the identification of vulnerable links in a real-size network, for instance using simulation. In [11], properties of vulnerable links are presented based on a simulation study. A problem arises when one wants to find the vulnerable parts in a big network. One of the methods is to do a quick, static simulation [12]. In this quick scan, the set of links that might be the most vulnerable of the network is reduced. The traffic dynamics are not captured correctly in this first simulation, though. Tamère et al. present a method is presented to select the most vulnerable links in different stages [13]. The first stage selects potentially vulnerable links and links that do not need to be reconsidered for being very vulnerable. This first selection is based on link flow properties in the every day situation.

Our research aims at problems in a real-world network. We will perform a simulation study in which we will incorporate the mathematical concepts of the analytical studies. Earlier, we presented a study to see the importance of spillback for finding the vulnerability of a link [14]. In that study, the route choice was taken fixed. In this paper, we show that also if the route choice varies over time, spillback effects are important for the robustness and the identification of the vulnerable links.

MATHEMATICAL APPROACH

There are two (groups of) actors in the situation at hand: on the one hand the travelers all wanting the best path through the network and on the other hand an “evil entity” wanting to harm the performance most by blocking one link. The network robustness studied here can be easily related to the performance of the evil entity. If a link is blocked, the evil entity wants the network performance to reduce a lot. Road users, on the other hand, all aim for a short travel time independent of the link blocking.

The performance indicator needs to be lower if people need to take a detour, come later or do not arrive at their destination at all. Often used indicators are the total time spent in the network or the average travel time. Both have the disadvantage of not including the people that cannot access in the network. As performance index, we opt for the total time people are at their destination before the end of the simulation.
Figure 1 shows the relationship between different performance indices. It shows the cumulative demand curve rising from the origin. Travelers will leave from the start of the simulation. After a while, the first people arrive at their destination, so the cumulative arrival curve rises as well. If from a certain moment in time the demand (rate) is larger than the capacity at the entrance links, people will queue at the entrance. From this moment on, there is a difference in the cumulative demand (including queued vehicles) and cumulative departures (excluding the waiting vehicles). The difference at a certain time is indicated with a 3 in Figure 1. At each time, the number of travelers that have arrived is the value of the line of cumulative arrivals; that value is indicated with the number 1 in the figure. The number of travelers at a moment t is the difference between the number of people departed and the number of people that have arrived; in the figure, it is indicated with a 2. It also shows that if there is no queuing at the entrance, the total time at destination is directly related to the total time spent (the demand is fixed). The performance index we use, the total time people are at their destination, is the green shaded area.

The collective result of the trips of all travelers is a measure for the network performance. As road users want to optimize their individual trip under all circumstances, the aim for the collective of road users is a robust network. However, the individual optimal route choice does not lead to a collective optimum (e.g., Braess [15]).

The situation is studied in two ways here. The first way is that the road users choose the routes that are fastest without an incident and do not deviate from it, even if their route is congested or blocked. In the second way, a road is blocked and road users will adapt their routes according to the new situation.

**Fixed routes**

In this scenario, we assume that travelers use fixed routes to reach their destination. The routes should represent the everyday choice of the travelers. An equilibrium assignment would be suitable, but in our model, it would be too time consuming to compute. Instead, we choose for an en-route assignment in the incident-free network. Vehicles are distributed over the fastest paths. Every quarter of an hour, travelers are informed about the travel times on every link. These still are the everyday travel times without a link blocking. Half of the travelers will reconsider his route based on this information. The new routes are found using a probit assignment. For each node-destination pair, the fastest route is determined 20 times, each with another random error on link travel times (normally distributed, standard deviation 10% of the travel time). Link travel time is computed from link distance and average speed on the link, which, in turn, is the calculated flow divided by density. Based on the routes, destination specific split fractions are stored for each node. In this way, more than 20 routes are used from an origin to a destination. After a vehicle has set off for a certain route, at the next node it reaches, it will follow the routes found for that node. Split fractions differ per node, destination and
time interval. The split fractions are stored; in this scenario, they determine the routes also when a lane is blocked. These routes are referred to as $\pi^*(G)$, in which $G$ is the network these routes are based on. We sequentially block links. With link $b$ blocked, network $G$ changes into $G_b$. For every link blocked, a separate simulation is run.

The network flows and therefore also the network performance $A$ can be different for the scenario in which spillback is modeled and the scenario in which no spillback is modeled, so it depends also on the simulation of spillback, $ss$.

The performance of the network in this scenario can now mathematically be expressed as:

$$A\left(\pi^*(G), G_b, ss\right)$$

(1)

The most vulnerable link $b^*$ is the link for which the network performance is lowest if it is blocked:

$$b^* = \arg\min_b \left\{ A\left(\pi^*(G), G_b, ss\right) \right\}.$$  (2)

### A link is blocked, road users adapt their routes

Here, the simulation starts with blocking link $b$. The travelers respond to that action by changing their routes and therefore, their actions are different for every different link blocked. The road users adapt their behavior to the newly envisaged situation with the blocked network, and will also change their routes during the simulated time due to congestion.

Just as in the scenario with fixed routes, routes are chosen based on expected travel time. Contrary to that scenario, drivers now base their expected travel time on the actual envisaged situation. Routes are updated every time period of 15 minutes based on the congestion, including the congestion caused by the incident. A part of the travelers will be assigned to a new route; the other part will choose the old route for the coming period. If the fraction of people that take a different route is too small, the route choice effect cannot be seen. On the other hand, a very large share is unrealistic [1]. The path set found in this case, is called $\pi^*(G_b)$. It depends on the blocked link $b$.

As network flows will differ for scenarios with and without spillback, the network performance can also differ dependent on the simulation of spillback.

For this scenario, the network performance function that needs to be evaluated is

$$A\left(\pi^*(G_b), G_b, ss\right).$$

(3)

This function is calculated for each choice of blocked link $b$. The most vulnerable link $b^*$ is:

$$b^* = \arg\min_b \left\{ A\left(\pi^*(G_b), G_b, ss\right) \right\}.$$  (4)

In terms of a mathematical game, this link $b^*$ would be the Stackelberg optimum for an evil entity to block if he was given the opportunity to block one link.

### TRAFFIC FLOW MODELLING

Analysis to road network robustness can be done with two types of simulations. Both microscopic simulations, in which vehicles are individually simulated, and macroscopic simulations are used. In microscopic simulation models the quality of the simulation depends on the quality of the underlying simulation sub-models. These models describe the driver tasks, such as a car-following models or lane change models. In theory, spillback can be captured correctly as all cars occupy a finite space.

Simulation studies that cover a larger area often use macroscopic models to reduce computation time. With these models, it is an issue to implement the congestion flow crossing a link border in a correct way. Often, vertical queuing models are used to describe queues. In these vertical queuing models, traffic jams do not occupy any space. What matters for the traffic flow, is that flow at the upstream link is not influenced by the queues on the (downstream) link. In our representation of a non-spillback model, the queue will grow upstream but does not cross the link border. The queue dynamics of this model are plotted in Figure 2a. In the upper figure, the space-time diagram of the queue is plotted. The queuing area is shaded. In the lower figure, the number of cars in the queue is plotted.

In reality, the queue will grow more upstream than the end of a link. This can be captured by a horizontal queuing model. In this model, the head of the queue is fixed at the bottleneck and the queue
can only dissolve from the tail. Often, a fixed space per car is assumed. The queuing dynamics of a horizontal queuing model are indicated in Figure 2b. In reality, the queue will, possibly dependent on traffic conditions, dissolve from the head, while the tail of the queue still moves upstream, as seen in Figure 2c. This study compares the situation with vertical queues (as in Figure 2a) with the situation of full spillback and tail solving from the head of the queue (Figure 2c).

Shockwave theory is needed to describe the real traffic state (Figure 2c) behavior sufficiently accurate. We use a continuum LWR-model that we solve with a Godunov scheme [16]. Second order effects (e.g. from synchronized flow to wide moving jams) have some minor influences [17] that we will neglect in the remainder of this paper.

**CASE STUDY DESCRIPTION**

The best way to compare the results of the vulnerability of links in a spillback and in a non-spillback simulator is to use one simulator that can run simulations both with and without spillback. In this way, there are no differences in systematic errors.

We perform a case study on a regional size network with both motorways and underlying roads. A morning peak period from 6.30 to 9.30 is simulated. 468 links with different link properties (capacity, speed limit) and link connections give insight to which properties are relevant for the network impact of a link being blocked. The network we used is the ring road around the city of Rotterdam (around 600,000 inhabitants). A map of the area is given in Figure 3.
RESULTS

Figure 4 shows the performance of the network in the different cases. The x-axis shows the performance of the network without spillback and on the y-axis the network performance with spillback is indicated. This is done for the case with and without route information. We find the network performance to be lower if spillback is included: all points lay below the line x=y (indicated green in the figure), as we would expect.

Figure 4 Correlation of the impacts of link blocking in scenarios between spillback simulations and non-spillback simulations.

One dot indicates one specific blocked link. We fitted a linear relationship for both the rerouting and for the fixed route case. The correlation shows how well a simulation without spillback can foretell the impact of the blocking of a link. For this purpose, we fitted the relationship

\[ A_{\text{spillback}} = \alpha + \beta A_{\text{no spillback}} \]  

and found the following parameters

Table 1

<table>
<thead>
<tr>
<th>Path update scenario</th>
<th>Fixed paths update scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>(-1.61E+06)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>2.08</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.38</td>
</tr>
</tbody>
</table>

The regression lines are plotted as dotted lines in Figure 4. The correlation coefficient \( R^2 \) indicates how much of the variance in performance reduction in a spillback case can be explained by the variation in arrivals in the corresponding scenario without spillback. We see that this value is low,
around 35%; that it is low, can also be seen by the big cloud of points in the figure. Therefore a static, non-spillback simulation cannot be used to identify the vulnerable parts for a road network.

Figure 5 shows the impact of blocking for the individual links and where these links are located. If a link is red, the impact of blocking that link is large. Figure 4 showed that the magnitude of variance of the performance reductions differ among the different scenarios. Therefore, the color scales in subplots in Figure 5 are not the same. Figure 5 shows the same area as Figure 3.

We see that in cases without spillback, the motorways are particularly important for the network performance. If one of these links is blocked, the performance reduces. Once spillback is included, the motorways are less critical (compared to the average of all links). When travelers are not informed about the routes, the urban roads are important (in case of realistic spillback modeling). Since it is at maximum two lanes wide, the urban link is completely blocked. Even in low intensity traffic, the queue builds up. As the tail propagates through the network, many links are blocked. When travelers are informed, they will be rerouted quite early already, since the queue starts building up immediately. The most vulnerable parts in the network in this scenario are not the urban links (as it was without information); the destination links are now vulnerable since people cannot there is no alternative for the exit links.

Four different scenarios can be mutually compared:

- no spillback, daily route choice;
- no spillback, route choice adapted to incident;
- spillback, daily route choice;
- spillback, route choice adapted to incident;
- spillback, route choice adapted to incident.

From the results we computed the relative advantage of updating the paths:

$$adv(b, ss) = \frac{A(\pi^*(G_b), G_b, ss) - A(\pi_0^*(G), G_b, ss)}{A(\pi_0^*(G), G_b, ss)}.$$  \hspace{1cm} \text{(6)}

Each blocked link $b$ leads to an advantage. The 468 numbers are ordered and plotted below in Figure 6.

![Figure 6 Predicted advantage of routing information](image)

In many cases (i.e., for a large share of the 468 possible locations of an incident), the advantage of route information is in the spillback scenario much larger than in the non-spillback scenario. That could also be derived from Figure 4, which shows a big performance decrease for the spillback scenario without rerouting. If there is rerouting, the performance reduction is much less. The new advices lead people around the bottleneck and hence the delays, but, more important, also the secondary delays (i.e., the delays induced by spillback) are much less.

We also investigated the (relative) performance of the network if a randomly chosen link is blocked. We compute the impact of the blocking of a link:

$$Ipct(\pi, b, ss) = \frac{A(\pi, G_b, ss)}{A(\pi, G, ss)}.$$  \hspace{1cm} \text{(7)}

For all 468 we get an impact value. This can be related to robustness in the following way. We take a threshold value for the network performance decrease which is acceptable for users. Then, we find the probability that the decrease is more, given that one randomly chosen link is blocked. The figure below shows how well the network performs compared to the case in which no link is blocked.

![Figure 7 Robustness of all scenarios](image)
The blocking is assessed less important if spillback is not simulated. For both the case with and without rerouting, the robustness is about the same. In none of the cases, the performance drops more than 10%. If spillback is taken into account, there are more links causing a large performance drop. If paths are updated, the performance drops by maximum 20%; with fixed routes, the performance might drop by more than 40%. So, robustness is overestimated if it is assessed by a non-spillback simulator and robustness can be increased by giving proper route information.

CONCLUSIONS AND FURTHER RESEARCH

We simulated a morning traffic flow on a real, regional sized, network for which sequentially one of the links was blocked. The traffic simulator had the possibility to make choices about both path update and spillback. Paths could be adapted to the situation or not and, independently, spillback could be switched on and off.

The links that are considered vulnerable differ per scenario. Motorways have the most impact if blocked if spillback is not taken into account. With spillback, it depends on the information which links are most vulnerable. Without information, the urban links in the city cause many problems if being blocked; a blocking leads to a total grid lock. If people are informed, the most vulnerable links are the links for which there is no route alternative, the destination links.

The vulnerable links in a network cannot be identified by a non-spillback simulator. Only 35% of the variations of impact of link blocking in realistic spillback simulations can be derived by performing a simplified, non-spillback simulation. Spillback is also important in assessing robustness of a network. Without spillback, impact of blocking one link is much less and therefore the network is considered more robust in a non-spillback simulator.

Finally, when a non-spillback simulator is used, the advantage of giving route information is highly underestimated. With spillback, in around 40% of the locations for a link blocking, route information can increase the performance considerably. Therefore, in reality, where spillback always occurs, robustness can be increased by informing road users properly.

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