Robust Control of Traffic Networks under Uncertain Conditions

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Abstract

Uncertainty of traffic network operations has been a subject of lively debate in the last decade. However, little effort has been put in developing control frameworks that are not only aimed at improving the average performance of the system, but also at improving the system robustness and reliability. In fact, it can be argued that most of the current control approaches are only aimed at improving the efficiency, which can even be counterproductive from a robustness point of view.

The main contributions of this article is the proposition of a new control framework based on the notion of controlled Markov processes, which explicitly takes into account the uncertainty in predicted traffic conditions and system performance. Furthermore, in contrast to traditional optimal control approaches, the objective function can include general statistic of the random system performance, such as the mean, standard deviation or 95-percentile.

The contribution aims to make clear how different performance function specifications yield different control strategies. This is shown for a relatively simple case study.

Keywords
Stochastic control, reliability, robust networks.

1 INTRODUCTION

Dutch transportation policy is not only aimed at achieving the largest throughput, but also at increasing transport reliability. In fact, current policy aims to achieve that 95% of all motorway trips in the peak hour will be on time. From the perspective of the direct users of the transportation users, this will mean a considerable improvement, because behavioral studies have already shown that besides mean travel time, travel time reliability plays an considerable role in the valuation of a trip (Bogers et al,2006).

Despite of this, for the deployment of most of the Dynamic Traffic Management (DTM) measures, we generally only take consider efficiency impacts (in terms of maximizing throughput, reducing emissions and noise, etc.). Little research has been done how DTM can be put to use to increase reliability. Amongst the few examples is the work of Liu (2006), showing how tolling can be used to improve reliability. Also, in Akamatsu and Nagae (2007) a stochastic optimal control framework is put forward, in which an optimal system optimum assignment is performed in case of stochastic system dynamics. The research shows how the problem can be formulated by formulating it as a dynamic programming problem. The paper focuses on minimizing the expected value of the objective function given the uncertain system dynamics. In Hoogendoorn et al (2007), a similar problem was tackled but for generic objective functions.

In this contribution we put forward a new control methodology showing how to *control for reliability* for generic control inputs and objectives. Instead of using a deterministic prediction model, which is done in traditional optimal control theory, a stochastic model is used instead. The uncertain system dynamics imply that the predicted performance is a random variable as well, rather than a single deterministic value. The control objective which is optimized can then be specified by the used, enabling not only the consideration of the average system performance, but any statistic of the random system performance or combinations thereof. The concepts are illustrated by a number of relatively simple examples.

2 FORMULATION OF THE TRAFFIC CONTROL PROBLEM

Dynamic Traffic Management offers many possibilities to influence traffic flow operations in networks. Examples are providing route information or guidance, ramp-metering, mainline metering, tidal flow, dynamics speed limits, intersection control, etc.

We assume that the *status* or *control settings* of these measures can be represented by some control input $\mathbf{u}$. This vector can include all kinds of control settings, such as the green-time, whether or not a specific lane is closed, the route advice people receive via the VMS, etc. Furthermore, the control will be dynamic, i.e. $\mathbf{u} = \mathbf{u}(t)$. The control $\mathbf{u}(t)$ influences the (current and future) state $\mathbf{x}(t)$ of the system (this is formulated mathematically in the next section). We generally assume that the next state (say, $\mathbf{x}(t+dt)$) is determined by the current state $\mathbf{x}(t)$, the control $\mathbf{u}(t)$, and any 'disturbances' that may be applied (including the boundary conditions, such as traffic flowing into the considered network). This implies that the state captures the entire history of the system.

In the remainder, we set out to determine the optimal control settings $\mathbf{u}^*$, i.e. the control that steers the state of the traffic network in some optimal way, according to some performance criteria chosen. In case the system dynamics are deterministic, optimal control theory can be effectively used to find the optimal control settings minimizing the choice performance criteria. We refer to Hegyi (2004) for (one of many) thorough overviews of application of optimal control applications in ITS. Rather that considering deterministic models to predict network flow operations, a stochastic model is put forward. In doing so, we can explicitly consider the impact of uncertainty in determining the optimal control laws, which yield for robust system operations.

Mathematical formulation of the control problem

In this section, we will formally describe the stochastic traffic control problem. First of all, we will consider the stochastics describing of the system dynamics. This is achieved by writing the system as a controlled Markov process in the so-called *continuous stochastic state-space form*:

$$\begin{aligned} d\mathbf{x} &= \mathbf{F}(t,\mathbf{x},\mathbf{u},\boldsymbol{\omega})dt \\ \mathbf{x}(t_k) &= \mathbf{x}_k \end{aligned} \tag{1}$$

In Eq. (1), $\mathbf{F}$ denotes the so-called right-hand-side, $\mathbf{x}$ denotes the (random) state, $\mathbf{u}$ denotes the control vector. Furthermore, $\boldsymbol{\omega} = \boldsymbol{\omega}(t)$ denotes some general stochastic process affecting the system dynamics. In transportation networks, it can for instance describe the fact that the capacity is a random variable (Botma,1999), or the fact that a link is susceptible to incidents. Note that we assume that at instant t_k, the state $\mathbf{x}_k$ of the transportation network is assumed known.

In many stochastic optimal control problems, the following less generic form is used:

$$\begin{aligned} d\mathbf{x} &= \mathbf{f}(t,\mathbf{x},\mathbf{u})dt + \sigma(t,\mathbf{x},\mathbf{u})d\boldsymbol{\omega} \\ \mathbf{x}(t_k) &= \mathbf{x}_k \end{aligned} \tag{2}$$

In Eq. (2), **x** = **x**(t) denotes the state vector of the system at time instant t, **u** = **u**(t) denote the control vector, and **ω** = **ω**(t) denotes the white noise term. The term **σ** denotes the error variance term. Note that the covariance matrix is defined as follows:

$$\Theta = \sigma\sigma' \tag{3}$$

We emphasize that in the remainder of this contribution, the general form Eq. (1) will be used unless explicitly stated differently (which will be the case for the analytical results).

QUAST stochastic queuing model

In the case study presented in this contribution, the QUAST model (see (Stembord,1991), and (Botma,1999)) will be used to model the traffic operations on a link stochastically. The QUAST model is a relatively simple stochastic generalization of the so-called *Store-and-Forward* model.

Let j denote the link number. The link has a bottleneck with capacity C_j. Let $r_j(t)$ denote the number of vehicles in the queue; $q_j(t)$ is the inflow at the entry of the queue, and finally T_j^0 denotes the free travel time. The queue dynamics are then given by:

$$\frac{d}{dt} r_j(t) = \begin{cases} p_j(t) + q_j(t - T_j^0) - C_j(t) & r_j(t) \geq 0 \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

In Eq. (4), $p_j(t)$ denotes the autonomous traffic demand *arriving at the link bottleneck* irrespectively of current flow conditions and control settings. In the route guidance problem which will be considered in the ensuing of the contribution, the inflow $q_j(t)$ may be controlled (e.g. via route guidance). For the outgoing flow $d_j(t)$ of link j, we have:

$$d_j(t) = \begin{cases} C_j(t) & r_j(t) \geq 0 \\ q_j(t - T_j^0) & \text{elsewhere} \end{cases} \tag{5}$$

Let I_i denote the set of incoming links, and let O_i denote the set of outgoing links of node i. Then, the total incoming flow $Q_i(t)$ into node i is equal to:

$$Q_i(t) = \sum_{j' \in I_i} d_{j'}(t) \tag{6}$$

For the considered route guidance problems, we control the flows $q_j(t)$ into the outgoing links j:

$$q_j(t) = u_j(t) \cdot Q_i(t) \ \text{ s.t. } \sum_{j' \in O_i} u_{j'} = 1 \tag{7}$$

In Eq. (7), $u_j(t)$ denotes the fraction of the flow $Q_i(t)$ at decision node i which is going to use the outgoing link j.

Note that in the QUAST model, the capacity $C_j(t)$ is a random variable. The variations in the capacity can stem from many sources (inter-driver differences, weather conditions, incidents, lighting conditions, etc.). Furthermore, we assume that the capacity is less once congestion has set in (capacity drop). This is modeled by checking if the number of vehicles $r_j(t)$ queuing in front of the bottleneck is larger than zero. More specifically, the following formulation is used:

$$C_j(t) = c_j(r_j) \cdot (1 - \alpha(t)) + \sigma_j(r_i)\zeta_j(t) \ \text{ where } \ \{c_j(r), \sigma_j(r)\} = \begin{cases} \{c_{free}, \sigma_{free}\} & r_j = 0 \\ \{c_{cong}, \sigma_{cong}\} & r_j > 0 \end{cases} \tag{8}$$

In Eq. (8), the term α denotes the (random) capacity reduction due to incidents; $\zeta_j(t)$ denotes some standard white noise process.

Also the autonomous demands $p_j(t)$ for link j are random process. In general, the serial correlation in the noise term of p_j will be much larger in case of the capacity. This will be modeled as follows:

$$p_j(t) = p_j^0(t) + \varepsilon_j(t) \ \text{ where } \ d\varepsilon_j(t) = \beta\varepsilon_j(t)dt + (1-\beta)d\varepsilon_j^0(t) \tag{9}$$

In Eq. (9), $p_j^0(t)$ denotes the expected autonomous demand for link j at time instant t; β denotes the serial correlation factor. Finally, ε_j^0 denotes the zero mean noise term. We refer to (Botma,1999) for details on the QUAST model and its characteristics.

Concept of random costs

Recall that at time instant $t = t_k$, the state $\mathbf{x}(t_k)$ of the system is assumed known. From that time instant onward, we compute the distribution of $\mathbf{x}(t)$ for $t > t_k$ using Eq. (2). Clearly, since $\{t,\mathbf{x}(t)\}$ for $t > t_k$ is a random process, so is the cost J defined by:

$$J(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) = \int_{t_k}^{t_k+H} L(s, \mathbf{x}(s), \mathbf{u}(s))ds + \phi(t_k + H, \mathbf{x}(t_k + H)) \tag{10}$$

where L denotes the *running cost,* and ϕ denotes the terminal cost; H denotes the prediction horizon.

The cost J defined by Eq. (10) is a *random variable describing the performance of the system*, for instance in terms of cumulative travel times or delays, throughput, average speeds, etc., given that from time t_k to time t_k+H we have applied a certain control $\mathbf{u}$.

Let $E(X)$ denote the expected value of the random variable X. For stochastic control problems, the *objective function J* is generally defined by the expected cost, conditional on the control $\mathbf{u}$ that is applied (Fleming,1993), (Akamatsu and Nagae,2007):

$$\overline{J}(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) = E\left[J(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) \right] \tag{11}$$

Besides the expected cost, we can for instance also define the *conditional cost variability*:

$$\Sigma(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) = \mathrm{var}\left[J(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) \right] \tag{12}$$

Clearly, any other statistic (median, mode, skewness, kurtosis, 90% percentile, etc.) *or combinations of statistics* can be used to express the future performance of the system, given the current state $\mathbf{x}(t_k)$ and the control $\mathbf{u}(s)$ for $s > t_k$. In the remainder, we will use the symbol ϑ to express the chosen statistic describing the system performance:

$$\vartheta(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) = \left\langle J(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) \right\rangle \tag{13}$$

where $\langle X \rangle$ denotes the chosen statistic expressing the future system performance (mean, standard deviation, etc.). In the remainder, we will illustrate the consequences of using different objective function specifications.

Mathematical formulation of the stochastic control problem

The resulting control problem is similar to the deterministic control problem: given the currently available state $\hat{\mathbf{x}}(t_k)$, the aim is to find the control $\mathbf{u}$ optimizing the chosen system performance, i.e.:

$$\mathbf{u}^*_{[t_k, t_k+H)} = \arg\min \vartheta(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) \tag{14}$$

subject to:

$$d\mathbf{x} = \mathbf{F}(t, \mathbf{x}, \mathbf{u}, \boldsymbol{\omega})dt \quad \text{given} \quad \mathbf{x}(t_k) = \hat{\mathbf{x}}(t_k) \tag{15}$$

It is important to note that the problem has been formulated as a *rolling horizon problem*, where it is assumed that the optimal control is recomputed each time a new measurement or state estimate becomes available. As such, the system can be made responsive to unpredicted changes, such as incidents, bad weather conditions, etc.

3 DYNAMIC PROGRAMMING SOLUTIONS

In this section, we discuss some analytical results from stochastic control theory (see Fleming,1993) which hold if the problem can be formulated in the additive form Eq. (11) and if the noise-term $\boldsymbol{\omega}$ is a standard Wiener process. It is stressed that in the remainder, these assumptions will be relaxed and that the results from dynamic programming shown here serve mainly to illustrate some of the properties of the solutions and the impact of introducing uncertainty on these properties.

If the conditions mentioned above are met, the problem can be formulated as a *dynamic programming problem with terminal boundary* conditions:

$$\begin{gathered}\frac{\partial W}{\partial t} + H(t, \mathbf{x}, \nabla W, \Delta W) = 0 \\ W(t_k + H, \mathbf{x}(t_k + H)) = \phi(t_k + H, \mathbf{x}(t_k + H))\end{gathered} \tag{16}$$

where $W = W(t,\mathbf{x})$ denotes the so-called *value function*, which is a function of both the time t and the state $\mathbf{x}$ and is defined by the *minimum value of the cost function* Eq. (13). The value function thus describes the minimum cost when starting from the state $\mathbf{x}(t)$ at instant t and applying the optimal control $\mathbf{u}_{[t,t+H)}^*$ from that point onward.

In Eq. (16), the Hamilton function H is defined by:

$$\begin{gathered}H(t, \mathbf{x}, \nabla W, \Delta W) = \min_{u \in U} h(t, \mathbf{x}, \mathbf{u}, \nabla W, \Delta W) \quad \text{with} \\ h(t, \mathbf{x}, \mathbf{u}, \nabla W, \Delta W) = L(t, x, u) + \sum_i f_i(t, x, u) \frac{\partial W}{\partial x_i} + \frac{1}{2} \sum_{i,j} \sigma_i \sigma_j \frac{\partial^2 W}{\partial x_i \partial x_j}\end{gathered} \tag{17}$$

Note that since W denotes the optimal cost, the partial derivative of W to the state elements reflects the *marginal optimal cost* due to applying small changes to the state.

Eq. (17) shows that the introduction of uncertainty is reflected by a second order term which effectively introduces smoothing of solutions to the dynamic programming equation. A direct consequence of this smoothing is that the value function W in the vicinity of networks states $\mathbf{x}$ having a high cost (e.g. due to the grid-lock effects, or because a capacity drop occurs when the traffic demand near a bottleneck becomes too high) will also have an average high cost. As a result, optimal controls will try to steer the state $\mathbf{x}$ away from such poorly performing situations. For instance, consider a bottleneck which has a free capacity of 4400 veh/h and a queue discharge rate of say 3800 veh/h. In the deterministic case, the best performance is achieved when the total demand is 4400 veh/h. In the stochastic case, however, it will be proficient to maintain the demand substantially lower than 4400 veh/h to ensure that oversaturation (and hence the capacity drop) does not occur.

Different techniques exist to numerically solve Eq. (16) (see for instance (Hoogendoorn and Bovy,2004)) thus allowing calculation of the optimal solution. However, application is limited to a specific type of small-scale control problems. First of all, numerically solving Eq. (16) is computationally demanding, in particular in case of high dimensional state variables $\mathbf{x}$. Second of all, the objective function must be of the form Eq. (11), that is, the *expected value of the cost function* is to be minimized.

4 CASE STUDIES SHOWING APPROACH APPLICATION

In this section, two application examples will be shown. Both examples pertain to routing control to small scale networks, and serve merely to present the results of the approach.

4.1 Example application using two-link network

Before discussing application results to a more complex example, we will first consider some analytical results. To this end, we need to assume that the optimization objective is equal to Eq. (11), while the state dynamics are of the form (2) where the noise term is a standard Wiener process.

Let us consider guidance control of a two-link network (see Figure 1) with one decision node (node A). Let $u_1(t)$ denote the fraction of traffic using link 1. In this case, let $Q(t)$ denote the total traffic demand at the entry node A. The inflow to link 1 and 2 are given by respectively:

$$q_1(t) = u_1(t)\cdot Q(t) \;\text{ and }\; q_2(t) = (1-u_1(t))\cdot Q(t) \tag{18}$$

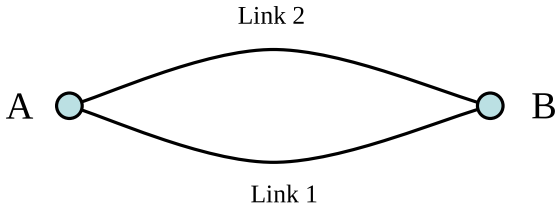

Figure 1 Example of a two-link network. Free travel times of link 1 and 2 are respectively equal to 16 and 18 minutes; the bottleneck capacities are equal to 4300 and 2300 veh/h. On average, the autonomous traffic demand of link 1 is much larger than of link 2, making the congestion on the latter link more sever.

Let us furthermore assume that the free travel times of both links can be neglected in the queue modeling dynamics (point-queue modeling), due to which the dynamics simplify to:

$$dr_j(t) = \left(p_j(t) + q_j(t) - C_j(t)\cdot dt + \sigma(r_j(t))d\omega_j \right)\cdot 1_{r_j(t)\geq 0} \tag{19}$$

Let us assume that the objective of the route guidance is to minimize collective travel times given the uncertain traffic operations, i.e.:

$$J(t,\mathbf{x}(t),\mathbf{u}_{[t,t+H)}) = \int_t^{t+H} \sum_j (q_j(s)+p_j(s))\cdot\left(TT_j^0 + \frac{r_j(s)}{C_j(s)} \right)\cdot ds \tag{20}$$

where $\mathbf{x} = (x_1,x_2) = (r_1,r_2)$. Let us first consider the Hamilton function H. We get:

$$\begin{aligned} h(t,\mathbf{x},u,\nabla W,\Delta W) &= (u\cdot Q(t)+p_1(t))\cdot\left(TT_1^0 + \frac{r_1(t)}{C_1(t)} \right) + ((1-u)\cdot Q(t)+p_2(t))\cdot\left(TT_2^0 + \frac{r_2(t)}{C_2(t)} \right) \\ &+ \frac{\partial W}{\partial x_1}(u\cdot Q(t)+p_1(t)-C_1(t))\cdot 1_{r_1(t)\geq 0} + \frac{\partial W}{\partial x_2}((1-u)\cdot Q(t)+p_2(t)-C_2(t))\cdot 1_{r_2(t)\geq 0} \\ &+ \frac{1}{2}\left(\sigma_1 \frac{\partial^2 W}{\partial x_1^2}\cdot 1_{r_1(t)\geq 0} + \sigma_2 \frac{\partial^2 W}{\partial x_2^2}\cdot 1_{r_2(t)\geq 0} \right) \end{aligned} \tag{21}$$

To find the optimal u (minimizing the Hamilton function), we need to determined u^* satisfying:

$$h(t,\mathbf{x},u^*,\nabla W,\Delta W) \leq h(t,\mathbf{x},u,\nabla W,\Delta W) \;\; \forall u \tag{22}$$

It can be easily shown that if Eq. (22) must be satisfied, that this implies that, for all u we have:

$$u^*\cdot\gamma(t,\mathbf{x},\nabla W) \leq u\cdot\gamma(t,\mathbf{x},\nabla W) \tag{23}$$

where the function

$$\gamma = \gamma(t,\mathbf{x},\nabla W) = \left(TT_1^0 + \frac{r_1(t)}{C_1(t)} + \frac{\partial W}{\partial x_1} 1_{r_1(t)\geq 0} \right) - \left(TT_2^0 + \frac{r_2(t)}{C_2(t)} + \frac{\partial W}{\partial x_2} 1_{r_2(t)\geq 0} \right) \tag{24}$$

is referred to as the *switching function* (Hoogendoorn and Bovy,1998). Note that the switching function describes *the difference in the marginal cost* of adding one vehicle to link 1 and link 2. This marginal cost is given by the travel cost incurred by the vehicle itself (i.e. $TT_j^0 + r_j(t)/C_j(t)$) and the cost induced by the vehicle onto the other vehicles arriving thereafter (i.e. $\partial W/\partial x_j$). Using the switching function, we can now see easily that the optimal control satisfies:

$$u^* = \begin{cases} 1 & \gamma < 0 \\ 0 & \gamma > 0 \\ \text{undetermined} & \gamma = 0 \end{cases} \tag{25}$$

In other words, when the marginal cost of link 1 is less than the marginal cost of link 2, *all* traffic will be redirected over link 1 and vice versa. Only in case that the marginal costs of link 1 and link 2 are the same, the optimal control is undetermined. In optimal control theory, this is referred to as the critical path. A special control strategy is so-called MRAP control (Most Rapid Approach Path) involving steering towards the critical path as quickly as possible and staying there.

In Fleming (1993), numerical solution approaches are described in detail. By applying these, we are able to compute the value function and the relation optimal control function. In the results shown in the ensuing, the traffic demands are shown in Figure 2.

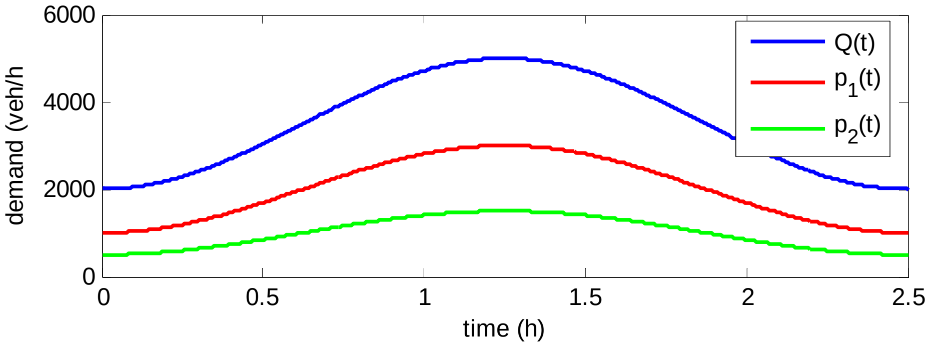

Figure 2 Specification of the traffic demand $Q(t)$ for entry node and autonomous demands $p_1(t)$ and $p_2(t)$ for links 1 and 2 respectively. The capacities for link 1 and 2 are respectively given by 4300 veh/h and 2300 veh/h.

Figure 3 shows the results of computing the value function and the optimal control in case of deterministic traffic conditions. From the value function plot, we can see that if we have a certain initial state at t = 0, what the cost of optimally controlling would be. The control function shows which control to apply. The figure shows that due to the shorter travel time and higher capacity, it is often beneficial to guide traffic along link 1 initially ($u(0)$ = 1). Only if (initially) the queues on link 1 are much larger than on link 2, then it is beneficial to guide traffic along link 2.

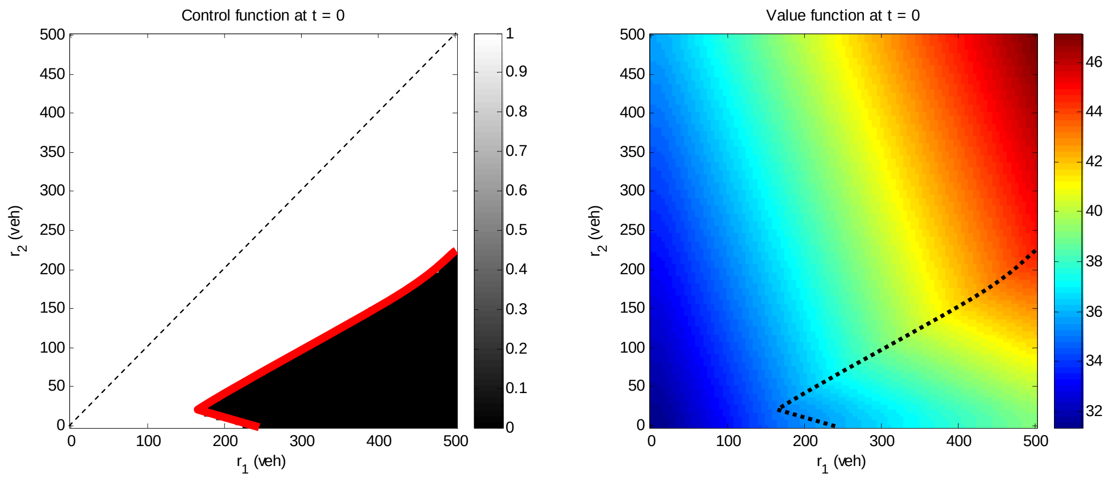

Figure 3 Control function and value function at t = 0 in case of deterministic traffic operations.

Figure 4 shows the same results but for uncertain traffic conditions. The uncertainty which is introduced is reflected by the noise term in Eq. (19). More specifically, the standard deviation of the noise σ for route 1 was chosen equal to 400 veh/h while for route 2 is was equal to 100 veh/h. The resulting figure clearly reveals the differences between the two situations (deterministic and uncertain case). In fact, the region for which guiding traffic along link 2 has reduced further due to the uncertainty of traffic operations.

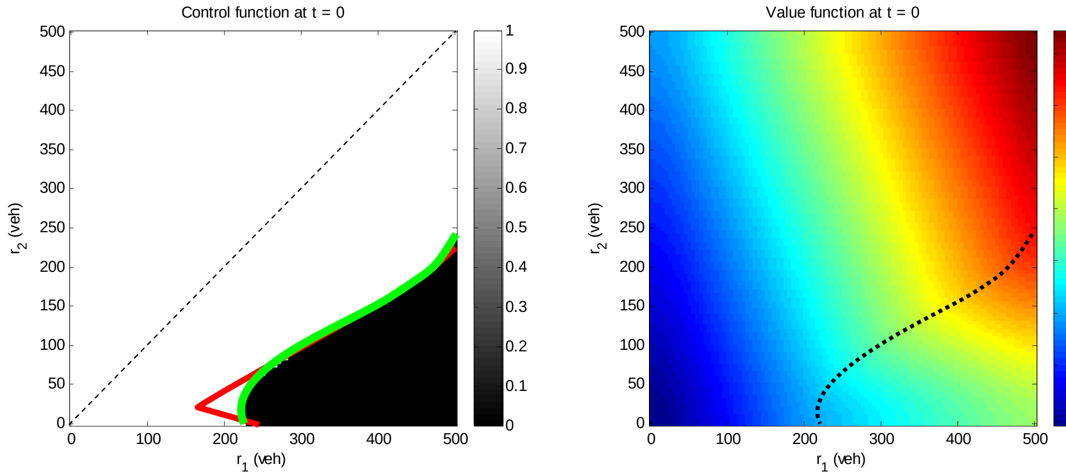

Figure 4 Control function and value function at t = 0 (initial time) for stochastic control problem where uncertainty of route 1 is larger than uncertainty of route 2.

4.2 Example application using more complex networks and flow dynamics

Let us now further the concepts developed in this paper by a simple but not trivial, hypothetical example. Figure 5 shows the network considered in the case study, and the origin-destination relation that is considered. Note that for the sake of illustration, we will only consider a single origin-destination relation and the control thereof. Autonomous demand is present on the two motorway links 3 and 4. Generalization to multiple origin-destination pairs is conceptually straightforward.

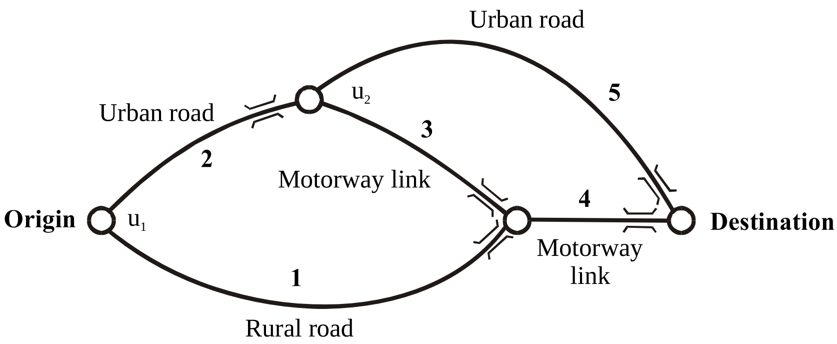

Figure 5 Hypothetical network with controls u_1 and u_2.

In the case, we consider route guidance control in which the splitfractions at the network decision nodes are optimized. However, the concept presented in this contribution can be applied to any other kind of control (ramp-metering, main-line metering, intersection control, etc.) using different control variables in the stochastic optimization process (e.g. metering rate, green times, etc.). In order to do so, the underlying probabilistic model needs to be modified to correctly include the impact of these ITS measures on traffic flow operations and the uncertainty therein.

The traffic in the network is guided over the links of the network according to the controls u_1 and u_2 respectively denoting the share of traffic going into link 1, and the fraction of traffic flowing into link 3. Note that different link types are present in the example. We will assume that all links have a different (stochastic) capacity and travel time, amongst other things depending on the type of link. With respect to the capacity, two kinds of uncertainty will be considered. The 'regular' uncertainty due to natural variations in traffic composition, driver behavior, etc., and the 'irregular' uncertainty due to incidents, accidents, changing weather conditions, etc. Also note that the capacity drop is considered in the modeling as described in the preceding sections.

A large part of the randomness in the capacity is for the case study determined by the probability of an incident occurring. For the sake of illustration, we assume that the probability of an incident occurring sometime during day *d* is a linear function of the volume to capacity ratio, i.e.:

$$P_{inc} = P_0 \cdot \frac{Q}{C} \tag{26}$$

When an incident occurs, it is assumed that the link is fully blocked for a random period of time (in this case, we assume 30 minutes). Note that the incident probability is determined by the link type (rural link: P_0 = 0.25, urban link: P_0 = 0.5 and motorway link P_0 = 0.05). Spillback is not modeled in this simple example, since it is not necessary to illustrate the concept.

Table 1 gives an overview of the link characteristics. From the table we can see that under free flowing conditions, route 2 (links 2-3-4) is the shortest alternative. Note that the autonomous traffic demands on the motorway links will cause link 3 and 4 to initially have a large probability of an incident occurring.

Table 1 Overview of link and route characteristics.

Link index	Free travel time	Capacity	P_0	Autonomous demand
1	25	1500	0.25	0
2	15	1000	0.50	0
3	5	4000	0.10	3500
4	12	6000	0.10	1000
5	20	1000	0.50	0
Route	**Links**	**Free travel time**	**Control (u_1,u_2)**	
1	1-4	37	(1,-)	
2	2-3-4	32	(0,1)	
3	2-5	35	(0,0)	

Random traffic performance

For a fixed traffic demand and for a fixed control law **u**, the dynamic stochastic model is run *N* times, leading to *N* different performances *J*, using a fixed random seed for realization *i* (meaning that realization *i* will always yield the same incident characteristics). Based on these *N* realized performances, the different relevant statistics are computed (e.g. mean performance, performance standard deviation, 90% percentile, etc.).

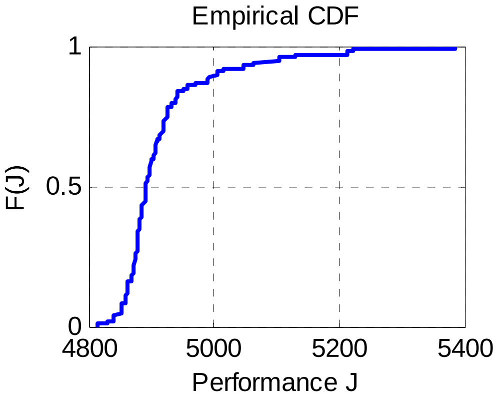

Figure 6 Example showing random performances (100 repetitions). Figure results pertain to optimal control *minimizing* the standard deviation of the performance *J*.

Case study results

Table 2 shows an overview of the performance of the network when applying optimal control with four different performance objectives. In all cases, the *collective travel times* have been used as the basic performance indicator. The table thus shows the mean and the median collective travel times, the 90% percentile of the collective travel times, and the standard deviation.

The table clearly shows that when the minimum value of for instance the mean is indeed achieved when minimizing the mean performance. From the table we can clearly observe the trade-off between average efficiency (e.g. minimizing the mean collective travel times) and the reliability (minimizing the 90-percentile collective travel times or the standard deviation). In particular we can observe that when we minimize the standard deviation, the other performance measures increase substantially. In other words: while the performance becomes more reliable, it does – on average – decrease.

Table 2 Network performance for different control objectives (for performance functions: mean, median, 90-percentile and standard deviation); values are given in 1000 veh-h.

Performance function	Mean	Median	90-perc.	Std.dev
Mean	4.143	4.076	4.432	0.178
Median	4.172	4.055	4.480	0.357
90-percentile	4.185	4.113	4.417	0.169
Std.dev.	4.865	4.841	4.996	0.086

Figure 8 provides some insight into the reasons for the trade-off illustrated by this example. It shows the value of the difference performance functions *assuming a static control function* $\mathbf{u}(t) = \mathbf{u}$. We can see that the mean and the median contours are quite similar, although small differences can be identified. The optimal controls for both situations are approximately the same.

If we compare the results when optimizing the median to the results when optimizing the 90-percentile objective function, the differences are more distinct. In particular we see that when optimizing the median, the optimal u_2 is larger than in case the 90-percentile is optimized. This means that in the latter case, a relatively large share of traffic is diverted over the urban link 5. Although this implies a detour, the probabilities of an incident occurring on this link are small as long as the traffic demand is sufficiently small. Thus, the network operations become more reliable.

This effect becomes even more apparent if we optimize the standard deviation. In this case, much more traffic will be diverted using link 5. Also, we see that more traffic is will be using link 1 (and subsequently link 4): although this implies a detour, the reliability increases substantially (until the demand of link 1 is such that the probability an incident occurs is comparable to the other links). Also note that the standard deviation has several local minima (on the contrary to the mean and the median).

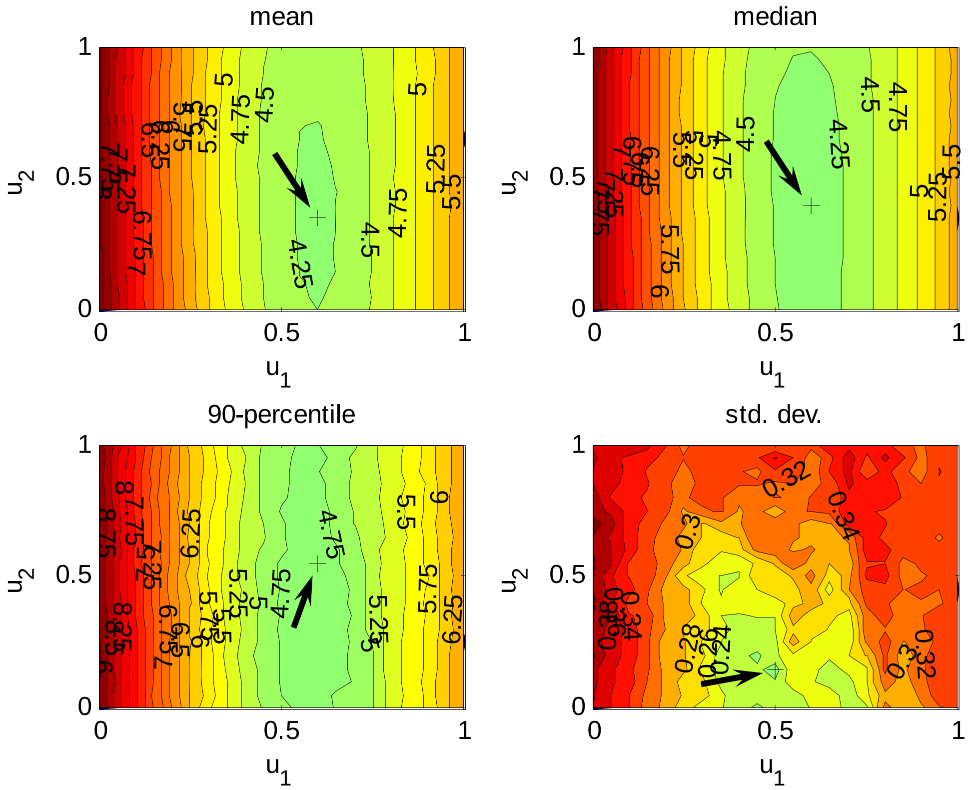

Figure 7 Overview of control functions u_1 and u_2 resulting when optimizing different performance indicators (mean, median, 90-percentile and standard deviation). The crosses indicate the optimal values.

5 CONCLUSIONS AND FUTURE WORK

We have proposed a new optimal control paradigm for traffic networks explicitly including the uncertainty of the resulting traffic operations. To this end, the problem is formulated as a controlled Markov process, in which the stochastic predictions are conditional on a candidate control law, describing the settings of the DTM measures in the network. The resulting objective function is thus also a random variable, the distribution of which is also dependent on the candidate control law.

In the control methodology, from all candidate control laws, we choose the law that optimizes some user-defined statistic (mean, median, 95% percentile) of the stochastic control function. The approach was illustrated by means of two simple application examples. Both examples show clearly the impact of the control objective function on the resulting optimal control strategies.

Future research will be directed towards finding numerical solutions to the stochastic optimal control problem more efficiently. This is needed if the approach is to be applied in a real-time control setting. Stochastic optimal control theory may be of use to find efficient numerical solution schemes.

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