A model of car-following behavior at sags

B. Goñi Ros, V. L. Knoop, W. J. Schakel, B. van Arem, and S. P. Hoogendoorn

Abstract Sags are bottlenecks in freeway networks. The main reason is that the increase in slope has a negative effect on vehicle acceleration, which results in local changes in car-following behavior that reduce traffic flow capacity. Existing car-following models are not able to reproduce the acceleration behavior of drivers at sags and the resulting traffic flow dynamics in a sufficiently realistic way. This paper presents a new car-following model that aims to fill that gap. The model assumes that drivers have a limited ability to compensate for the negative effect that an increase in gradient has on vehicle acceleration. Compensation is assumed to be linear over time; the maximum compensation rate is defined as a parameter. The paper presents the results of a case study using the proposed car-following model. The study site is a particular sag in Japan. Similar traffic flow patterns are observed in simulation and in empirical data from that site. In particular, the model generates a bottleneck caused by the increase in freeway slope, reproducing its location very accurately.

1 Introduction

Sags are freeway sections along which the slope changes significantly from downwards to upwards. Sags are bottlenecks in freeway networks [1, 2]. The main cause is that the increase in freeway slope has a negative effect on vehicle acceleration, which results in local changes in car-following behavior that reduce traffic flow capacity [3, 4, 5]. In this contribution, we identify two characteristics of traffic flow at sags that existing car-following models are not able to reproduce in a sufficiently realistic way. The first characteristic is that drivers change their car-following behavior only on the lower part of the uphill section, regaining their normal driving behav-

---

Delft University of Technology, Faculty of Civil Engineering and Geosciences, Department of Transport and Planning, Stevinweg 1, 2628 CN, Delft, The Netherlands, e-mail: {b.goniros, v.l.knoop, w.j.schakel, b.vanarem, s.p.hoogendoorn}@tudelft.nl

ior farther up the hill [4, 5]. The second characteristic is that, at sags, the capacity bottleneck is located 500-1000 m downstream of the bottom of the sag [2, 4]. In this paper, we present a new car-following model that aims to reproduce traffic flow dynamics at sags, including the two phenomena mentioned above. We present the results of a case study using the new model. The study site is the Yamato sag (Tomei Expressway, Japan). Analyses of empirical traffic data from that site are available [2, 6]. The results of the case study show that the model is able to reproduce the traffic flow patterns observed in empirical data. Particularly, the model generates a bottleneck caused by the increase in freeway slope, and it predicts the location of that bottleneck very accurately. We conclude that the proposed model is able to reproduce traffic flow dynamics at sags more realistically than existing models.

2 Background

Empirical studies show that drivers change their car-following behavior when they reach the uphill section of sags. Drivers tend to reduce speed [4, 7] and keep longer headways than expected given their speed [3, 6]. These local changes in car-following behavior occur because drivers are unable to accelerate sufficiently to immediately compensate for the increase in resistance force resulting from the increase in freeway slope [5]. Capacity decreases as a result of the above-mentioned changes in car-following behavior, which causes congestion in conditions of high traffic demand [1, 2]. Generally, in congested conditions, the head of the queue (i.e., the bottleneck) is located 500-1000 m downstream of the bottom of the sag [2, 4].

Several car-following models have been developed in the last decades with the objective of reproducing traffic flow dynamics at sags. Koshi *et al.* [1] and Komada *et al.* [8] propose two different models that assume that drivers do not explicitly compensate for the limiting effect that a positive freeway gradient has on vehicle acceleration. Therefore, those models assume that a constant slope has a constant influence on vehicle acceleration. This assumption is not consistent with empirical observations, which show that drivers generally regain their normal car-following behavior as they climb the uphill section [4, 5]. Yokota *et al.* [9] present a car-following model that assumes that drivers explicitly compensate for the limiting effect that an increase in gradient has on vehicle acceleration. The model assumes that drivers are able to fully compensate for changes in slope with a certain time delay. Hence a constant slope has a decreasing influence on vehicle acceleration. This is more in line with empirical observations. However, an important disadvantage of that model is that it does not accurately reproduce the location of the bottleneck at sags. The model generates a bottleneck around the bottom of the sag [9].

In order to model traffic operations at sags, it is necessary to develop a car-following model that reproduces the following two phenomena: i) drivers regain their normal car-following behavior as they climb the uphill section of sags; ii) at sags, the bottleneck is located 500-1000 m downstream of the bottom of the sag. In the next section, we present a car-following model that aims to fill that gap.

3 Car-following model

The model describes vehicle acceleration ($\dot{v}$) based on a two-term additive function:

$$\dot{v}(t) = f_{\mathrm{r}}(t) + f_{\mathrm{g}}(t) \; . \tag{1}$$

The first term (f_{r}) describes regular car-following behavior. Its formulation is based on the IDM model [10], and it accounts for the influence of speed (v), relative speed to the leading vehicle (Δv) and net distance headway (s) on acceleration:

$$f_{\mathrm{r}}(t) = a \cdot \left[1 - \left(\frac{v(t)}{v_{\mathrm{des}}(t)} \right)^4 - \left(\frac{s_{\mathrm{des}}(t)}{s(t)} \right)^2 \right] \; , \tag{2}$$

where the dynamic desired net distance headway (s_{des}) is:

$$s_{\mathrm{des}}(t) = s_0 + v(t) \cdot T(t) + \frac{v(t) \cdot \Delta v(t)}{2 \cdot \sqrt{ab}} \; . \tag{3}$$

The parameters in Eqs. 2-3 are: desired speed (v_{des}), maximum acceleration (a), maximum comfortable deceleration (b), net distance headway at standstill (s_0), and safe time headway (T). Note that we specified a different value for T depending on the traffic state. If $v(t) \geq v_{\mathrm{crit}}$ (uncongested traffic conditions), $T = T_0$. If $v(t) < v_{\mathrm{crit}}$ (congested conditions), the value of T is higher ($T = \gamma \cdot T_0$, where $\gamma > 1$).

The second term in Eq. 1 (f_{g}) accounts for the influence of freeway gradient on vehicle acceleration. At a given time t, that influence is the gravity acceleration (g = 9.81 m/s^2) multiplied by the difference between the gradient at the location where the vehicle is at that time ($G(t)$) and the gradient compensated by the driver until that time ($G_{\mathrm{c}}(t)$):

$$f_{\mathrm{g}}(t) = -g \cdot [G(t) - G_{\mathrm{c}}(t)] \; . \tag{4}$$

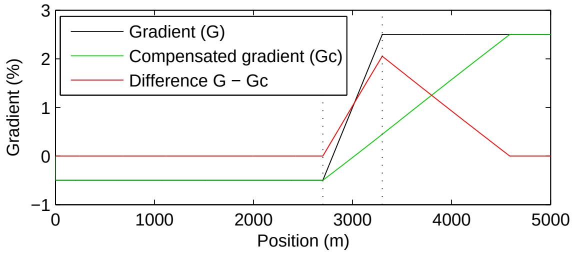
The *compensated gradient* (G_{c}) is a variable that accounts for the fact that drivers have a limited ability to compensate for the negative effect that an increase in freeway slope has on acceleration [5]. The model assumes that drivers compensate for any increase in slope linearly over time with a maximum gradient compensation rate defined by parameter c (and they fully compensate for any decrease in slope):

$$G_c(t) = \begin{cases} G(t) & \text{if } G(t) \leq G(t_{\mathrm{c}}) + c \cdot (t - t_{\mathrm{c}}) \\ G(t_{\mathrm{c}}) + c \cdot (t - t_{\mathrm{c}}) & \text{if } G(t) > G(t_{\mathrm{c}}) + c \cdot (t - t_{\mathrm{c}}) \; , \end{cases} \tag{5}$$

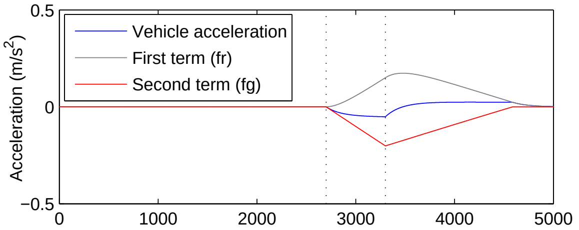
where:

$$t_{\mathrm{c}} = \max(t \mid G_{\mathrm{c}}(t) = G(t)) \; . \tag{6}$$

The properties of the model are as follows. If the gradient profile of a sag is such that the rate at which the freeway slope increases is lower than the driver's maximum gradient compensation rate (c), then $G_{\mathrm{c}}(t) = G(t)$ at any time t. Therefore, in such a sag, $f_{\mathrm{g}}(t) = 0$ at any time t, which implies that vehicle acceleration is not affected at all by the increase in slope. However, at sags where the gradient increase rate is higher than the driver's maximum gradient compensation rate (see example in Fig. 1(a)), $G_c(t) < G(t)$ for a certain period of time $t = [t_c,\ t_f]$. Time t_c is the time at which the driver could no longer fully compensate for the increase in gradient, and time t_f is the time at which the driver fully compensates for the whole increase in gradient. In Fig. 1(a), t_c corresponds to the time at which the vehicle is at location $x =$2.7 km, and t_f corresponds to the time at which the vehicle is at location $x \approx 4.8$ km. During period $t = [t_c,\ t_f]$, the compensated gradient (G_c) increases linearly over time (see Fig. 1(a)), but f_g is negative (see Fig. 1(b)). Note that the acceleration limitation caused by a negative f_g may reduce the vehicle speed and/or increase the distance to the leading vehicle, which are regular car-following behavior incentives to accelerate. Therefore, in our model, a negative f_g generally causes an increase in f_r. Acceleration is the combination of the influence of regular car-following behavior incentives and the influence of gradient (Eq. 1). As shown in Fig. 1(b), $f_r(t) + f_g(t) < 0$ during the time the vehicle is within the freeway section with increasing degree of gradient; therefore, the vehicle decelerates on that section. Once the vehicle gets on the freeway section with constant positive slope, $f_r(t) + f_g(t)$ becomes positive; hence the vehicle re-accelerates and normal car-following behavior is eventually restored. Note that the acceleration limitation is maximum at the location where the gradient increase rate becomes lower than the driver's maximum gradient compensation rate (i.e., $x = 3.3$ km in Fig. 1(b)), because the difference between G and G_c is maximum at that location (see Fig. 1(a)).



(a) Gradient and compensated gradient over distance



(b) Vehicle acceleration over distance

Fig. 1 Influence of an increase in gradient on the acceleration of a single vehicle driving on a sag

4 Case study

4.1 Site

The study site is the Yamato sag (Tomei Expressway, Japan), which has 3 lanes. Analyses of empirical traffic data from that site are available [2, 6]. According to those data, the bottleneck at the Yamato sag is located on the uphill section, 500 m downstream of the bottom of the sag. The free flow capacity and the queue discharge capacity of the bottleneck are around 5400 veh/h and 4800 veh/h, respectively. Therefore, the capacity drop is approximately 11%. The process of congestion formation consists of two phases: first, traffic flow becomes congested on the median lane (which is the busiest lane in high demand conditions); then, congestion spreads to the other lanes as a result of lane changes.

4.2 Model setup

We simulate traffic on a 5-km freeway stretch that has a layout similar to that of the Yamato sag. The freeway stretch has 3 lanes and the following gradient profile (Fig. 1(a)): a) 2.7-km section with a constant slope of -0.5%; b) 0.6-km section where slope increases linearly from -0.5% to $+2.5\%$; and c) 1.7-km section with a constant slope of $+2.5\%$. We defined two vehicle classes (i.e., cars and trucks), which have different vehicle length (4 m and 15 m, respectively) and different speed limit (100 km/h and 85 km/h, respectively). Furthermore, we defined three classes of car drivers and one class of truck driver. The drivers' car-following behavior is determined by the model presented in Section 3. Lane-changing behavior is determined by the LMRS model [11]. The values of the parameters of the car-following and lane-changing models for each vehicle-driver class are shown in Table 1. Those values were selected based on [11]; however, it is important to remark that the car-following and lane-changing models have not been specifically calibrated for the study site. Note that some of the parameters are stochastic. More specifically, the values of parameters v_{des}, a, b, T_0, c, $T_{\min}$ for car drivers (only parameter v_{des} for truck drivers) are Gaussian distributed. The values of the stochastic parameters shown in Table 1 correspond to their mean values. The simulation period is 100 min. Total demand increases linearly from 3000 to 5200 veh/h between $t = 0$ and $t = 75$ min. From $t = 75$ min on, total demand is constant (5200 veh/h). The distribution of the total demand across the three lanes at location $x = 0$ is determined by Wu's lane flow distribution model [12]. The composition of the traffic demand is the following: a) 10% trucks and 90% cars with drivers of class I on the shoulder lane; b) 5% trucks and 95% cars with drivers of class II on the center lane; and c) 100% cars with drivers of class III on the median lane. After entering the network, drivers are free to change lanes according to the lane-changing model.

Table 1 Parameters of the car-following model and the lane-changing model[a]

Vehicle class		Car		Truck
Driver class	Class I	Class II	Class III	
a (m/s^2)	1.15	1.21	1.29	0.50
b (m/s^2)	1.66	1.75	1.85	1.50
T_0 (s)	1.58	1.24	1.12	1.50
s_0 (m)	3	3	3	3
v_{des} (km/h)	92	97	103	85
v_{crit} (km/h)	60	60	60	60
c (s^{-1})	0.00039	0.00041	0.00043	0.00042
γ (-)	1.15	1.15	1.15	1.15
T_{min} (s)	0.61	0.58	0.54	0.56
τ (s)	25	25	25	25
x_0 (m)	200	200	200	200
v_{gain} (km/h)	70	50	50	70
d^{ij}_{free} (-)	0.365	0.365	0.365	0.365
d^{ij}_{sync} (-)	0.577	0.577	0.577	0.577
d^{ij}_{coop} (-)	0.788	0.788	0.788	0.788

[a] A description of the lane-changing model parameters can be found in [11].

4.3 Results

The model generates a bottleneck around the location where gradient becomes constant on the uphill section, i.e., 500 m downstream of the bottom of the sag (see Figs. 1(a) and 2). Traffic breaks down at that location when the total flow approaches 5200 veh/h (see Fig. 3(b)), which can be considered as the free flow capacity of the bottleneck. The process of congestion formation is as follows. First, traffic breaks down on the median lane (see Figs. 2 and 3(a)). Then, some drivers move from the median lane to the other lanes in order to avoid queuing. This increases the flow on the center and shoulder lanes, as observed in Fig. 3(a). The capacity of those lanes is exceeded and traffic breaks down there as well (see Fig. 3(a)). Note that the occurrence of congestion on a lane causes a reduction in lane flow (capacity drop). When congestion has spread to all lanes, the total outflow from the bottleneck is around 5000 veh/h, i.e., 4% lower than the total demand (see Fig. 3(b)). As a result, a queue forms upstream of the bottleneck. The head of the queue stays at $x \approx 3.3$ km, as observed in the speed contour plot of the median lane (Fig. 2). The speed contour plots of the other lanes (not shown) are very similar to that of the median lane.

The traffic flow patterns observed in simulation are similar to those observed in empirical data (see Section 4.1). The model generates a bottleneck and reproduces its location very accurately, although the simulated free flow capacity (5200 veh/h) is lower than in empirical data (5400 veh/h). Also, the model reproduces the capacity drop in congestion, although the magnitude of the drop (around 4%) is lower than in empirical observations (11%). Finally, the car-following model (combined with a lane-changing model) is able to reproduce the process of congestion formation.

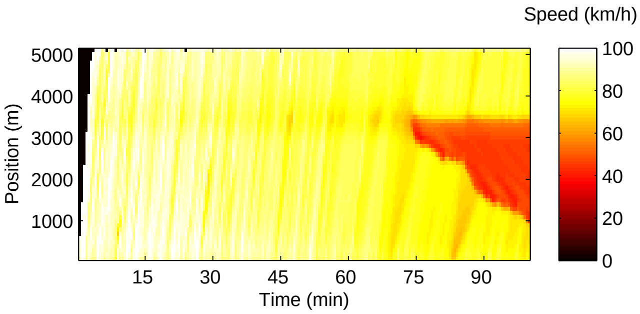
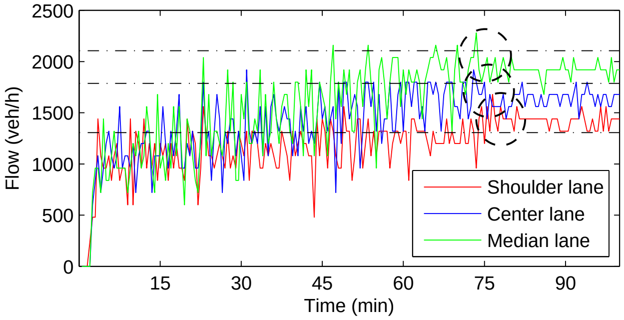
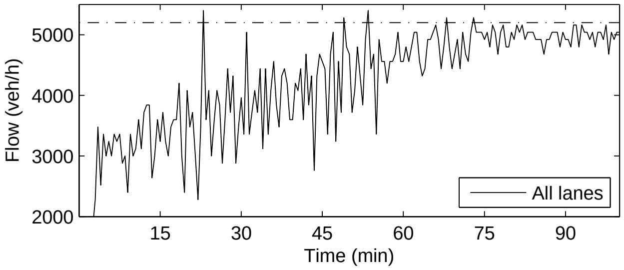


Fig. 2 Speed contour plot of the median lane



(a) Flow per lane



(b) Total flow on all lanes

Fig. 3 Flows at $x = 3.2$ km (dashed lines indicate the average inflow at $x = 0$ after $t = 75$ min)

5 Conclusions

This contribution presented a new model describing car-following behavior at sags. The model assumes that drivers have a limited ability to compensate for the negative effect that an increase in freeway slope has on vehicle acceleration. Compensation is assumed to be linear over time. The maximum compensation rate is defined as a parameter. The difference between actual gradient and compensated gradient is the variable that limits vehicle acceleration. A case study was carried out in order to test the model. The study site is a particular sag of a Japanese freeway. Similar traffic flow patterns are observed in the simulation output data and empirical data from the study site. More specifically, the model generates a capacity bottleneck caused by the increase in gradient, reproducing its location very accurately. Although the proposed model still needs to be calibrated, we conclude that it is able to reproduce traffic flow dynamics at sags more realistically than existing models. The assumption of a bounded gradient compensation rate appears to be more realistic than the behavioral assumptions of other models, such as a fixed compensation time delay [9]. This finding suggests that the magnitude of the gradient change over distance has a strong influence on the location and capacity of the bottleneck at sags.

Acknowledgements This research was sponsored by Toyota Motor Europe.

References

1. M. Koshi, M. Kuwahara, H. Akahane: Capacity of Sags and Tunnels on Japanese Motorways. *ITE Journal*, **62**(5), 17–22, 1992.
2. A.D. Patire, M.J. Cassidy: Lane changing patterns of bane and benefit: Observations of an uphill expressway. *Trans. Res. B*, **45**(4), 656–666, 2011.
3. M. Koshi: An Interpretation of a Traffic Engineer on Vehicular Traffic Flow. In: M. Fukui, Y. Sugiyama, M. Schreckenberg, D.E. Wolf (eds.): *Traffic and Granular Flow'01*, pp. 199–210, Springer, Berlin, 2003.
4. W. Brilon, A. Bressler: Traffic Flow on Freeway Upgrades. *Trans. Res. Rec.*, **1883**, 112–121, 2004.
5. R. Yoshizawa, Y. Shiomi, N. Uno, K. Iida, M. Yamaguchi: Analysis of Car-following Behavior on Sag and Curve Sections at Intercity Expressways with Driving Simulator. *Int. J. Intel. Trans. Sys. Res.*, **10**(2), 56–65, 2012.
6. B. Goñi Ros, V.L. Knoop, B. van Arem, S.P. Hoogendoorn: Car-following Behavior at Sags and its Impacts on Traffic Flow. Proc. 92nd Annual Meeting Trans. Res. Board, Washington, D.C., 2013.
7. T. Furuichi, S. Yamamoto, M. Kotani, M. Iwasaki: Characteristics of spatial speed change at motorway sag sections and capacity bottlenecks. Proc. 82nd Annual Meeting Trans. Res. Board, Washington, D.C., 2003.
8. K. Komada, S. Masukura, T. Nagatani: Effect of gravitational force upon traffic flow with gradients. *Physica A*, **388**, 2880–2894, 2009.
9. T. Yokota, M. Kuwahara, H. Ozaki: A study of AHS effects on traffic flow at bottlenecks. Proc. 5th ITS World Congress, Seoul, South Korea, 1998.
10. M. Treiber, A. Hennecke, D. Helbing: Congested traffic states in empirical observations and microscopic simulations. *Phys. Rev. E*, **62** (2), 1805–1824, 2000.
11. W. Schakel, V.L. Knoop, B. van Arem: Integrated Lane Change Model with Relaxation and Synchronization. *Trans. Res. Rec.*, **2316**, 47–57, 2012.
12. N. Wu: Equilibrium of Lane Flow Distribution on Motorways. *Trans. Res. Rec.*, **1965**, 48–59, 2006.