Traffic Flow Theory

Mijn presentatie spreekt over de verkeersstroom theorie!

Ludovic Leclercq, University of Lyon, IFSTTAR / ENTPE
July, 16th 2013
Outline

• Experimental evidences
  – Traffic behavior on freeways
  – The fundamental diagram

• Traffic modeling
  – The three representation of traffic flow
  – The three kinds of traffic models
  – Equilibrium (first order) model

• Overview of first order model solutions

• The variational theory
  – General basis
  – Connections between the three traffic representations

• Some extensions to the theory
Experimental evidences
Traffic flow on a motorways (M6 in England)

Data were kindly provided by the Highway Agency
Traffic representation

NGSIM Data – I80/lane 4 – USA

Microscopic vision
- Vehicle dynamics
  - position $x$
  - speed $v$
- Interactions
  - spacing $s$
  - Headway $h$

Macroscopic vision
- density $k$
- flow $q$
- Shockwaves
Flow / occupancy plot on a motorway (M6)

The fundamental diagram (FD)
Different definitions of the FD

FD for monitoring

FD for simulation

Aggregation / impacts of local behavior (lane-changing, traffic composition, ...)

L. Lédercq (2013)
Impact of the lane aggregation on the FD

Simulations are figures were kindly provided by Prof. Jorge Laval
Traffic Modelling
From discrete to continuous representations

Moskowitz's surface

Eulerian coordinates
\( N(t,x) \)

Lagrangian coordinates
\( X(t,n) \)

T coordinates
\( T(t,n) \)
From micro to macro: Edie’s definitions

\[ q_i = \frac{\sum_k d'_k}{\Delta x \Delta t} \quad \text{and} \quad k_i = \frac{\sum_k \tau'_k}{\Delta x \Delta t} \]
The three representation of traffic flow

<table>
<thead>
<tr>
<th>Partials</th>
<th>Symbol</th>
<th>Name</th>
<th>Eulerian</th>
<th>T coordinates</th>
<th>Lagrangian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N(t, x)$</td>
<td># of vehicles that have crossed location $x$ by time $t$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X(t, n)$</td>
<td>position of vehicle $n$ at time $t$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T(n, x)$</td>
<td>time vehicle $n$ crosses location $x$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Laval and Leclercq, 2013, part B)
Classical classification of traffic models

- Macroscopic models
  - Continuous representation
  - Mainly deterministic
  - Global behavior (may be distinguished per class)
  - Equilibrium (1st order) / + transition states (2nd order)

- Microscopic models
  - Discrete representation
  - Mainly stochastic
  - Local interactions (car-following)

- Mesoscopic models
  - Discrete or semi-discrete representation
  - Intermediate level for traffic representation (vehicle clusters or link servers)

For further details see the model tree from (van Wageningen-Kessels, PhD, 2013)
Equilibrium macroscopic model (1)

• The PDE expression
  – in Eulerian coordinates
    \[ k_t + Q(k)_x = 0 \]
  – in Lagrangian coordinates
    \[ s_t + V(s)_x = 0 \]
  – in T coordinates
    \[ r_t + H(r)_x = 0 \]
Equilibrium macroscopic model (2)

- The Hamilton-Jacobi (HJ) expression
  - In Eulerian coordinates
    \[ q = Q(k) \]
  - In Lagrangian coordinates
    \[ v = V(s) \]
  - In T coordinates
    \[ h = H(r) \]
Overview of first order model solutions
Solutions for an unsaturated traffic signal (1)

General solution methods: Hyperbolic equations (EDP), characteristics, waves, ...
Solutions for an unsaturated traffic signal (2)
The Variational Theory
General considerations on the variations of $N$

Legendre’s transformation:
\[ r(y'(t), k) \leq R(y'(t)) = \sup_k \left( r(y'(t), k) \right) \]

This makes costs independent from traffic states but no longer from the paths

Equality is observed on the optimal wave paths

\[ \Delta N_{AB} = \int_{t_A}^{t_B} d_t N = \int_{t_A}^{t_B} \partial_t N + y'(t) \partial_n N \]
\[ \Delta N_{AB} = \int_{t_A}^{t_B} Q(k(t)) - y'(t)k(t) \frac{r(y'(t), k)}{r(y'(t), k)} \]
Variational theory (VT) in Eulerian – General basis

HJ Equation: \[ q = Q(k) \iff \partial_t k = Q(-\partial_x k) \]

General expression for the solutions:

\[ N_P = \min_{\Gamma \in D_p} \left( N_{O(\Gamma)} + \Delta(\Gamma) \right) \]
\[ \Delta(\Gamma) = \int_{t_o(\Gamma)}^{t_p} r(y'(t), k) dt \]

Key VT result using the Legendre’s transformation

\[ N_P = \min_{\Gamma \in D_p} \left( N_{O(\Gamma)} + \Delta'(\Gamma) \right) \]
\[ \Delta'(\Gamma) = \int_{t_o(\Gamma)}^{t_p} R(y'(t)) dt \]

VT is really useful with PWL FD (and especially triangular one)
VT in Eulerian – The Highway Problem

\[ N(x, t) = \min \left( N\left(x_u, t - \frac{(x - x_u)}{u}\right), N\left(x_d, t - \frac{(x_d - x)}{w}\right) + \kappa(x_d - x) \right) \]

Newell’s model (1993) !!!

Triangular FD

L. Leclercq (2013)
Classical formulation of the Newell’s N-curve model

Well-known as the three detectors problem
VT in Lagangian - the IVP problem

\[ X_B = \min \left( X_{O(\Gamma_0)} + \Delta(\Gamma_0), X_{O(\Gamma_{wk})} + \Delta(\Gamma_{wk}) \right) \]

\[ X_B = X(t,i) = \min \left\{ \frac{X(t_0,i) + u(t-t_0)}{\text{free-flow}}, \frac{X(t-1/(wk),i-1) - w(1/wk)}{\text{congestion}} \right\} \]

Newell’s model again !!! (2002)
The simplest car-following rule
Account for driver reaction time

\( \vec{w} \left( \frac{1}{w \kappa}, -\frac{1}{\kappa} \right) \)

(Newell, 2002)
VT in T coordinates

\[ T(x,n) = \max \left( T(x_u,n) + \frac{(x-x_u)}{u} , T(x_d,n-k(x_d-x)) - \frac{(x_d-x)}{w} \right) \]

Mesoscopic model
(Mahut, 2000; Leclercq & Becarie, 2012)
The mesoscopic LWR model
Variational theory - summary

• Variational theory exhibits the connections between the three traffic representations for the LWR model

• A unique model that leads to three solution methods (numerical scheme) corresponding to the three different vision on traffic flow
  (macroscopic / mesoscopic / microscopic)

• Some previous models appears to be particular cases for the LWR model and a triangular fundamental diagram in different systems of coordinates
Extensions to the theory
Diverge: Newell’s FIFO model

FIFO => Travel times should be equal whatever the destination is

(Newell, 1993)
Merge: Daganzo’s model

This model has been proved consistent with experimental observations a multitude of times

(Daganzo, Transportation Research part B, 1995)
Other extensions

- Fixed and moving bottlenecks
- Bounded acceleration
- Multiclass
- Multilanes
- Lane-changing and relaxation
- Complex intersections
- Hybridation

Need to be coupled with a route choice model to simulate a network
The network fundamental diagram

Macroscopic Fundamental Diagram (MFD)

(Daganzo & Geroliminis, 2008)
Thank you for your attention

LICIT – Ifsttar/Bron and ENTPE
25, avenue François Mitterand
69675 Bron Cedex - FRANCE
leclercq@entpe.fr
References

Exercices
Problem statement

Let consider a freeway with two lanes and the following FD: $u=30$ m/s ; $w=4.28$ m/s ; $\kappa=0.28$ veh/m. Two points a et b are respectively located at $x=0$ m and $x=3600$ m.

The flow at a is constant and equal to 3000 veh/h. At time $t=120$ s, the capacity at is reduced from 1800 veh/h during 10 minutes.

• Draw the fundamental diagram
• Determine the $N$-curve at $x=3600$, $x=1800$, $x=600$ and $x=0$ m
• Provide an estimate for the maximal length of the congestion
The fundamental diagram
$N$-curve at $x=3600$ m
$N$-curve at $x=1800$ m
$N$-curve at $x=600$ m
$N$-curve at $x=0$ m